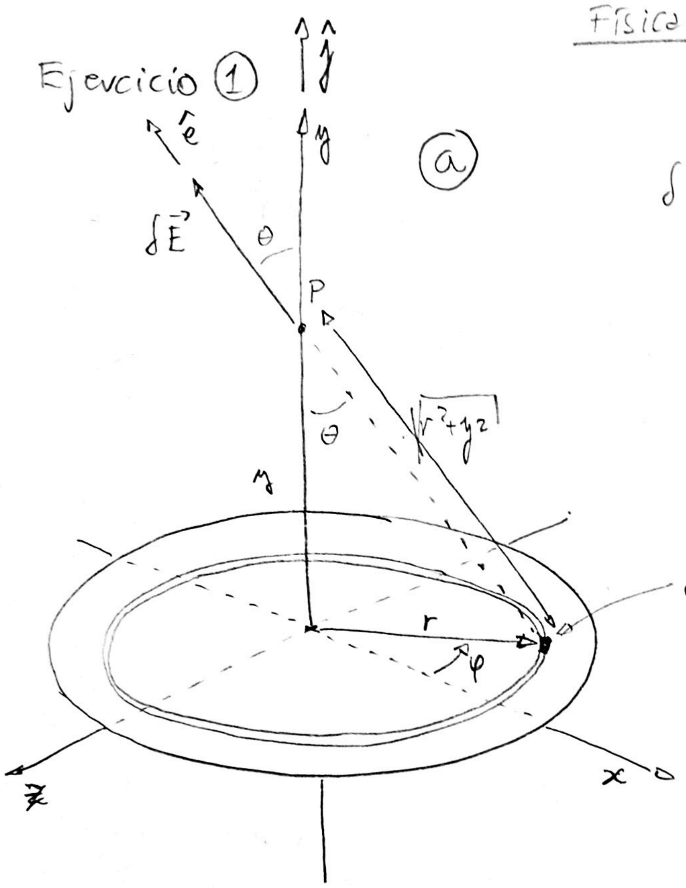
Ejercicio (1)



(a)

$$\delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\delta q}{(r^2+y^2)}\hat{e}$$ campo para una carga puntual

$$\delta q = \overbrace{\sigma . \delta r . r . \delta\varphi}^{\delta A} \qquad \boxed{\sigma = \frac{Q}{R^2\pi}}$$

$\vec{E}_r$: Campo para un aro de carga de radio r

Por simetría $\boxed{E_{rx} = E_{rz} = 0}$

$$\delta E_{ry} = \frac{(\sigma r \delta r \delta\varphi)}{4\pi\varepsilon_0(r^2+y^2)} . \cos\theta$$

$$\cos\theta = \frac{y}{\sqrt{r^2+y^2}}$$

$$E_{ry} = \int_{\varphi} \delta E_{ry} = \int_0^{2\pi} \frac{\sigma r \delta r y}{4\pi\varepsilon_0(r^2+y^2)^{3/2}} d\varphi$$

$$E_{ry} = \frac{\sigma r y \delta r}{2\varepsilon_0 (r^2+y^2)^{3/2}}$$

$\vec{E}$: campo de todo el disco $\quad \vec{E} = E_y\hat{j}$

$$E_y = \int_0^R \frac{\sigma y r dr}{2\varepsilon_0(r^2+y^2)^{3/2}} = \frac{\sigma y}{2\varepsilon_0} . \int_{y^2}^{R^2+y^2} \frac{1/2\, d\eta}{\eta^{3/2}} = \frac{\sigma y}{4\varepsilon_0} . \left(\frac{-2}{\eta^{1/2}}\right)\Bigg|_{y^2}^{R^2+y^2}$$

$\eta = r^2+y^2$

$d\eta = 2r dr$

$$E_y = \frac{\sigma y}{2\varepsilon_0}\left(\frac{1}{\sqrt{y^2}} - \frac{1}{\sqrt{R^2+y^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(\underbrace{\frac{y}{|y|}}_{\text{Signo}(y)} - \frac{y}{\sqrt{R^2+y^2}}\right)$$

$$\boxed{E_y = \frac{Q}{2\pi\varepsilon_0 R^2}\left(\text{signo}(y) - \frac{y}{\sqrt{R^2+y^2}}\right)}$$

