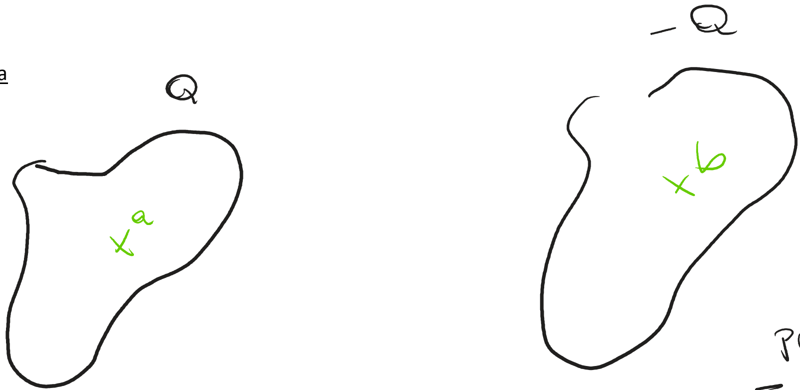
P4 $\longrightarrow$ Hallar Capacitancias de diferentes sistemas

$\longrightarrow$ Analizar circuitos con capacitores

Capacitancia

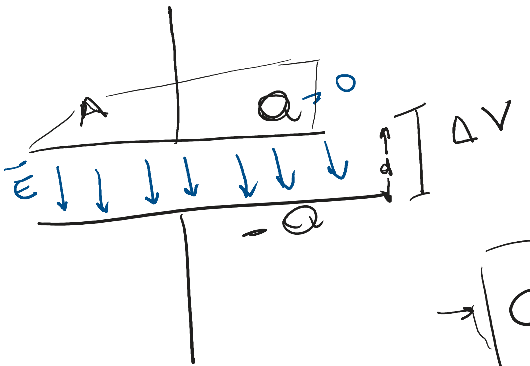


$$\boxed{C \equiv \frac{|Q|}{|{}_a\Delta V_b|}}$$

Placas paralelas

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

ej



$$\hookrightarrow \Delta V = -\int \vec{E}\, d\vec{l} = -E d = -\frac{Q d}{\epsilon_0 A}$$

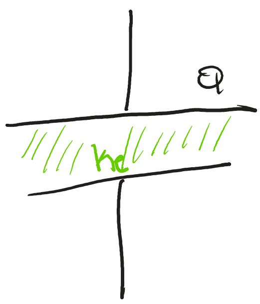
$\uparrow$ E cte

$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{\epsilon_0 A}{d}}$$

$[C] = [\epsilon_0] \cdot [L]$

Solo depende de parametros geometricos

Capacitancia con dielectricos



$$k_e = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0}$$

con material

vacio

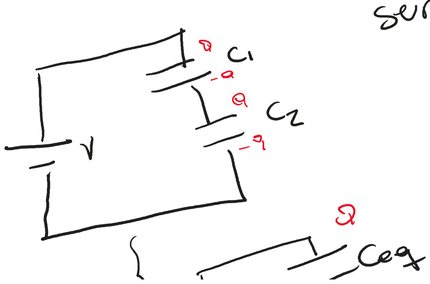
$$E = \frac{\sigma}{\epsilon} = \frac{\sigma}{\epsilon_0 k_e} = \frac{Q}{\epsilon_0 k_e A}$$

$$\rightarrow |\Delta V| = E d \Rightarrow C = \frac{Q}{|\Delta V|} = k_e \cdot \underbrace{\frac{\epsilon_0 A}{d}}_{C_0}$$

