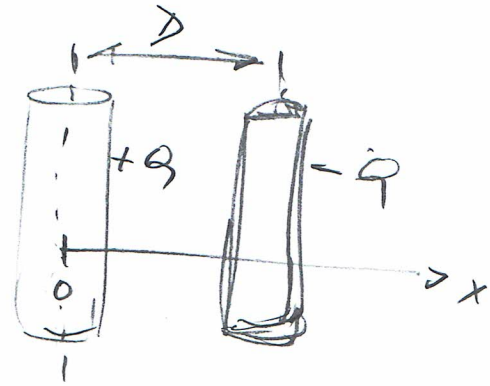
Física 2. Examen 14/08/2020

Problema 1



a)

Interior: $2\pi r L E = \frac{q_{encerrada}}{\varepsilon_0}$

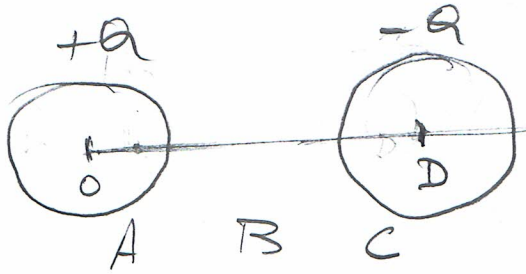
$$\frac{q_{encerrada}}{\text{Volumen encerrado}} = \frac{Q}{\text{Vol. total}} \Rightarrow \frac{q_{enc}}{\pi r^2 L} = \frac{Q}{\pi R^2 L}$$

$$q_{enc} = Q\frac{r^2}{R^2} \Rightarrow 2\pi r L E = \frac{Q r^2}{\varepsilon_0 R^2} \Rightarrow$$

$$\Rightarrow \boxed{E = \frac{Q \cdot r}{2\pi\varepsilon_0 R^2 L} \quad (0 \le r \le R)}$$

Exterior: $2\pi r L E = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{Q}{2\pi\varepsilon_0 L r} \left(R \le r < \infty\right)}$

b)



Zona A: $0 \le x \le R$

Zona B: $R \le x \le D-R$

Zona C: $D-R \le x \le D$

Zona A: $\vec{E}_- \rightarrow \leftarrow \vec{E}_+$ $E_A = E_+ + E_- = \frac{Q x}{2\pi\varepsilon_0 R^2 L} + \frac{Q}{2\pi\varepsilon_0 L (D-x)}$

$$\boxed{E_A = \frac{Q}{2\pi\varepsilon_0 L}\left(\frac{x}{R^2} + \frac{1}{D-x}\right)}$$

Zona B: $\vec{E}_+ \rightarrow \leftarrow \vec{E}_-$ $E_B = E_+ + E_- = \frac{Q}{2\pi\varepsilon_0 L x} + \frac{Q}{2\varepsilon_0 L (D-x)}$

$$\boxed{E_B = \frac{Q}{2\pi\varepsilon_0 L}\left(\frac{1}{x} + \frac{1}{D-x}\right)}$$

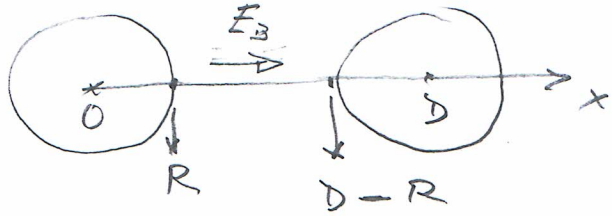
(continuación Problema 1)

Zona C $\rightarrow \vec{E}_+$ $\rightarrow \vec{E}_-$

$$\vec{E}_c = \vec{E}_+ + \vec{E}_- = \frac{Q}{2\pi\varepsilon_0 L x} + \frac{Q(D-x)}{2\pi\varepsilon_0 L R^2}$$

$$\boxed{\vec{E}_c = \frac{Q}{2\pi\varepsilon_0 L}\left(\frac{1}{x} + \frac{D-x}{R^2}\right)}$$

c)