(b)

$$\delta V = \frac{1}{4\pi\varepsilon_0} \frac{\delta q}{\sqrt{r^2+y^2}}$$

V_r: potencial debido a un aro de carga de radio r

$$V_r = \int \delta V = \int_0^{2\pi} \frac{\sigma\, \delta r\, r\, \delta\varphi}{4\pi\varepsilon_0 \sqrt{r^2+y^2}} = \frac{\sigma\, r\, \delta r}{2\varepsilon_0 \sqrt{r^2+y^2}}$$

V: potencial de todo el disco

$$V = \int V_r = \int_0^R \frac{\sigma\, r\, dr}{2\varepsilon_0\sqrt{r^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\int_{y^2}^{R^2+y^2} \frac{1/2\, d\eta}{\eta^{1/2}} = \frac{\sigma}{4\varepsilon_0}\left(\eta^{1/2}\right)\Big|_{y^2}^{R^2+y^2} =$$

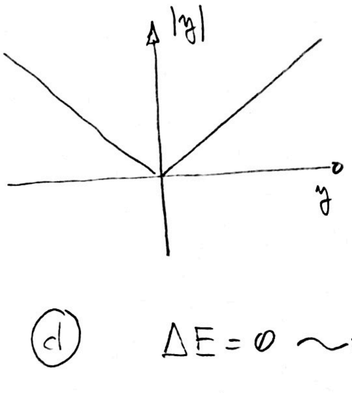
$$\eta = r^2+y^2$$
$$d\eta = 2r\,dr$$

$$= \frac{\sigma}{4\varepsilon_0}\left(\sqrt{R^2+y^2} - |y|\right) = \boxed{\frac{Q}{4\pi\varepsilon_0 R^2}\left(\sqrt{R^2+y^2} - |y|\right)}$$

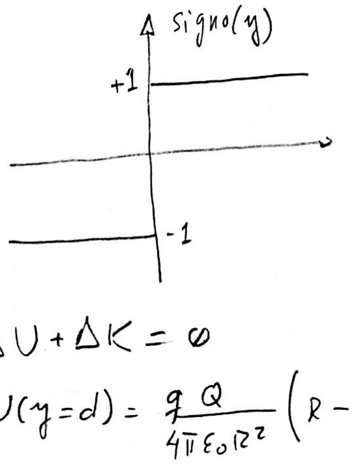
(c)

$$V = V(y) \implies E_x = -\frac{\partial V}{\partial x} = 0 \;,\; E_z = -\frac{\partial V}{\partial z} = 0$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{Q}{4\pi\varepsilon_0 R^2}\left(\text{signo}(y) - \frac{y}{\sqrt{R^2+y^2}}\right)$$



$\frac{\partial |y|}{\partial y} \rightsquigarrow$



(d)

$$\Delta E = 0 \rightsquigarrow \Delta U + \Delta K = 0 \qquad (U = qV)$$

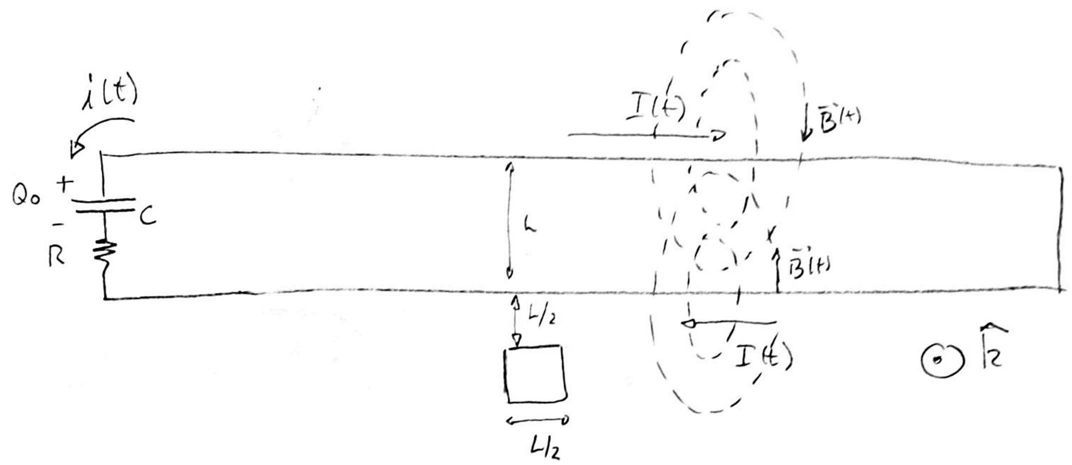
$$\Delta U = U(y=0) - U(y=d) = \frac{qQ}{4\pi\varepsilon_0 R^2}\left(R - \sqrt{R^2+d^2} + d\right)$$

$$\Delta K = K(y=0) - K(y=d) = 0 - \frac{1}{2} m v_{max}^2$$

$$\Delta U = -\Delta K \longrightarrow \qquad v_{max} = \sqrt{\frac{qQ}{2\pi\varepsilon_0 R^2 m}\left(R + d - \sqrt{R^2+d^2}\right)}$$