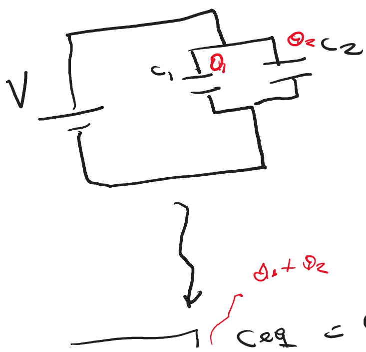
$$\Rightarrow C = k_e C_0$$

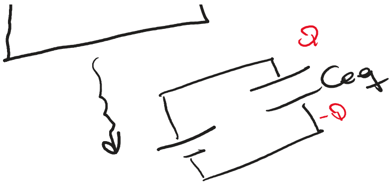
Condensadores en circuitos

serie

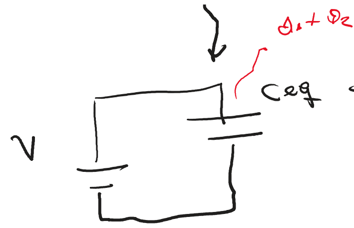


paralelo





$$1/C_{eq} = 1/C_1 + 1/C_2$$

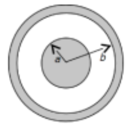


$$C_{eq} = C_1 + C_2$$

P4Ej5

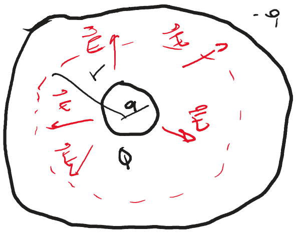
martes, 1 de abril de 2025 9:39

Ejercicio 5 Un condensador cilíndrico tiene radios a y b como en la figura.



a. Calcule su capacidad.

b. Demuestre que la mitad de la energía potencial eléctrica almacenada se encuentra dentro de un cilindro cuyo radio es $r = \sqrt{ab}$.



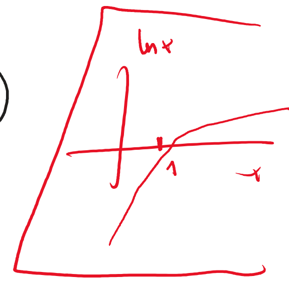
$$\phi_E = ES = E \cdot (2\pi r L)$$

$$\phi_E = \frac{\cancel{Q_{enc}}\ Q}{\epsilon_0}$$

$$\boxed{E = \frac{Q}{2\pi\epsilon_0 r L}}$$

$$\Rightarrow {}_a\Delta V_b = -\int_a^b E\,dr = -\frac{Q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r} = -\frac{Q}{2\pi\epsilon_0 L}\left(\ln b - \ln a\right)$$

$$= -\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

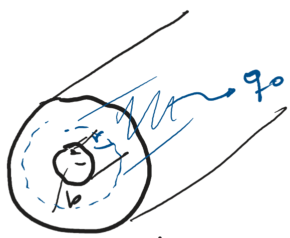


$$|{}_a\Delta V_b| = \frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{|{}_a\Delta V_b|} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

$$[\epsilon_0]\cdot[L] = [C]\ \checkmark$$

b -



$$U(r) = q_0\ {}_a\Delta V_r$$

Toda la carga q_0 se encuentra bajo el mismo potencial

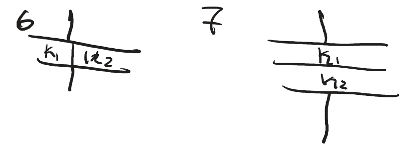
$$\frac{U(r)}{U(b)} = \frac{1}{2}\ ?$$

$$\frac{U(r)}{U(b)} = \frac{\cancel{q_0}\ {}_a\Delta V_r}{\cancel{q_0}\ {}_a\Delta V_b} = \frac{\ln(r/a)}{\ln(b/a)}$$

$$N \cdot \ln(x) = \ln(x^N)$$

$$\frac{\ln(r/a)}{\ln(b/a)} = \frac{1}{2} \rightarrow \ln(r/a) = \frac{1}{2}\ln(b/a) = \ln\left(\sqrt{b/a}\right)$$

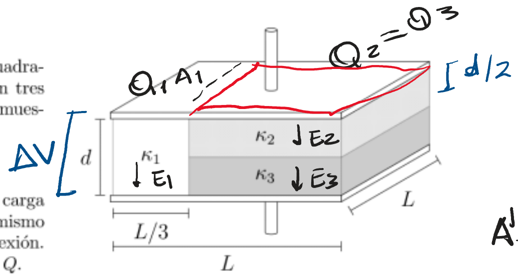
$$\Rightarrow r/a = \sqrt{b/a} \Rightarrow \boxed{r = \sqrt{ab}}$$



Ejercicio 8 Un capacitor de placas paralelas cuadradas de lado L separadas una distancia d se llena con tres dieléctricos distintos con constantes κ_1, κ_2, κ_3, como muestra la figura.

a) Calcule la capacitancia de dicho capacitor.

