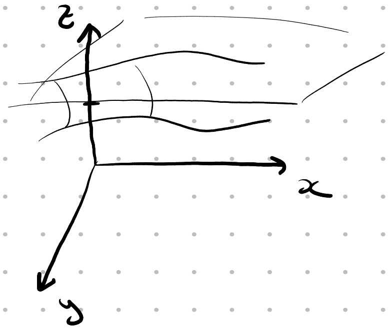
FUNCIONES DE VARIAS VARIABLES

$f: U \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x,y) \longmapsto f(x,y)$



$k \in \mathbb{R}$, el conjunto de nivel k

$C_k = \{(x,y) : f(x,y) = k\}$

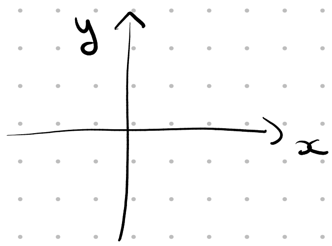
= la intersección del gráfico con el plano $z = k$

1. Dibuje el dominio, los conjuntos de nivel y la gráfica de las siguientes funciones:

(*a*) x^2+y^2 (*b*) x^2-y^2 (*c*) x^2 (*d*) y/x (*e*) xy (*f*) $\max\{x^2,y^3\}$ (*g*) $\max\{x^2,x+y\}$

a) $f(x,y) = \underbrace{x^2+y^2}_{\geq 0}$

DOMINIO: $Dom(f) = \mathbb{R}^2 = \{(x,y) : x,y \in \mathbb{R}\}$



CONJUNTOS DE NIVEL

* $k = -1$ $C_{-1} = \emptyset$

$f(x,y) = -1 \iff x^2+y^2 = -1$

* $k = 0$ $C_0 = \{(0,0)\}$

$f(x,y) = 0 \iff x^2+y^2 = 0 \iff (x,y) = (0,0)$

* $k = 1$ $C_1 = \{(x,y) \in \mathbb{R}^2 : \|(x,y)\| = 1\}$

$f(x,y) = 1 \Leftrightarrow \underbrace{x^2+y^2=1}_{\substack{\text{circunferencia} \\ \text{de centro } (0,0) \\ \text{y radio } 1}} \Leftrightarrow \|(x,y)\|^2 = 1$

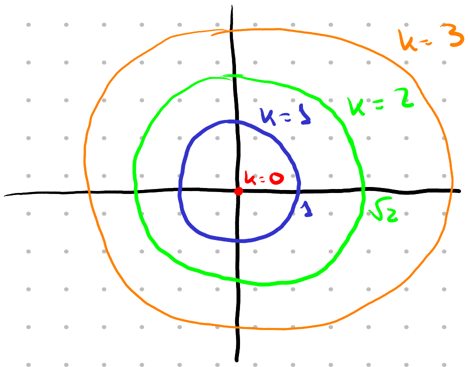
* $k=2$ $\quad C_2 = \{(x,y) \in \mathbb{R}^2 : \|(x,y)\| = \sqrt{2}\}$

$f(x,y) = 2 \Leftrightarrow \underbrace{x^2+y^2=2}_{\substack{\text{circunferencia} \\ \text{de centro } (0,0) \\ \text{y radio } \sqrt{2}}} \Leftrightarrow \|(x,y)\|^2 = 2 \Leftrightarrow \|(x,y)\| = \sqrt{2}$

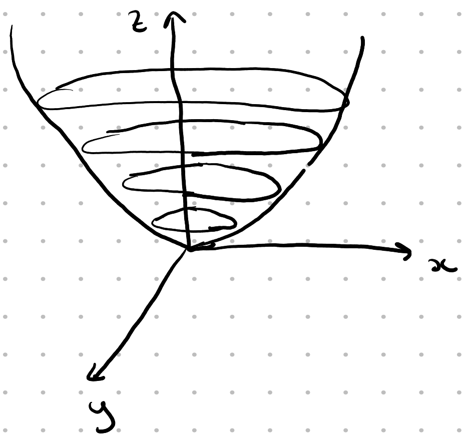
* $k>0$ $\quad C_k = \{(x,y) \in \mathbb{R}^2 : \|(x,y)\| = \sqrt{k}\}$

