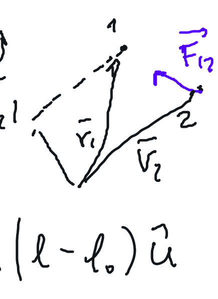
Práctico 2: Dinámica

Repaso: 2ª Ley de Newton: $\boxed{\sum \vec{F} = m\vec{a}}$ (SÓLO en ref. inerciales) → No acelerados

→ Ec vectorial (3 ecs) $F_x = m a_x$, $F_y = m a_y$, $F_z = m a_z$

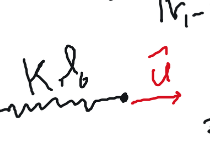
Fuerzas "típicas": • Peso: $\vec{P} = -mg\hat{j}$ ($\hat{j}$, $\hat{i}$)

• Gravitatoria: $\vec{F}_{12} = \frac{G m_1 m_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^2 \, |\vec{r}_1 - \vec{r}_2|}$



RSO

• Elástica



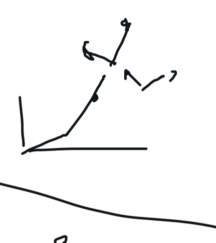
$\vec{F}_{el} = -K(\ell - \ell_0)\hat{u}$

Vínculos ← • $\vec{N}$ y $\vec{F}_r$:

→ No desliza: $|\vec{F}_r| \leq \mu_E |\vec{N}|$

→ Desliza: $|\vec{F}_r| = \mu_D |\vec{N}|$

¿Cómo usamos esto?:
0) Dibujito!!!
1) Identifico el objeto.
2) Qué fuerzas actúan sobre él? $\boxed{DCL}$
3) Tomar coord y versores apropiados.
4) Qué vínculos tengo → Apoyan? → $\vec{N}$; Deslizo? → $\vec{F}_r$
5) Newton VECTORIAL → Ec de mov
6) Resolverla → Leyes horarias
Sino puedo → Preintegro v(Posición)
7) Compatibilidad vínculos.



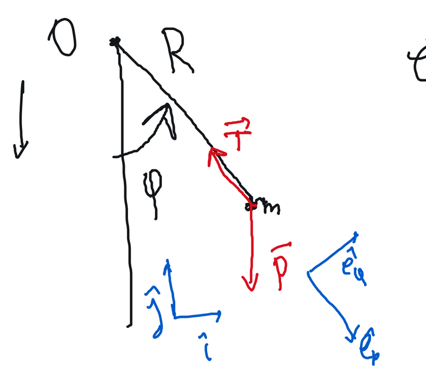
$\ddot{\theta} + \frac{g}{h}\theta$ ...

$\theta(t)$

Ejercicio 6:

$\sum \vec{F} = m\vec{a}$ | $\vec{r} = R\hat{e}_\rho \rightarrow \vec{v} = R\dot{\hat{e}}_\rho = R\dot{\varphi}\hat{e}_\varphi$

$\boxed{\vec{a} = R\ddot{\varphi}\hat{e}_\varphi - R\dot{\varphi}^2\hat{e}_\rho}$



$\dot{\hat{e}}_\rho = \dot{\varphi}\hat{k} \times \hat{e}_\rho = \dot{\varphi}\hat{e}_\varphi$

$\dot{\hat{e}}_\varphi = \dot{\varphi}\hat{k} \times \hat{e}_\varphi = -\dot{\varphi}\hat{e}_\rho$

$\vec{F} = -mg\hat{j} - T\hat{e}_\rho$
$= mg\cos\varphi\,\hat{e}_\rho - mg\sin\varphi\,\hat{e}_\varphi - T\hat{e}_\rho$

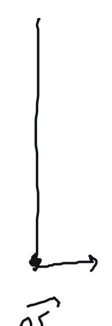
$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$ | $\hat{j} = -\cos\varphi\,\hat{e}_\rho + \sin\varphi\,\hat{e}_\varphi$

$\Longrightarrow \vec{F} = m\vec{a} \iff mg\cos\varphi\,\hat{e}_\rho - mg\sin\varphi\,\hat{e}_\varphi - T\hat{e}_\rho = mR\ddot{\varphi}\hat{e}_\varphi - mR\dot{\varphi}^2\hat{e}_\rho$

$$\begin{cases} mg\cos\varphi - T = -mR\dot{\varphi}^2 \\ -mg\sin\varphi = mR\ddot{\varphi} \end{cases}$$

Ec de mov: $\boxed{mR\ddot{\varphi} = -mg\sin\varphi}$

b)



$\boxed{mR\ddot{\varphi} = -mg\sin\varphi}$ Para pre-integrar multiplico por $\dot{\varphi}$

