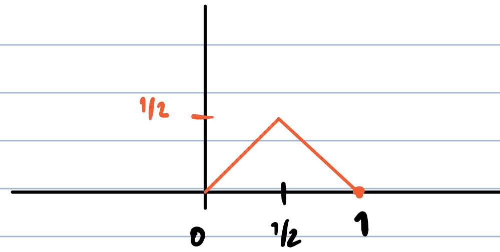
6) b) $$f(x) = \begin{cases} x & si \quad x \in [0, 1/2] \\ 1-x & si \quad x \in [1/2, 1] \end{cases}$$



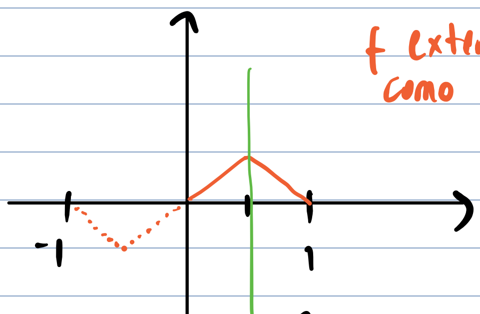
Ej pide probar que

$$\left[\frac{4}{\pi^2} \sum_{0}^{+\infty} \frac{(-1)^{k+1}}{(2k+1)^2} \cdot sen((2k+1)\pi x) = f(x), \quad x \in [0,1]. \right]$$

Series de Fourier 2L-periódicas: $V = \{ f : \mathbb{R} \to \mathbb{R} \text{ cont. a trozos } 2L\text{-per} \}$

$$\langle f, g \rangle = \frac{1}{L} \int_{-L}^{L} f(x) g(x)\, dx, \quad \mathcal{B} = \left\{ \frac{1}{\sqrt{2}}, \cos\left(x \cdot \frac{\pi}{L}\right), sen\left(x \cdot \frac{\pi}{L}\right), \cos\left(x \cdot \frac{2\pi}{L}\right), sen\left(x \frac{2\pi}{L}\right), \dots \cos\left(x \cdot \frac{n\pi}{L}\right), \dots \right\}$$

Para este ejercicio necesitamos calcular serie de Fourier para funciones 2-periódicas



f extendida como función impar.

Para la f extendida:

$$a_0 = 1 \cdot \int_{-1}^{1} f(x)\, dx = 0$$

$$a_n = 1 \cdot \int_{-1}^{1} f(x) \cdot \cos(n\pi x)\, dx = 0$$

$$b_n = 2 \int_0^1 f(x) \cdot sen(n\pi x) = 2 \left[\int_0^{1/2} x\, sen(n\pi x)\, dx + \int_{1/2}^{1} (1-x) \cdot sen(n\pi x)\, dx \right]$$

$$= 2 \left[\int_0^{1/2} x\, sen(n\pi x)\, dx - \int_{1/2}^{1} x\, sen(n\pi x)\, dx + \int_{1/2}^{1} sen(n\pi x)\, dx \right]$$

$$= 2 \left[\left(-\frac{1}{2} \cdot \frac{\cos(n\pi/2)}{n\pi} + \frac{sen\left(\frac{n\pi}{2}\right)}{(n\pi)^2} - 0 \right) - \left(-1 \cdot \frac{\cos(n\pi)}{n\pi} + \frac{sen(n\pi)}{(n\pi)^2} + \frac{1}{2} \cdot \frac{\cos\left(\frac{n\pi}{2}\right)}{n\pi} - \frac{sen(n\pi/2)}{(n\pi)^2} \right) - \frac{\cos(n\pi x)}{n\pi} \Bigg|_{1/2}^{1} \right]$$

$$= 2 \left[\cancel{- \frac{\cos(n\pi/2)}{n\pi}} + \frac{2\, sen\left(\frac{n\pi}{2}\right)}{(n\pi)^2} + \cancel{\frac{\cos(n\pi)}{n\pi}} - \cancel{\frac{sen(n\pi)}{(n\pi)^2}} - \cancel{\frac{\cos(n\pi)}{n\pi}} + \cancel{\frac{\cos(n\pi/2)}{n\pi}} \right]$$

$$\left(\int x\, \text{sen}(n\pi x) = -\frac{x\cos(n\pi x)}{n\pi}\Big| + \int \frac{\cos(n\pi x)}{n\pi} = -\frac{x\cos(n\pi x)}{n\pi} + \frac{\text{sen}(n\pi x)}{(n\pi)^2}\right)$$

$$= 4\left[\frac{\text{sen}(n\pi/2)}{(n\pi)^2}\right] = \begin{cases} 0 & \text{si } n = 2k \\ \dfrac{4\,\text{sen}(k\pi + \frac{\pi}{2})}{(2k+1)^2\pi^2} & \text{si } n = 2k+1 \end{cases}$$

$$\text{sen}(k\pi + \pi/2) = (-1)^k$$

Concluyendo que $b_{2k} = 0$, $b_{2k+1} = \frac{4}{\pi^2}(-1)^{k+1}$.

$$\Rightarrow \left[f(x) = \frac{4}{\pi^2}\sum_{1}^{+\infty} \frac{(-1)^{k+1}}{(2k+1)^2}\cdot \text{sen}((2k+1)\pi x) \quad , x \in [0,1]. \right]$$