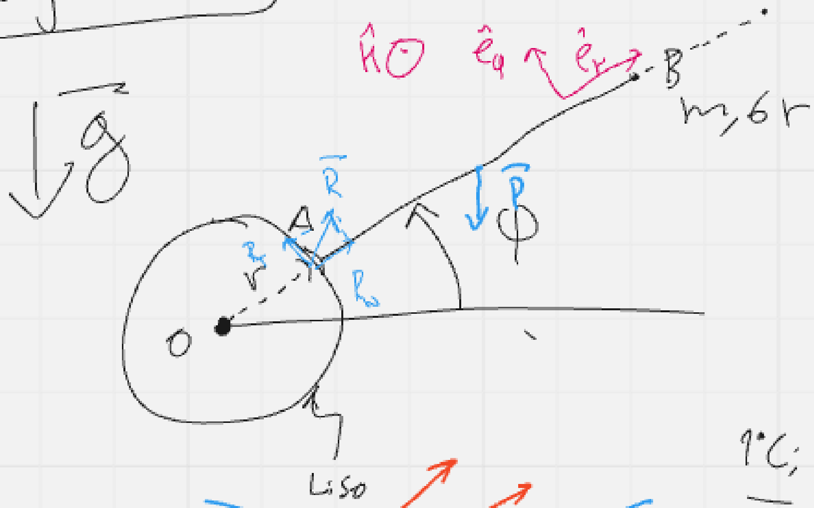
Ejercicio 7

PVI



a) Ec de mov.

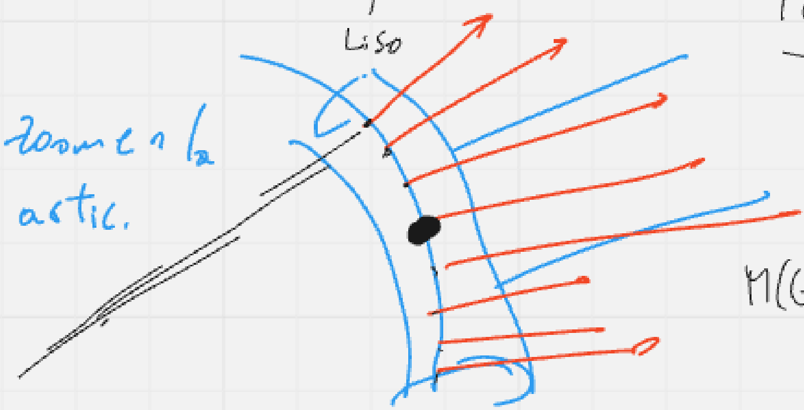
1ª Card + 2ª Card } Fuerzas?

o

Energía

1ºC; $m\bar a_c = \bar R^{ext} = \bar R - mg(\cos\varphi\,\hat e_\varphi + \sin\varphi\,\hat e_r)$

zoom en la artic.



2ºC: ¿Dónde me paro? } Si me paro en ⊛, los torques en A dan cero!!

→ Todas las fuerzas son radiales.

$$M(G-Q)\times \bar a_Q + \frac{d(\mathbb{I}_Q\bar\omega)}{dt} = \bar M_Q^{Ext}$$

⊛ O es parte de la extensión rígida de la barra.

Camino 1: 2ª Card. $\dfrac{d\,\mathbb{I}_O\bar\omega}{dt} = \bar M_O^{Ext}$

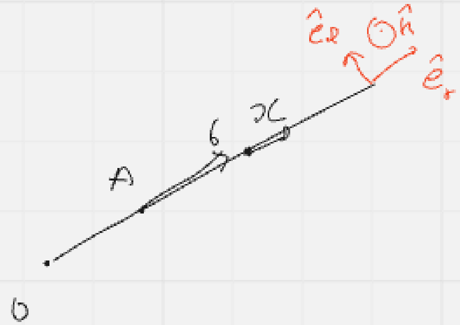
Hallo $\mathbb{I}_G$ y después lo muevo a O con Steiner $\mathbb{I}_O^{\{\hat e_r \hat e_\varphi \hat k\}} = \mathbb{I}_G^{\{\hat e_r \hat e_\varphi \hat k\}} + \mathbb{J}_O^{\{\hat e_r \hat e_\varphi \hat k\}}$

Hallo $\mathbb{I}_G$ y después lo muevo a O con Steiner $\mathbb{I}_O^{\{\hat{e}_r,\hat{e}_\varphi,\hat{n}\}} = \mathbb{I}_G^{\{\hat{e}_r,\hat{e}_\varphi,\hat{n}\}} + \mathbb{J}_O^{\{\hat{e}_r,\hat{e}_\varphi,\hat{n}\}}$

$$\begin{pmatrix} \cancel{\int \lambda dx (y^2+z^2)} & -\cancel{\int \lambda dx\, xy} & -\cancel{\int \lambda dx\, xz} \\ 0 & \int \lambda dx (x^2+z^2) & -\cancel{\int \lambda dx\, yz} \\ 0 & 0 & \int \lambda dx (y^2+x^2) \end{pmatrix}$$

