2) $u_{tt} - c^2 u_{xx} = 0 \qquad (x,t) \in [0,\pi] \times [0,+\infty)$

- Extremos fijos : $\left[\begin{array}{l} u(0,t) = 0 \\ u(\pi,T) = 0 \end{array}\right.$

- Posición inicial : $u(x,0) = u_0(x)$
- Velocidad inicial : $u_t(x,0) = v_0(x)$

Por método de var sep : $u(x,t) = X(x).T(t) = sen(nx)\left(a\cos(nct) + b\,sen(nct)\right)$

$$\boxed{\begin{array}{l} \cdot\ u_0(x) = a\,sen(nx) \\ \cdot\ v_0(x) = bnc.sen(nx) \end{array}}$$

Qué pasa si quiero resolver (*)
$$\left|\begin{array}{l} \cdot\ u_{tt} - c^2 u_{xx} = 0 \\ \cdot\ u(0,t) = 0 = u(\pi,t) \\ \cdot\ u_0 = 2\,sen(3x) \\ \cdot\ v_0 = sen(2x) + sen(4x) \end{array}\right.$$

$$u_0(x) = 0.sen(x) + 0.sen(2x) + 2.sen(3x) + 0.sen(4x)$$
$$v_0(x) = 0.sen(x) + 1.sen(2x) + 0.sen(3x) + 1.sen(4x)$$

$n=2$	$n=3$	$n=4$
$a=0$	$a=2$	$a=0$
$b=1/2c$	$b=0$	$b=1/4c$

La solución a (*) es la suma de 3 soluc :

notación (no estoy elev. al cuadrado)

$$u^2(x,t) = sen(2x)\left(\frac{1}{2c}\cdot sen(2ct)\right)$$
$$u^3(x,t) = sen(3x)\left(2.\cos(3ct)\right).$$
$$u^4(x,t) = sen(4x)\left(\frac{1}{4c}sen(4ct)\right)$$

$$u(x,t) = u^2(x,t) + u^3(x,t) + u^4(x,t)$$

b) $g(x) = x.(x-\pi), \quad g:[0,\pi] \to \mathbb{R}.$

Observar que g no puede ser suma finita de funciones de la forma $b_n.sen(nx)$ (pues $g'''(x) \equiv 0$).

c)

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(0,t) = 0 = u(L,t) \\ u_0(x) = x(x-\pi) = \sum_1^{\infty} \underbrace{\left[\frac{4((-1)^k - 1)}{\pi k^3}\right]}_{a_n} \cdot \operatorname{sen}(kx) = \sum_1^{+\infty} a_k \cdot \operatorname{sen}(kx) \\ v_0(x) = 0 \end{cases}$$

$$u_0 = u_0^1 + u_0^2 + u_0^3 + \dots = \overbrace{a_1 \operatorname{sen}(x)}^{u_0^1} + \overbrace{a_2 \operatorname{sen}(2x)}^{u_0^2} + \dots$$

$$v_0 = v_0^1 + v_0^2 + v_0^3 + \dots = \underbrace{0 \operatorname{sen}(x)}_{\substack{n=1 \\ a=a_1 \\ b=0}} + \underbrace{0 \operatorname{sen}(2x)}_{\substack{n=2 \\ a=a_2 \\ b=0}} + \dots$$

La solve u

$$u(x,t) = \operatorname{sen}(x) \cdot \left(a_1 \cos(ct)\right) + \operatorname{sen}(2x)\left(a_2 \cdot \cos(2ct)\right) + \dots$$

$$\left[u(x,t) = \sum_1^{+\infty} \operatorname{sen}(kx) \cdot \left(a_k \cdot \cos(kct)\right) \right]$$