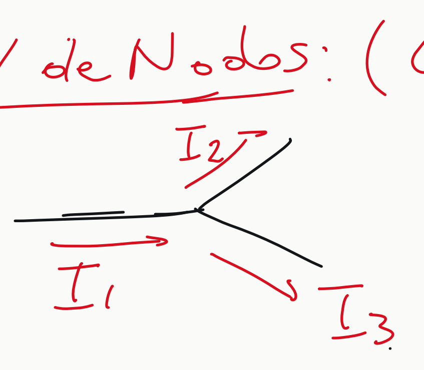
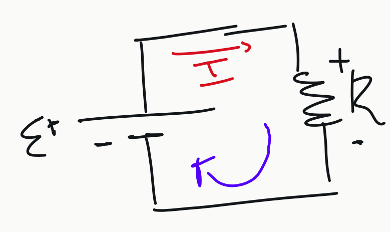
Práctico 5: Parte 2.

Ley de Nodos: (Conservación de la carga)



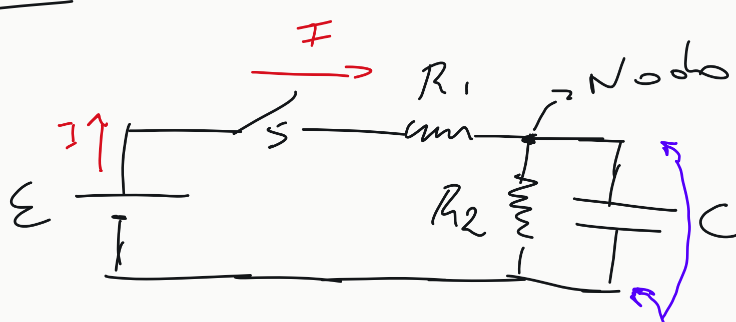
$\equiv \quad I_1 = I_2 + I_3$

Ley de Mallas



$\varepsilon - IR = 0$ (ohm)

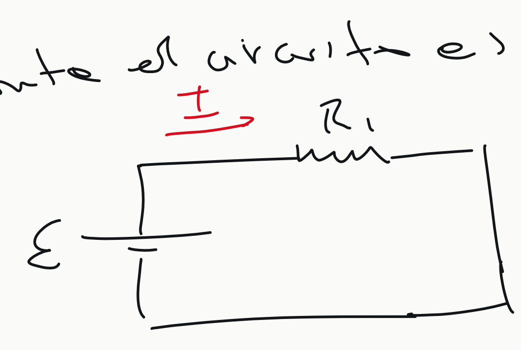
Ejercicio 6:



parte a: Queremos I en $t=0$

Cuánto vale $Q(t=0)=0 \Rightarrow \Delta V_c = C.Q = 0$

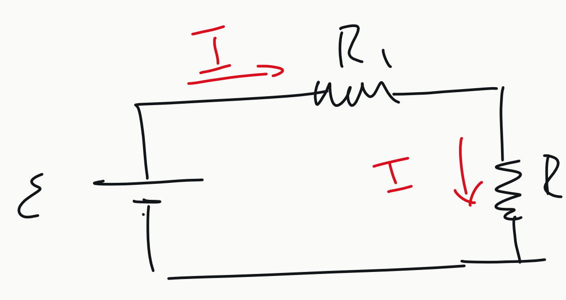
Inicialmente el circuito es eq:



$I = \frac{\varepsilon}{R_1} = 41{,}7\,\mu A$

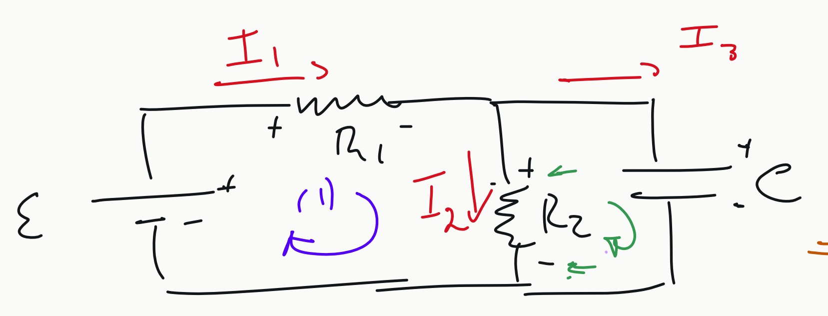
parte b: Queremos I en $t \to +\infty$

eq



$I = \frac{\varepsilon}{R_1+R_2} = 27{,}8\,\mu A$

parte c.



$\Delta V_2 = \frac{\varepsilon}{R_1+R_2} . R_2$

$\Rightarrow \boxed{Q_{max} = \frac{\varepsilon R_2 C}{R_1+R_2}}$

parte 1: Conocer la Qmáx del capacitor

(1): $\varepsilon - R_1 I_1 - R_2 I_2 = 0$

(2): $I_2 R_2 - \frac{Q}{C} = 0 \to I_2 = \frac{Q}{R_2 C}$

(3): $I_1 = I_2 + I_3$

(1); (3) $\quad \varepsilon - R_1(I_2+I_3) - R_2 I_2 = 0$

(2) $\quad \varepsilon - R_1\left(\frac{Q}{R_2C} + I_3\right) - \frac{R_2 Q}{R_2 C} = 0$

$I_3 = \frac{dQ}{dt} = \dot{Q}$

$\varepsilon - R_1\left(\frac{Q}{R_2C} + \dot{Q}\right) - \frac{Q}{C} = 0$

simplificando....