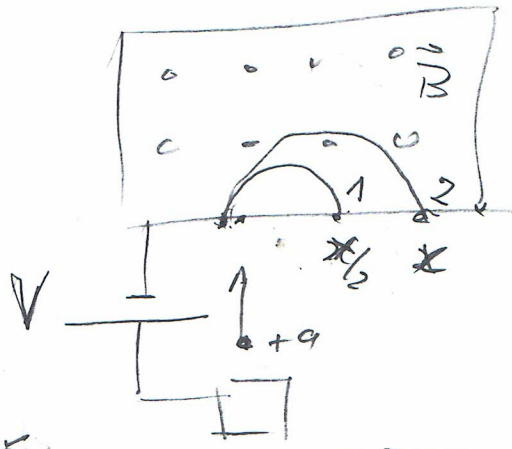
$$V = \int_R^{D-R} E_B \, dx$$

$$E_B = \frac{Q}{2\pi\varepsilon_0 L}\left(\frac{1}{x} + \frac{1}{D-x}\right) \Rightarrow V = \frac{Q}{2\pi\varepsilon_0 L}\int_R^{D-R}\left(\frac{1}{x} + \frac{1}{D-x}\right) dx$$

$$V = \frac{Q}{2\pi\varepsilon_0 L}\Big[Ln(x) - Ln(D-x)\Big]_R^{D-R}$$

$$\boxed{V = \frac{Q}{\pi\varepsilon_0 L} Ln\left(\frac{D-R}{R}\right)}$$

Problema 2



a) $\frac{mv^2}{2} = V_a q \Rightarrow$

$$\boxed{V_a = \frac{mv^2}{2q}}$$

b) $qv_b B = \frac{mv_b^2}{r} = \frac{mv_b}{(x/2)} \Rightarrow v_b = \frac{qBx}{2m}$ → cámara 2

$$\Rightarrow V_b = \frac{mv_b^2}{2q} = \frac{m}{2q}\left(\frac{qBx}{2m}\right)^2 = \boxed{\frac{qB^2x^2}{8m} = V_b}$$

c) Cámara 1: $\frac{m_c v_c^2}{x/4} = q v_c B \Rightarrow v_c = \frac{qBx}{4m_c}$, $V_b q = \frac{m_c v_c^2}{2} \Rightarrow$

$$\Rightarrow V_b q = \frac{m_c}{2}\left(\frac{qBx}{4m_b}\right)^2 = \frac{m_c}{32 m_b^2} q^2 B^2 x^2 \Rightarrow$$

$$\boxed{\frac{32 V_b}{qB^2x^2} = \frac{m_c}{m_b^2}} \Rightarrow \text{sustituyendo } V_b \text{ de b)}$$