$dm = \lambda dx$

$\hookrightarrow \lambda = \dfrac{m}{6r}$



$$\lambda \int_{-3r}^{3r} x^2 dx = \frac{m}{6r} \frac{x^3}{3}\Big|_{-3r}^{3r} = \frac{m}{6r} \cdot \frac{54r^3}{3} = 3mr^2$$

$$3mr^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\mathbb{J}_O\right)_{\alpha\beta}^{\{\hat{e}_r,\hat{e}_\varphi,\hat{n}\}} = m(G-O)^2 \delta_{\alpha\beta} - m(G-O)_\alpha (G-O)_\beta \qquad \delta_{\alpha\beta} \to \begin{cases} 1 & \text{si } \alpha=\beta \\ 0 & \text{si no} \end{cases}$$

$(G-O) = (4r,0,0)$

$(G-O)^2 = 16r^2$

$$\mathbb{J}_O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 16mr^2 & 0 \\ 0 & 0 & 16mr^2 \end{pmatrix}$$

$$\mathbb{I}_O^{\{\hat{e}_r,\hat{e}_\varphi,\hat{n}\}} = \mathbb{I}_G^{\{\}} + \mathbb{J}_O^{\{\}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} 19mr^2$$

Volvemos a la 2ª Card:

$$\bar{\omega} = \dot{\varphi}\hat{n} = \begin{pmatrix} 0 \\ 0 \\ \dot{\varphi} \end{pmatrix}$$

$$\frac{d}{dt}\left(\mathbb{I}_G \bar{\omega}\right) = \bar{M}_G^E = 4r\hat{e}_r \times mg(-\cos\varphi\, \hat{e}_\varphi - \sin\varphi\, \hat{e}_r)$$

$$= -4mgr\cos\varphi\, \hat{n}$$

$$\mathbb{I}_0 \bar{\omega} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} 19mr^2 \begin{pmatrix} 0 \\ 0 \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 19mr^2\dot{\varphi} \end{pmatrix} = 19mr^2\dot{\varphi}\hat{k} \Longrightarrow \frac{d}{dt}\left(\mathbb{I}_0 \bar{\omega}\right) = \underline{19mr^2\ddot{\varphi}\hat{k}}$$

$$\xRightarrow{2^{a}C} 19mr^2\ddot{\varphi} + 4mgr\cos\varphi = 0 \Rightarrow \boxed{\ddot{\varphi} = \frac{-4g}{19r}\cos\varphi}$$

Camino 2 (Energía): $P^{res} = 0 \Longrightarrow \frac{dE}{dt} = 0$ | $E = T + U$

En O

$$T = \cancelto{0}{\frac{1}{2} m v_Q^2} + \frac{1}{2}\bar{\omega}\cdot\mathbb{I}_Q\bar{\omega} + \cancelto{0}{m\bar{v}_Q}\cdot\left[\bar{\omega}\times(\bar{r}_0 - \bar{r}_Q)\right] = \frac{1}{2}\bar{\omega}\cdot\underbrace{\mathbb{I}_0\bar{\omega}}_{19\dot{\varphi}mr^2\hat{k}} = \frac{19\dot{\varphi}^2 mr^2}{2}$$

$$U = mg\,4r\sin\varphi$$

$$E = T + U = \frac{19}{2}\dot{\varphi}^2 mr^2 + mg\,4r\sin\varphi \longrightarrow \frac{dE}{dt} = 0 \Longleftrightarrow 19\cancel{\dot{\varphi}}\ddot{\varphi}mr^2 + mg\,4r\cancel{\dot{\varphi}}\cos\varphi = 0$$

$$\Longleftrightarrow \boxed{19\ddot{\varphi}mr^2 + mg\,4r\cos\varphi = 0}$$

b) Los momentos de las reacc desde O son 0: Son todas radiales.

Resultantes en A y Momento desde A.

Vuelvo a la 1° Card: $m\overline{a_G} = R_N \hat{e}_r + R_T \hat{e}_\varphi - mg(\cos\varphi \hat{e}_\varphi + \sin\varphi \hat{e}_r)$

$\overline{r_G} = 4r\hat{e}_r$

$\overline{v_G} = 4r\dot{\varphi}\hat{e}_\varphi$

$\overline{a_G} = 4r\ddot{\varphi}\hat{e}_\varphi - 4r\dot{\varphi}^2\hat{e}_r$

$m4r\ddot{\varphi}\hat{e}_\varphi - m4r\dot{\varphi}^2\hat{e}_r = R_N\hat{e}_r + R_T\hat{e}_\varphi - mg(\cos\varphi\hat{e}_\varphi + \sin\varphi\hat{e}_r)$

$$\begin{cases} R_N = mg\sin\varphi - m4r\dot{\varphi}^2 \\ R_T = mg\cos\varphi + m4r\ddot{\varphi} \end{cases}$$

$\ddot{\varphi} = -\dfrac{4g}{19r}\cos\varphi$

$\rightarrow R_T = mg\cos\varphi - \dfrac{16mg}{19}\cos\varphi = \boxed{\dfrac{3}{19}mg\cos\varphi}$

$\rightarrow$ Para R_N preciso $\dot{\varphi} \longrightarrow$ Pre integro la ec de mov.

integro la ec de mov.