Ejercicio ②

ⓐ



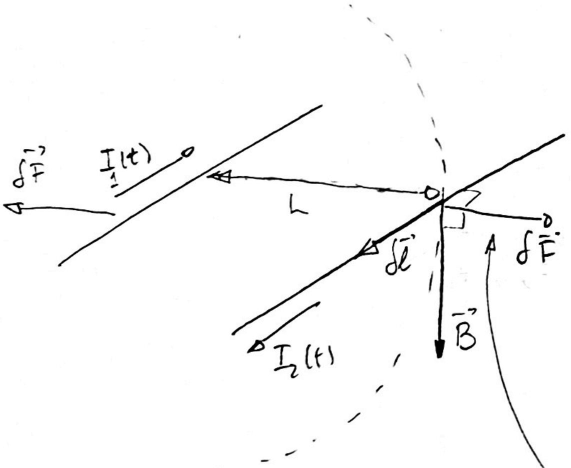
$$0 = Ri + \frac{Q}{C}$$

$$R\frac{dQ}{dt} = -\frac{Q}{C}$$

$$\frac{dQ}{Q} = -\frac{dt}{RC} \rightsquigarrow \ln\left(\frac{Q}{Q_0}\right) = \frac{-t}{RC} \rightsquigarrow Q = Q_0 e^{-t/RC}$$

$$i(t) = -\frac{Q_0}{RC} e^{-t/RC}$$

Notación: sea $I(t) = -i(t)$



$I_1(t) = I_2(t)$

Ampère $\Rightarrow \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I_1(t)$

$$B(t) \cdot 2\pi L = \mu_0 I_1(t)$$

$$\vec{B}(t) = -\frac{\mu_0 I_1(t)}{2\pi L}\hat{k}$$

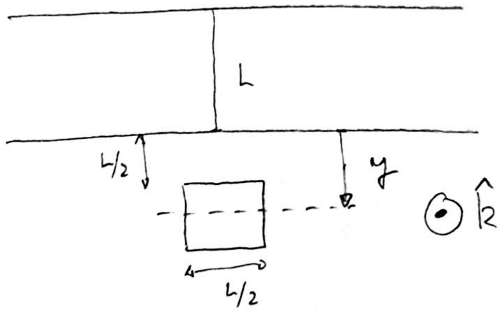
$$|\delta \vec{F}| = I_2\, \delta\vec{l} \times \vec{B} = I_2 \delta l B = \frac{\delta l\, I^2(t)\, \mu_0}{2\pi L}$$

$$\frac{|\delta \vec{F}|}{\delta l} = \frac{\mu_0 I(t)^2}{2\pi L} = \frac{\mu_0 Q_0^2 e^{-2t/RC}}{2\pi L (RC)^2}$$

Las fuerzas tienden a separar los conductores

$$\boxed{\frac{|\delta \vec{F}|}{\delta l} = \frac{\mu_0 Q_0^2 e^{-2t/RC}}{2\pi L (RC)^2}}$$

(b) Inducción sobre la espira.



Ley de Faraday $\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$

Cálculo del flujo ϕ_B:

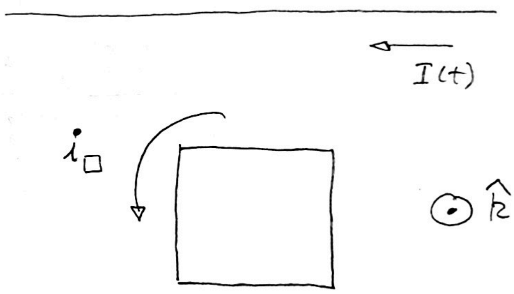
$$\vec{B} = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I_1(t)}{2\pi(y+L)}\hat{k} + \frac{\mu_0 I_2(t)}{2\pi y}\hat{k} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} - \frac{1}{y+L}\right)\hat{k}$$

$$\phi_B = \int_{L/2}^{L} B\cdot\frac{L}{2}\,dy = \frac{\mu_0 I L}{4\pi}\cdot\left(\ln y - \ln(y+L)\right)\Big|_{L/2}^{L} = \frac{\mu_0 I L}{4\pi}\underbrace{\left(\ln 2 - \ln(4/3)\right)}_{\beta}$$

$$\phi_B = \frac{\mu_0 \beta I L}{4\pi} \leadsto \frac{d\phi_B}{dt} = \frac{\mu_0 \beta L}{4\pi}\frac{dI}{dt} = -\frac{\mu_0 \beta L}{4\pi}\frac{Q_0}{(RC)^2}e^{-t/RC}$$

$$\oint_{\square} \vec{E}\cdot d\vec{l} = 2Lr\,i_{\square}(t) = -\frac{\mu_0 \beta L Q_0}{4\pi (RC)^2}e^{-t/RC}$$