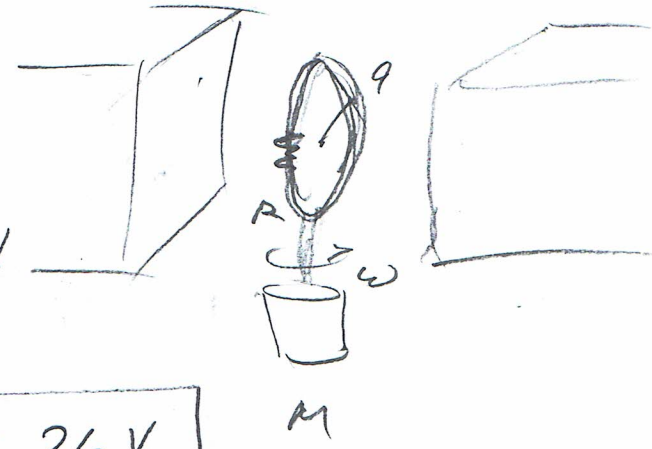
$$\boxed{4 = \frac{m_c}{m_b}}$$

Problema 3

a) $\phi_B(t) = B \cdot S_x \cos \omega t$

$\phi_B(t) = \pi a^2 B \cos \omega t$

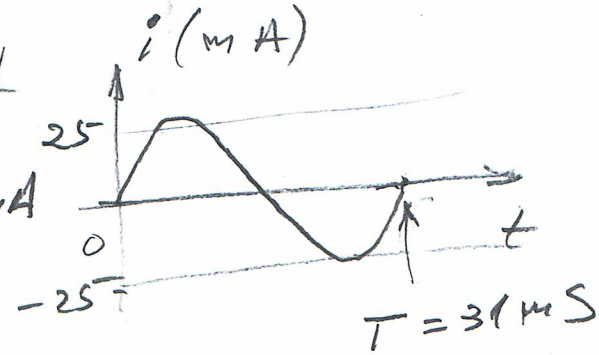
$\mathcal{E} = -\frac{d\phi_B}{dt} = \underbrace{\pi a^2 \omega B \operatorname{sen} \omega t}_{\mathcal{E}}$



$$\boxed{\mathcal{E}_{máx} = \mathcal{E}_0 = \pi a^2 \omega B \cong 1{,}26\ V}$$

b) $i(t) = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_0}{R} \operatorname{sen} \omega t$

$i(t) = i_0 \operatorname{sen} \omega t \quad i_0 = 25\ mA$

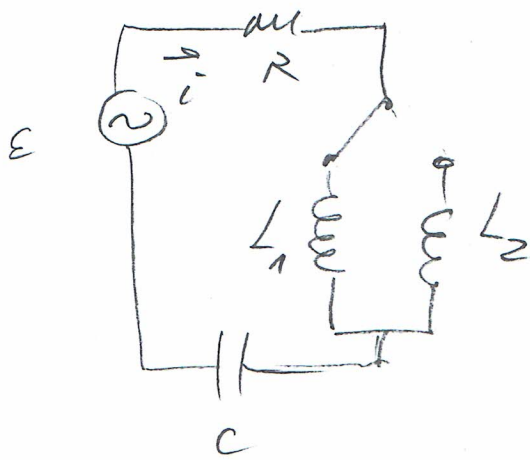


$T = \frac{2\pi}{\omega} = 31\ ms$

c) $P = i_{rms}^2 R \qquad i_{rms} = \frac{i_0}{\sqrt{2}} \Rightarrow P = \frac{i_0^2 R}{2}$

$$\boxed{P = 15{,}6\ mW}$$

PROBLEMA 4

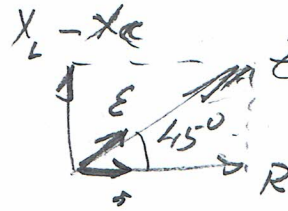


a) Con L_1 $\phi = 0 \Rightarrow$ resonancia

$$\Rightarrow \omega = \frac{1}{\sqrt{L_1 C}} = 1000 \frac{rad}{s}$$

$$\boxed{f \simeq 159\ Hz}$$

b) Con L_2:



$$\Rightarrow X_L - X_C = R \Rightarrow$$

$$X_L = X_C + R = \frac{1}{\omega C} + R = 300\ \Omega = \omega L_2$$

$$\Rightarrow L_2 = \frac{300}{\omega} = \boxed{0{,}30\ H = L_2}$$

c) $\mathcal{E}_{rms} = 220\ V$ $\quad P = \mathcal{E}_{rms}\, i_{rms} \cos\varphi = \frac{\mathcal{E}_{rms}^2}{Z} \cos\varphi$

$\varphi = 45°$ $\quad Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(300-200)^2 + 100^2} = 141.4$

$$\Rightarrow P = \frac{220^2}{141.4} \cos 45° \quad \boxed{P = 242\ W}$$