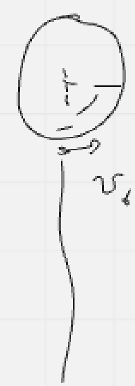
$$\int \ddot{\varphi}\,\dot{\varphi}\,dt = -\frac{4g}{19r}\int \cos\varphi\,\dot{\varphi}\,dt$$

$$\left(\begin{aligned} \dot{\varphi} &= \frac{d\varphi}{dt} \longrightarrow \dot{\varphi}\,dt = d\varphi \\ \ddot{\varphi} &= \frac{d\dot{\varphi}}{dt} \longrightarrow \ddot{\varphi}\,dt = d\dot{\varphi} \end{aligned}\right)$$

$$\int_{\dot{\varphi}_0}^{\dot{\varphi}} \dot{\varphi}\,d\dot{\varphi} = -\frac{4g}{19r}\int_{\varphi_0}^{\varphi} \cos\varphi\,d\varphi$$

$$v_A(\varphi = -\pi/2) = v_0 \Rightarrow \boxed{\dot{\varphi}_0 = \frac{v_0}{r}}$$

$$\frac{\dot{\varphi}^2}{2} - \frac{\dot{\varphi}_0^2}{2} = -\frac{4g}{19r}\left(\sin\varphi - \sin\underbrace{\varphi_0}_{-\pi/2}\right)$$



$$\bar{r}_A = r\hat{e}_r$$

$$\bar{v}_A = r\dot{\varphi}\,\hat{e}_\varphi$$

$$v_A(\varphi = -\pi/2) = r\dot{\varphi}(\varphi = -\pi/2)\,\hat{e}_\varphi = v_0\hat{e}_\varphi$$

$$\boxed{\dot{\varphi}^2 = \frac{v_0^2}{r^2} - \frac{8g}{19r}(\sin\varphi + 1)}$$

$$R_N = mg\sin\varphi - 4mr\dot{\varphi}^2$$

$$= mg\sin\varphi + \frac{32mg}{19}(\sin\varphi + 1) - \frac{4mv_0^2}{r}$$

$$R_N = mg\sin\varphi - 4mr\dot{\varphi}^2$$

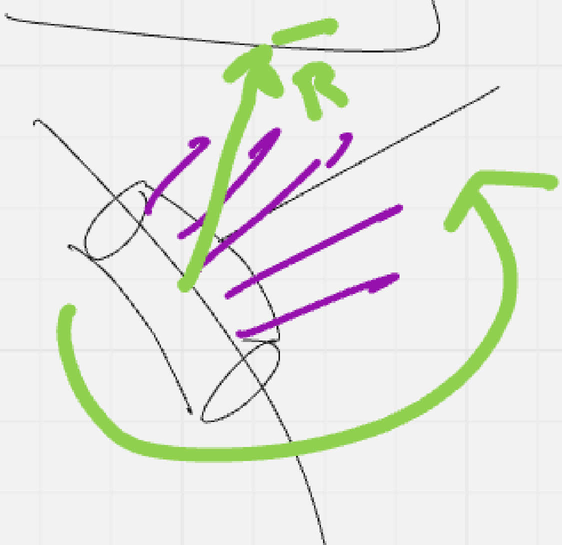
$$= mg\sin\varphi + \frac{32}{19}mg(\sin\varphi + 1) - \frac{4mv_0^2}{r}$$

$$\boxed{R_N = \frac{51}{19}mg\sin\varphi + \frac{32mg}{19} - \frac{4mv_0^2}{r}}$$

$$\boxed{\bar{R} = \left(\frac{51}{19}mg\sin\varphi + \frac{32}{19}mg - \frac{4mv_0^2}{r}\right)\hat{e}_r + \frac{3}{19}mg\cos\varphi\,\hat{e}_\varphi}$$

$$\boxed{R_T = \frac{3}{19}mg\cos\varphi}$$

$$\bar{M}^{ext}_{O_1} = \bar{M}^{ext}_{O_2} + \bar{R}^{ext} \times (O_1 - O_2)$$



$$\bar{M}^R_O = 0$$

$$\boxed{\bar{M}^R_A = \overbrace{\bar{M}^R_O}^{=0} + \bar{R} \times (r\hat{e}_r) = -\frac{3}{19}mgr\cos\varphi\,\hat{k}}$$