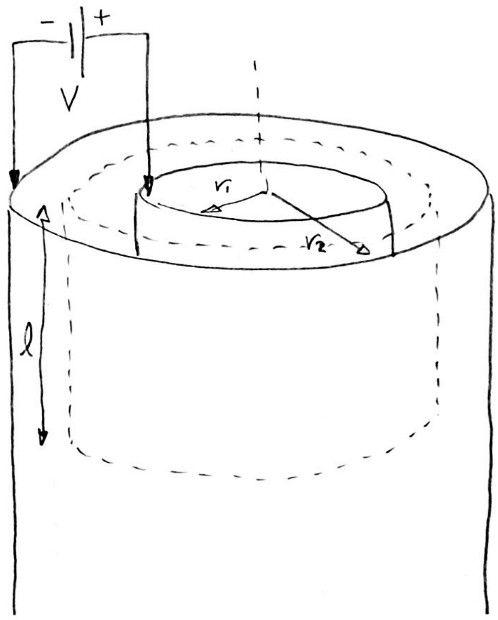
$$\boxed{i_{\square}(t) = \frac{\mu_0 \beta Q_0 e^{-t/RC}}{8\pi r (RC)^2}}$$



El flujo es positivo según $\hat{k}$ y decreciente en módulo. La corriente inducida buscará evitar dicho decrecimiento (Lenz).

Ejercicio 3

a)



$$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\varepsilon}$$

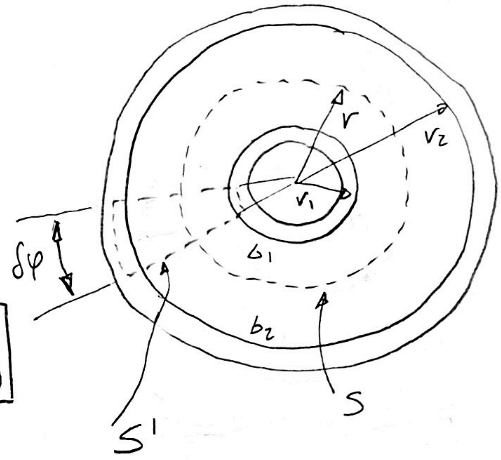
σ_1: densidad de carga superficial

$$E(r)\cdot 2\pi r \ell = \frac{\sigma_1 2\pi r_1 \ell}{\varepsilon_0}$$

$$\vec{E}(r) = \frac{\sigma_1 r_1}{\varepsilon_0 r}\hat{e}_r$$

$$V = -\int_{r_2}^{r_1} \vec{E}\cdot\vec{dr} = -\int_{r_2}^{r_1} \frac{\sigma_1 r_1}{\varepsilon_0 r}\hat{e}_r\cdot dr\,\hat{e}_r =$$

$$= -\int_{r_2}^{r_1} \frac{\sigma_1 r_1}{\varepsilon_0 r}dr = -\frac{\sigma_1 r_1}{\varepsilon_0}Ln\left(r_1/r_2\right)$$



$$V = \frac{\sigma_1 r_1}{\varepsilon_0}Ln\left(r_2/r_1\right) \rightsquigarrow \boxed{\sigma_1 = \frac{\varepsilon_0 V}{r_1 Ln(r_2/r_1)}}$$