$f(x,y) = k \Leftrightarrow x^2+y^2 = k \Leftrightarrow \|(x,y)\| = \sqrt{k}$

* $k<0$, $\quad C_k = \emptyset$



GRÁFICO



corte adicional : $y=0$

$f(x,0) = x^2 \leftarrow$ parábola

b) $f(x,y) = x^2 - y^2$

DOMINIO: $\text{Dom}(f) = \mathbb{R}^2$

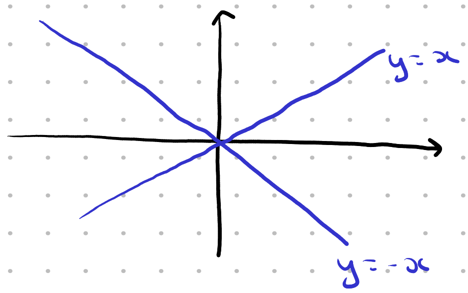
CONJUNTOS DE NIVEL

$k \in \mathbb{R} \rightsquigarrow f(x,y) = k$

* $k = 0$

$$f(x,y) = 0 \iff x^2 - y^2 = 0 \iff x^2 = y^2$$
$$\iff y = \pm x$$

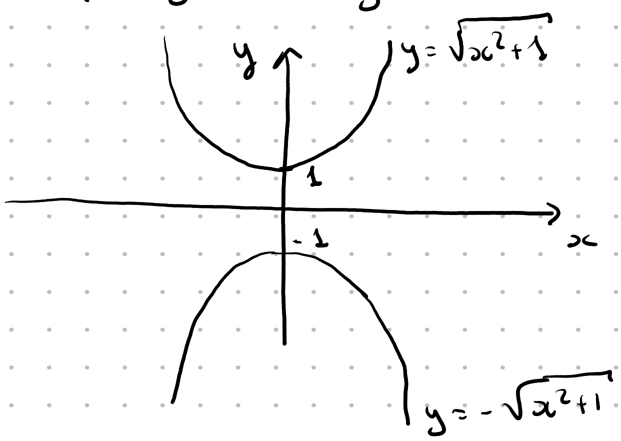
$$C_0 = \{(x,y) \in \mathbb{R}^2 : y = \pm x\}$$



* $k = -1$

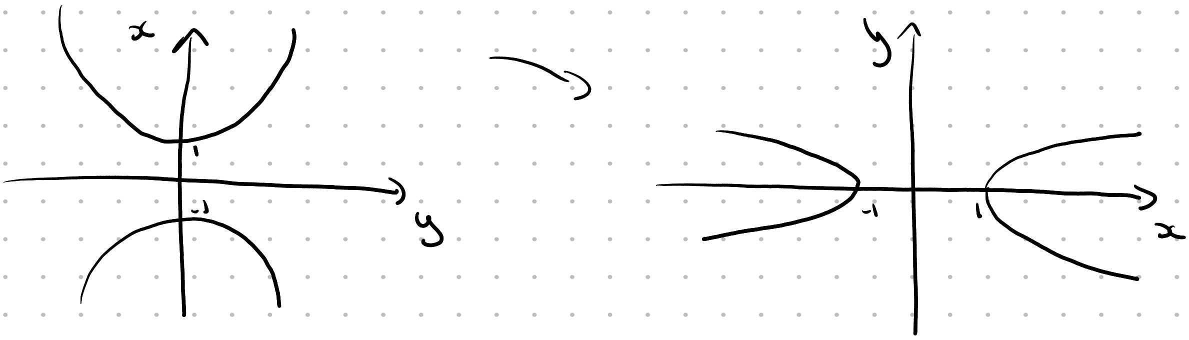
$$f(x,y) = -1 \iff x^2 - y^2 = -1$$
$$\iff y^2 = x^2 + 1$$
$$\iff y = \pm\sqrt{x^2+1}$$