b) Calcule el valor de la densidad superficial de carga en el capacitor y la energía almacenada en el mismo después de un largo tiempo de realizada la conexión. Suponga que el capacitor tiene una carga total Q.



$$A_1 = L \cdot (L/3)$$

$$A' = A_2 = A_3 = L \cdot (2L/3)$$

$$\rightarrow E_1 = \frac{\sigma_1}{\epsilon_0 \kappa_1} = \frac{Q_1}{\epsilon_0 \kappa_1 A_1} \qquad E_2 = \frac{Q_2}{\epsilon_0 \kappa_2 A_2} \qquad E_3 = \frac{Q_3}{\epsilon_0 \kappa_3 A_3}$$

$$|\Delta V| = \left|\int_0^d E_1\, dl\right| = E_1 \cdot d = \frac{Q_1}{\epsilon_0 \kappa_1 A_1} d$$

$$Q_2 = Q_3 = Q'$$

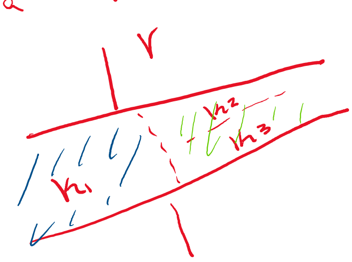
$$\Rightarrow \boxed{Q = Q_1 + Q'}$$

$$|\Delta V| = \int_0^{d/2} E_2\, dl + \int_{d/2}^{d} E_3\, dl = \frac{Q'}{\epsilon_0 \kappa_2 A'}\frac{d}{2} + \frac{Q'}{\epsilon_0 \kappa_3 A'}\frac{d}{2} = \frac{Q'}{\epsilon_0 A'}\frac{d}{2}\left(\frac{1}{\kappa_2} + \frac{1}{\kappa_3}\right)$$

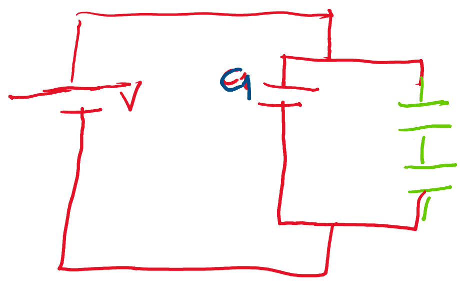
$$C = \frac{Q}{|\Delta V|} = \frac{Q_1 + Q'}{|\Delta V|} = \frac{Q_1}{|\Delta V|} + \frac{Q'}{|\Delta V|} = \frac{\epsilon_0 A_1}{d}\kappa_1 + \frac{2\epsilon_0 A'}{d}\frac{1}{1/\kappa_2 + 1/\kappa_3}$$

$$= \frac{\epsilon_0 L^2}{d}\left(\frac{\kappa_1}{3} + \frac{4}{3}\frac{1}{1/\kappa_2 + 1/\kappa_3}\right)$$

Analogía entre circuitos y dieléctricos



$\approx$

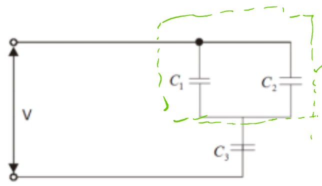


$$C' = \frac{1}{1/C_2 + 1/C_3}$$

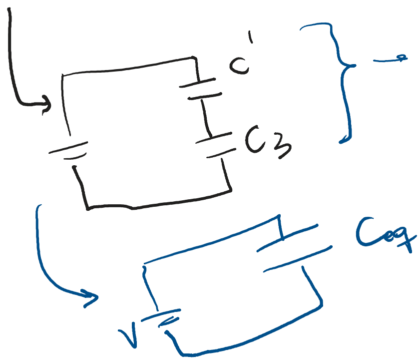
$$C_{eq} = C_1 + C' = C_1 + \frac{1}{1/C_2 + 1/C_3}$$

$$\text{"}C = \{\ \} \left(A\kappa_1 + \frac{B}{1/\kappa_2 + 1/\kappa_3}\right)$$

Ejercicio 2 Halle la capacidad del condensador equivalente de los tres condensadores de la figura. Considere que $C_1 = 10,3\ \mu F$, $C_2 = 4,80\ \mu F$ y $C_3 = 3,90\ \mu F$ y que $V = 115\ V$. Supóngase que el condensador C_3 se perfora eléctricamente, resultando equivalente a una trayectoria conductora. ¿Qué cambios ocurren en la carga y en la diferencia de potencial en el condensador C_1?