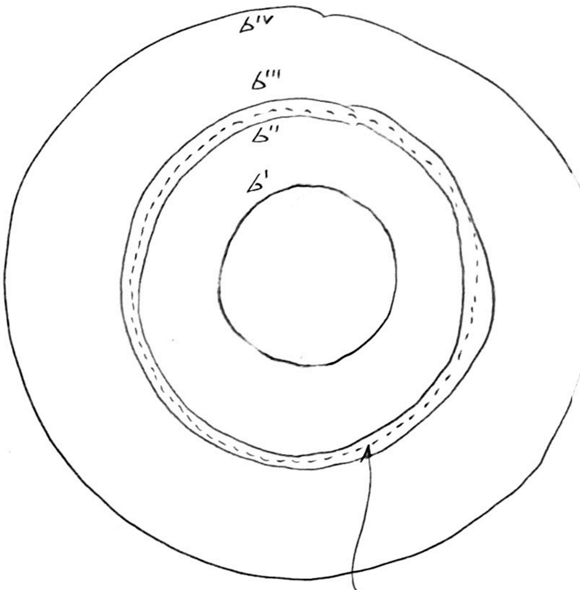
$$\oint_{S'} \vec{E}\cdot\vec{ds} = 0 \Rightarrow Q_{int} = 0 = \sigma_1 r_1 \delta\varphi \ell + \sigma_2 r_2 \delta\varphi \ell$$

$$\boxed{\sigma_2 = -\sigma_1 r_1/r_2}$$

$$C = \frac{Q}{V} = \frac{2\pi r_1 \ell \sigma_1}{V} = \frac{2\pi \cancel{r_1}\ell}{\cancel{V}}\cdot\frac{\varepsilon_0 \cancel{V}}{\cancel{r_1}Ln(r_2/r_1)}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 \ell}{Ln(r_2/r_1)}}$$

(b) Entre las placas la situación es análoga a la de (a)



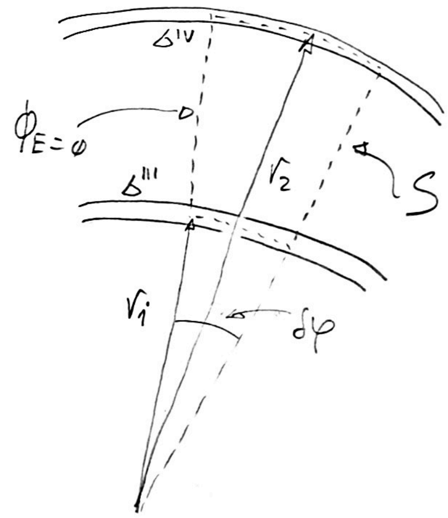
$\vec{E}=0$ en el conductor $\Rightarrow$ carga interna a esta superficie NULA.

$$\vec{E}_1(r) = \frac{\sigma' r_1}{\varepsilon_0 r}\,\hat{e}_r$$

$$V = \frac{\sigma' r_1}{\varepsilon_0} \operatorname{Ln}\left(r_i/r_1\right) \Rightarrow \boxed{\sigma' = \frac{\varepsilon_0 V}{r_1 \operatorname{Ln}\left(r_i/r_1\right)}}$$

$$\vec{E}_2(r) = \frac{\sigma''' r_i}{\varepsilon_0 r}\,\hat{e}_r \qquad \sigma''' < 0$$

$$V = -\frac{\sigma''' r_i}{\varepsilon_0} \operatorname{Ln}\left(r_2/r_i\right) \Rightarrow \boxed{\sigma''' = \frac{-\varepsilon_0 V}{r_i \operatorname{Ln}\left(r_2/r_i\right)}}$$



Para calcular Q^{IV} en la placa externa, calculo σ^{IV}.

$$\phi_{E_S} = 0 \Rightarrow Q_{int} = 0 \Rightarrow \sigma'''\,\delta\varphi\, r_i l = -\sigma^{IV}\delta\varphi\, r_2 l$$

$$\sigma''' r_i = -\sigma^{IV} r_2$$

$$\boxed{\sigma^{IV} = -\sigma''' \frac{r_i}{r_2}}$$

idem $\boxed{\sigma'' = -\sigma' \frac{r_1}{r_i}}$

(c)

$$C = \frac{2\pi r_1 l \sigma' + 2\pi r_2 l \sigma^{IV}}{V} = 2\pi l\left(\frac{1}{\cancel{V}} \cdot \frac{\varepsilon \cancel{V}}{\operatorname{Ln}(r_i/r_1)} + \frac{1}{\cancel{V}}\frac{\varepsilon \cancel{V}}{\operatorname{Ln}(r_2/r_i)}\right) =$$

$$= \underbrace{\frac{2\pi l\varepsilon}{\operatorname{Ln}\left(r_i/r_1\right)}}_{C'} + \underbrace{\frac{2\pi l \varepsilon}{\operatorname{Ln}\left(r_2/r_i\right)}}_{C''}$$

$$\left(C = \frac{Q}{V} = \frac{Q' + Q^{IV}}{V} = \frac{|Q'' + Q'''|}{V}\right)$$