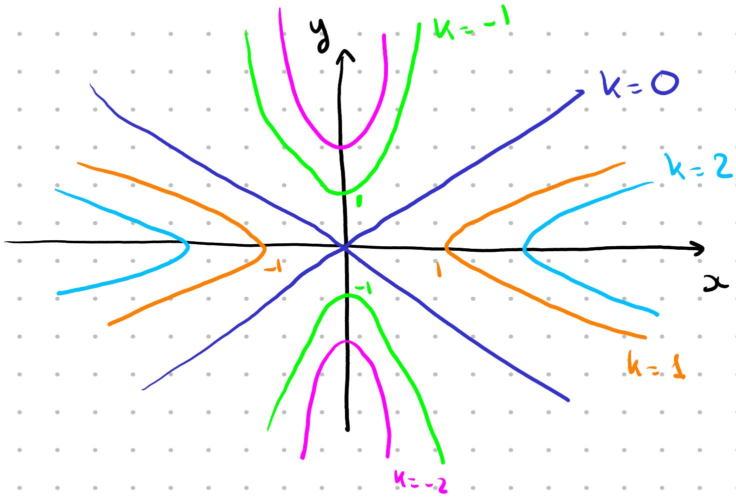
$$C_{-1} = \{(x,y) \in \mathbb{R}^2 : y = \pm\sqrt{x^2+1}\}$$



* $k = 1$

$$f(x,y) = 1 \iff x^2 - y^2 = 1$$
$$\iff x^2 = y^2 + 1$$
$$\iff x = \pm\sqrt{y^2+1}$$





* $k = -2$

$$f(x,y) = -2 \iff x^2 - y^2 = -2$$
$$\iff y^2 = x^2 + 2$$
$$\iff y = \pm\sqrt{x^2+2}$$

* $k < 0$

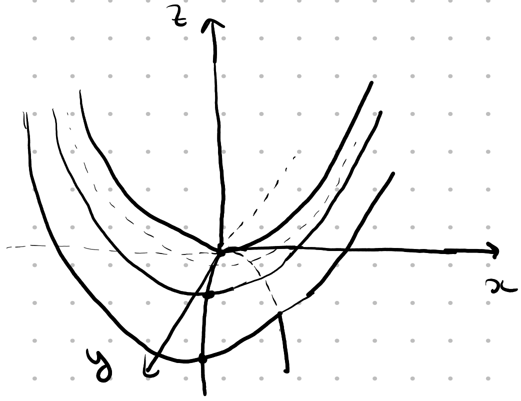
$$C_k = \left\{ (x,y) \in \mathbb{R}^2 : y = \pm\sqrt{x^2 - k} \right\}$$

* $k > 0$

$$f(x,y) = k \iff x^2 - y^2 = k$$
$$\iff x^2 = y^2 + k$$
$$\iff x = \pm\sqrt{y^2+k}$$

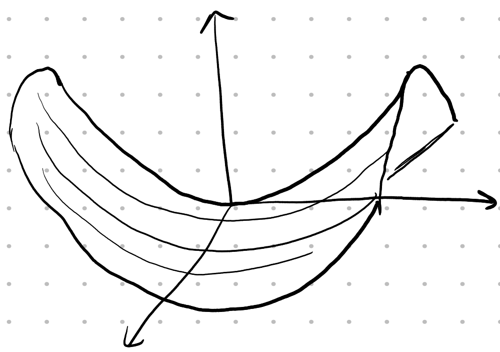
$$C_k = \left\{ (x,y) \in \mathbb{R}^2 : x = \pm\sqrt{y^2+k} \right\}$$

GRÁFICO



$f(x,y) = x^2 - y^2$

- corte con $y=0 \rightarrow f(x,0) = x^2$
- corte con $x=0 \rightarrow f(0,y) = -y^2$
- corte con $y=1 \rightarrow f(x,1) = x^2 - 1$
- corte con $y=2 \rightarrow f(x,2) = x^2 - 4$
- corte con $y=-1 \rightarrow f(x,-1) = x^2 - 1$



c) $f(x,y) = x^2$

DOMINIO: $Dom(f) = \mathbb{R}^2$

CONJUNTOS DE NIVEL

- $k<0$

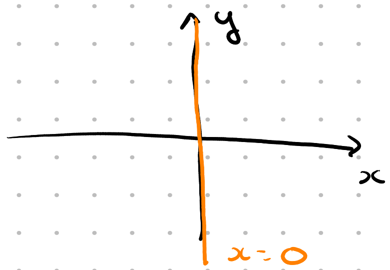
$f(x,y) = k \iff \underbrace{x^2}_{\text{positivo}} = \underbrace{k}_{\text{negativo}}$