$$\boxed{\dot{Q} + \underbrace{\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}_{\alpha} Q = \frac{\varepsilon}{R_1}}$$

EC. Dif: → lineal
→ No homogenea
→ 1° orden

Solución: $Q(t) = Q_H(t) + Q_P(t)$

homogenea: $\dot{Q} + \alpha Q = 0$

Buscamos soluciones en la forma
$Q_H(t) = A e^{\lambda t}$

verificamos: $A\lambda e^{\lambda t} + \alpha A e^{\lambda t} = 0$

$\lambda + \alpha = 0 \Rightarrow \lambda = -\alpha$

$\Rightarrow Q_H(t) = A e^{-\alpha t}$

particular: $Q_P(t) = k$

$\dot{Q}_P + \alpha Q_P = \frac{\varepsilon}{R_1} \Rightarrow \alpha k = \frac{\varepsilon}{R_1}$

$\Rightarrow k = \frac{\varepsilon}{\alpha R_1}$

$Q(t) = A e^{-\alpha t} + \frac{\varepsilon}{\alpha R_1} \qquad Q(t=0) = 0$

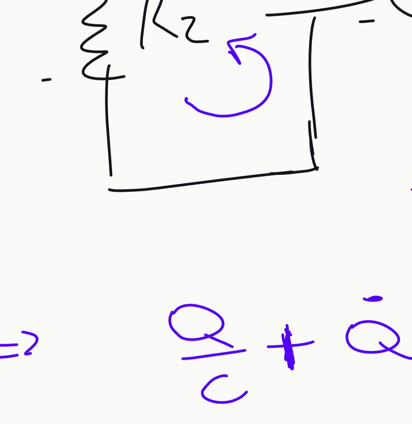
$\Rightarrow A + \frac{\varepsilon}{\alpha R_1} = 0 \Rightarrow A = -\frac{\varepsilon}{\alpha R_1}$

$$\Rightarrow \boxed{Q(t) = \frac{\varepsilon}{\alpha R_1}\left(1 - e^{-\alpha t}\right)}$$

$\Rightarrow \alpha = \frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \Rightarrow Q(t) = \frac{\varepsilon R_2 C}{R_1+R_2}\left(1 - e^{-\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t}\right)$

$$\boxed{Q_{max} = \lim_{t\to\infty} Q(t) = \frac{\varepsilon R_2 C}{R_1+R_2}}$$

parte 2: Conocer $Q(t) \Rightarrow Q(t=0) = \frac{\varepsilon R_2 C}{R_1+R_2}$



$\frac{Q(t)}{C} - I R_2 = 0$

$\Rightarrow I = -\frac{dQ}{dt} = -\dot{Q}$

$\Rightarrow \frac{Q}{C} + \dot{Q} R_2 = 0 \Rightarrow \boxed{\dot{Q} + \frac{1}{R_2 C} Q = 0}$

$\Rightarrow$ EC. Dif: Buscamos soluciones $Q(t) = Be^{\gamma t}$

$B\gamma e^{\gamma t} + \frac{1}{R_2 C} B e^{\gamma t} = 0 \Rightarrow \gamma + \frac{1}{R_2C} = 0$

$\Rightarrow \gamma = -\frac{1}{R_2 C}$

$\Rightarrow Q(t) = B e^{-\frac{t}{R_2C}}$

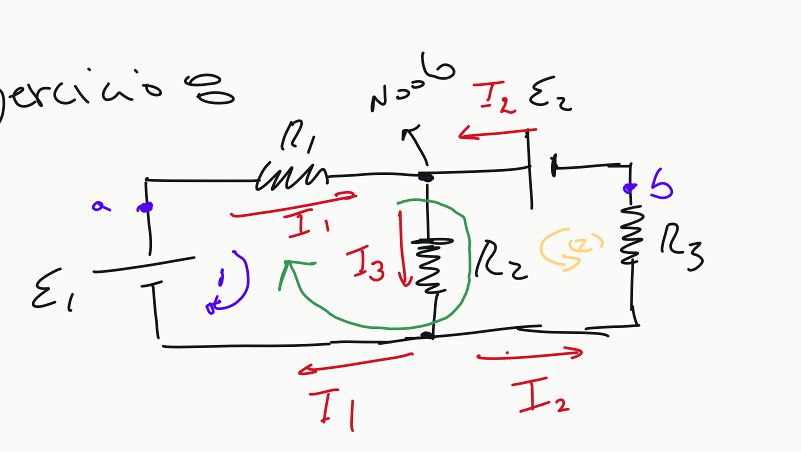
$Q(t=0) = B = Q_{max} = \frac{\varepsilon R_2 C}{R_1+R_2}$

$$\Rightarrow \boxed{Q(t) = \frac{\varepsilon R_2 C}{R_1+R_2} e^{-\frac{t}{R_2C}}}$$

$I = -\frac{dQ}{dt} = -\frac{\varepsilon R_2 C}{R_1+R_2} \cdot \frac{(-)}{R_2 C} e^{-\frac{t}{R_2C}}$

$$\boxed{I = \frac{\varepsilon}{R_1+R_2} e^{-\frac{t}{R_2C}}}$$

Ejercicio 8



$\Rightarrow I_1 + I_2 = I_3$

$\varepsilon_1 + \Delta V_{ab} + I_2 R_3 = 0$

$\Rightarrow \Delta V_{ab} = -\varepsilon_1 - I_2 R_3$

$\left.\begin{matrix}(1)\\(2)\end{matrix}\right\}$ Despejar I_2