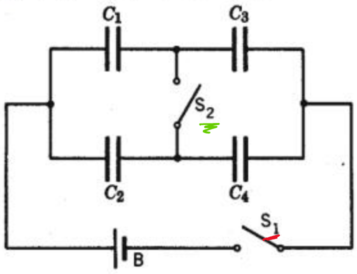
$$c' = C_1 + C_2$$



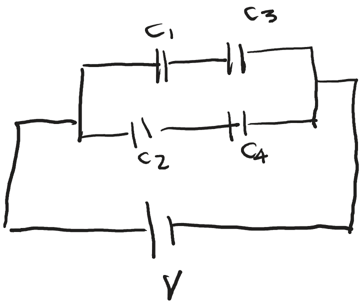
$$C_{eq} = \frac{1}{1/C' + 1/C_3}$$

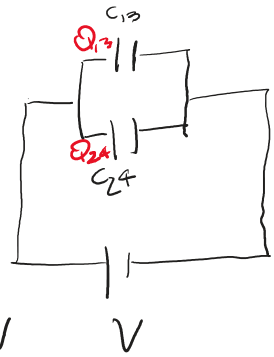
$$= \frac{1}{\frac{1}{C_1 + C_2} + 1/C_3}$$

Ejercicio 10 La batería de la figura suministra 12 V.
a. Halle la carga sobre cada condensador cuando el interruptor ~~S1~~ se cierra.
b. Halle la carga cuando (más tarde) el interruptor S2 también se cierra.
Considere $C_1 = 1,0\ \mu F$, $C_2 = 2,0\ \mu F$, $C_3 = 3,0\ \mu F$ y $C_4 = 4,0\ \mu F$.

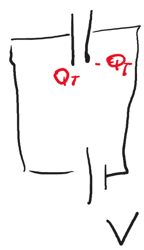


a)





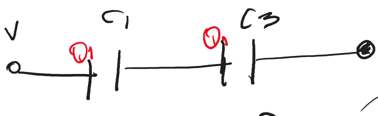
$C_{eq} = C_{13} + C_{24}$



$Q = CV$

$$\boxed{C = \frac{Q}{V}}$$

$$Q_T = Q_{13} + Q_{24}$$
$$C_{eq} \cdot V \qquad C_{13} \cdot V \qquad C_{24} V$$

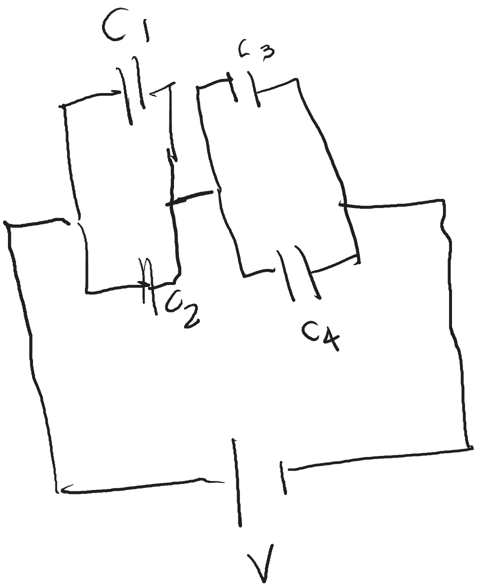


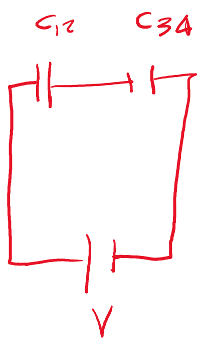
$Q_1 = Q_3 = Q_{13}$

$$V = \frac{Q_1}{C_1} + \frac{Q_3}{C_3} = \frac{Q_{13}}{C_1} + \frac{Q_{13}}{C_3} = Q_{13}\left(1/C_1 + 1/C_3\right) \rightarrow Q_{13} = \frac{V}{1/C_1 + 1/C_3} = C_{13}V$$

$$Q_{24} = C_{24}V = \frac{V}{1/C_2 + 1/C_4}$$

b)





Puesta en común la que viene...

Final

9:40