Problema 1

$(A_2 = A_3)$

1) $Q_2 = Q_3 \Rightarrow \cancel{A_2}\, V_2 = \cancel{A_3}\, V_3 \Rightarrow V_2 = V_3$

Continuidad

$$A_1 V_1 = \underbrace{A_2}_{=\frac{2}{3}A_1} V_2 + \underbrace{A_3}_{=\frac{2}{3}A_1} \underbrace{V_3}_{=V_2} \Rightarrow \cancel{A_1} V_1 = \frac{4}{3} \cancel{A_1} V_2$$

Por lo tanto $V_1 = \frac{4}{3} V_2 \Rightarrow \boxed{V_1 > V_2}$

2) **Bernoulli entre 1 y 2**

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g H$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g H$$

Pero $V_2 = \frac{3}{4} V_1 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho\Big(\overbrace{\frac{9}{16} - 1}^{=-\frac{7}{16}}\Big) V_1^2 + \rho g H$

Queremos que $P_1 > P_2 \Rightarrow P_1 - P_2 > 0$. Además, $P_1 - P_2 = \frac{1}{2}\rho\left(-\frac{7}{16}\right)V_1^2 + \rho g H$

$$\Rightarrow -\frac{7}{32}\cancel{\rho}\, V_1^2 > -\cancel{\rho}\, g H \Rightarrow \frac{7}{32} V_1^2 < g H \Rightarrow V_1 < \sqrt{\frac{32}{7} g H}$$

$$\therefore Q_{máx} = A_1 V_{1\,máx} = A_1 \sqrt{\frac{32}{7} g H} = \left(60 \times 10^{-4}\, m^2\right) \sqrt{\frac{32}{7}\left(9{,}81\, \frac{m}{s^2}\right)\left(1{,}2\, m\right)}$$

$$= \boxed{0{,}044\ m^3/s}$$

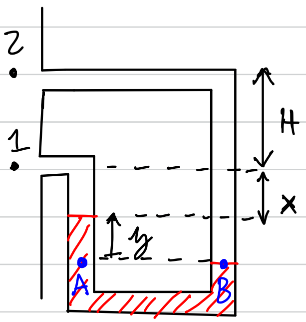
3) Queremos v_2

Bernoulli: $1 \leftrightarrow 2$

$$\bullet \; P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - \overbrace{v_1^2}^{=\frac{16}{9}v_2^2}\right) + \rho g H$$

$$= \frac{1}{2}\rho v_2^2\left(-\frac{7}{9}\right) + \rho g H$$

Hidroestática



$$P_A = P_B$$

$$\Rightarrow \quad P_1 + \cancel{\rho g x} + \rho_m g z = P_2 + \rho g(H + \cancel{x} + z)$$

$$\Rightarrow \quad \bullet \; P_1 - P_2 = (\rho - \rho_m) g z + \rho g H$$

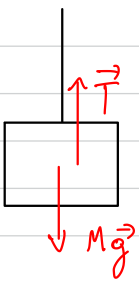
$$\left.\begin{aligned} P_1 - P_2 &= \frac{1}{2}\rho v_2^2\left(-\frac{7}{9}\right) + \cancel{\rho g H} \\ P_1 - P_2 &= (\rho - \rho_m) g z + \cancel{\rho g H} \end{aligned}\right\} \Rightarrow -\frac{7}{18}\rho v_2^2 = (\rho - \rho_m) g z$$

$$\therefore \quad \boxed{v_2 = \sqrt{\frac{18}{7}\frac{(\rho_m - \rho)}{\rho} g z} = 8{,}9 \; m/s}$$

Problema 2

1) $f = f_{OSCILADOR} = 140 Hz$, $f = \dfrac{v}{\lambda_n} = \dfrac{\sqrt{T/\mu}}{\lambda_n}$ I

Diagrama de cuepo libre



$\Longrightarrow$ $T = Mg$ II

Onda estacionaria

$\lambda_n = \dfrac{2L}{n}$, $n=2 \Rightarrow \lambda = L$ III

Usando I , II y III :

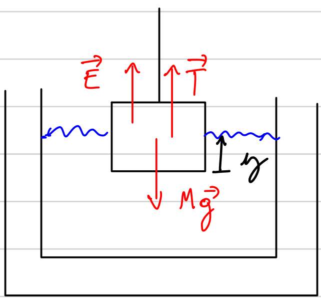
$$f = \frac{\sqrt{\frac{Mg}{\mu}}}{L}$$

$$\therefore \boxed{M = \frac{\mu f^2 L^2}{g} = 0{,}9\ kg = 900 g}$$

2) Teníamos que $f = \dfrac{\sqrt{T/\mu}}{\lambda_n}$, $\lambda_n = \dfrac{2L}{n}$

Si queremos $n = 3 \Rightarrow f = \dfrac{\sqrt{T/\mu}}{\frac{2}{3}L} \Rightarrow T = \frac{4}{9}\mu f^2 L^2$

Diagrama de cuerpo libre: Ahora aparece un empuje $\vec{E}$



$\Rightarrow T + E = Mg$, $E = \rho_{agua}\, g\, V_S$

- Volumen sumergido: $V_S = a^2 y$

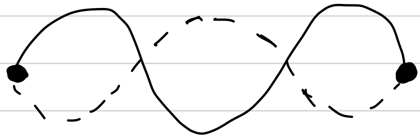
$\Rightarrow T = Mg - E = Mg - \rho_{agua}\, a^2 y g$

$$\left.\begin{array}{l} T = Mg - E = Mg - \rho_{agua}\, a^2 y g \\ \text{y usamos que } T = \frac{4}{9}\mu f^2 L^2 \end{array}\right\} \Rightarrow \frac{4}{9}\mu f^2 L^2 = Mg - \rho_{agua}\, a^2 y g$$

$$\therefore \boxed{y = \frac{M}{\rho_{agua}\, a^2} - \frac{4}{9}\frac{\mu f^2 L^2}{\rho_{agua}\, a^2 g} = 0{,}05 \text{ m}}$$

3)

a)



$n=3$

b)

$$y_{res}(x,t) = A\cos(\omega t)\,\mathrm{sen}(k_3 x)$$

Donde $\quad k_3 = \dfrac{2\pi}{\lambda_3} = \dfrac{\cancel{2}\pi}{\cancel{2}L/3} = \dfrac{3\pi}{L}$

$$\Rightarrow y_{res}(x,t) = A\cos(\omega t)\,\mathrm{sen}\left(\frac{3\pi}{L}x\right)$$

Condiciones de borde

$x=0)\quad y_{res}(x=0,t) = A\cos(\omega t)\,\cancelto{0}{\mathrm{sen}(0)} = 0 \quad \forall t$ ✓

$x=L)\quad y_{res}(x=L,t) = A\cos(\omega t)\,\mathrm{sen}\left(\frac{3\pi}{\cancel{L}}\cancel{L}\right) \underset{\mathrm{sen}(3\pi)=0}{=} 0 \quad \forall t$ ✓