$C_k = \emptyset$

- $k=0$

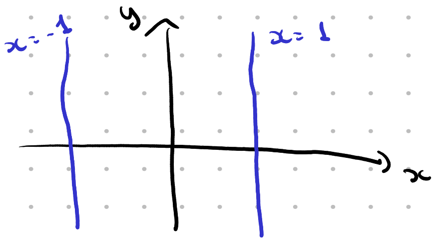
$f(x,y)=0 \iff x^2=0 \iff x=0$

$C_0=\{(x,y)\in\mathbb{R}^2 : x=0\}$



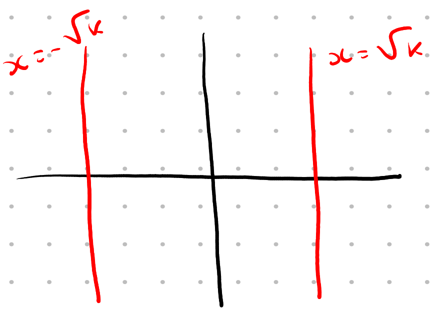
- $k=1$

$f(x,y)=1 \iff x^2=1 \iff x=\pm 1$



- $k>0$

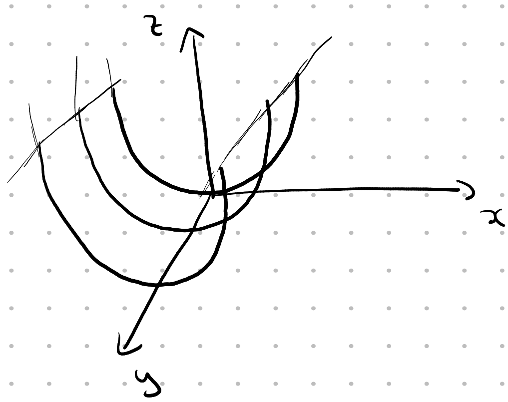
$f(x,y)=k \iff x^2=k \iff x=\pm\sqrt{k}$



$C_k=\{(x,y)\in\mathbb{R}^2 : x=\sqrt{k}\}\cup\{(x,y)\in\mathbb{R}^2 : x=-\sqrt{k}\}$

GRÁFICO

$f(x,y)=x^2$

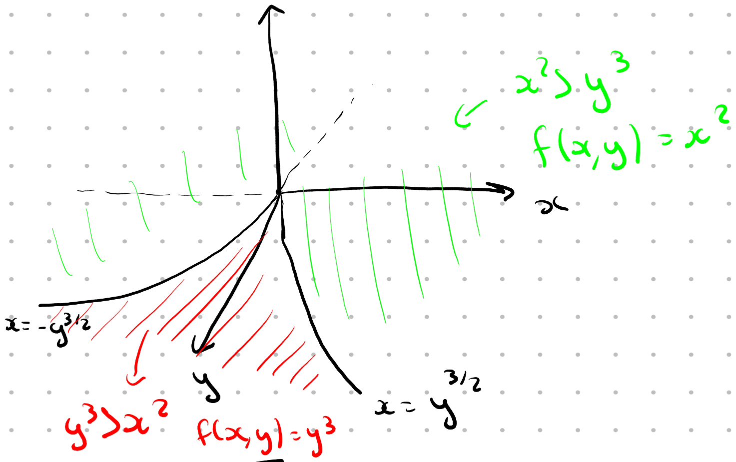


- corte con $y=0$: $f(x,0) = x^2$
- corte con $y=1$: $f(x,1) = x^2$

f) $f(x,y) = \max\{x^2, y^3\}$

<u>Dominio</u>: $Dom(f) = \mathbb{R}^2$

$$f(x,y) = \begin{cases} x^2 & \text{si } x^2 \geq y^3 \\ y^3 & \text{si } y^3 > x^2 \end{cases}$$



$x^2 = y^3 \Rightarrow x = \pm\sqrt{y^3} = \pm y^{3/2}$