$\int mR\ddot{\varphi}\dot{\varphi}\,dt = -\int mg\sin\varphi\,\dot{\varphi}\,dt$ (Ahora integro en t)

$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} \rightarrow \ddot{\varphi}\,dt = d\dot{\varphi}$

$\dot{\varphi} = \frac{d\varphi}{dt} \rightarrow \dot{\varphi}\,dt = d\varphi$

$\Rightarrow \int mR\dot{\varphi}\,d\dot{\varphi} = -mg\int \sin\varphi\,d\varphi$

$mR\frac{\dot{\varphi}^2}{2}\Big|_{\dot{\varphi}_0}^{\dot{\varphi}} = +mg\cos\varphi\Big|_{\varphi_0}^{\varphi} \Rightarrow \frac{\dot{\varphi}^2}{2} - \frac{\dot{\varphi}_0^2}{2} = \frac{g}{R}(\cos\varphi - \cos\varphi_0)$

$\dot{\varphi} = \sqrt{\frac{2g}{R}\left[\cos\varphi - 1\right] + \dot{\varphi}_0^2}$ $\boxed{\dot{\varphi}_0 = \frac{v_0}{R}}$

$\vec{v} = R\dot{\hat{e}}_\rho = R\dot{\varphi}\hat{e}_\varphi$

$\vec{v}(0) = R\dot{\varphi}_0\,\hat{e}_\varphi$

$\vec{v}(0) = v_0\hat{e}_\varphi$

$$\boxed{\dot{\varphi} = \sqrt{\frac{2g}{R}\left[\cos\varphi - 1\right] + \frac{v_0^2}{R^2}}}$$

c) En vez de hilo, tengo una barra; (T puede ser >0, <0).

Se detiene si: $\dot{\varphi} = 0 \iff \dot{\varphi} = \sqrt{\frac{2g}{R}\left[\cos\varphi - 1\right] + \frac{v_0^2}{R^2}} = 0$

$\frac{2g}{R}\left[\cos\varphi - 1\right] + \frac{v_0^2}{R^2} = 0 \iff \boxed{\cos\varphi = 1 - \frac{v_0^2}{2gR}}$

Ojo! $-1 \leq \cos\varphi \leq 1 \iff$ Esta ec sólo vale si

$-1 \leq 1 - \frac{v_0^2}{2gR} \leq 1$

✓ se cumple siempre

Impongo

$-1 \leq 1 - \frac{v_0^2}{2gR} \iff v_0^2 \leq 4gR$

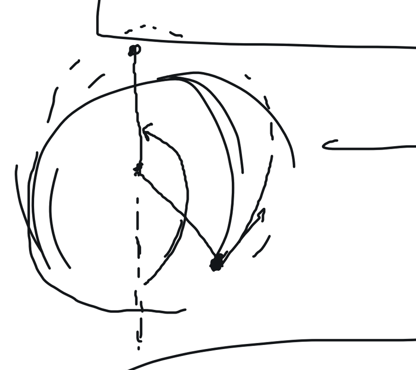
$\iff \boxed{v_0 \leq 2\sqrt{gR}}$

i) $\cos\varphi = 1 - \frac{v_0^2}{2gR} \longrightarrow \boxed{\varphi_{DET} = \text{Arccos}\left[1 - \frac{v_0^2}{2gR}\right]}$

ii) Si: $v_0 > 2\sqrt{gR}$

$\dot{\varphi} \neq 0$

$\Rightarrow$ Nunca para.



$\longrightarrow$ Mov Circ, R.

iii) Si: $v_0 = 2\sqrt{gR}$, $\varphi_{DET} = \text{Arccos}\left(1 - \frac{v_0^2}{2gR}\right)$

$= \text{Arccos}(-1) = \pi$

$\Delta t = \int_{t_0}^{t} dt \longrightarrow \dot{\varphi} = \sqrt{\frac{2g}{R}\left[\cos\varphi - 1\right] + \frac{v_0^2}{R^2}}$

$\dot{\varphi} = \frac{d\varphi}{dt} \Rightarrow dt = \frac{d\varphi}{\dot{\varphi}}$

$\Delta t = \int_0^{\pi} \frac{d\varphi}{\sqrt{\frac{2g}{R}(\cos\varphi - 1) + \frac{v_0^2}{R^2}}}$ → Vale infinito

WOLFRAMALPHA

d) Hallo T: $mg\cos\varphi - T = -mR\dot{\varphi}^2$

$T = mg\cos\varphi + mR\dot{\varphi}^2$

$\dot{\varphi} = \sqrt{\frac{2g}{R}\left[\cos\varphi - 1\right] + \frac{v_0^2}{R^2}}$

$$\boxed{T = mg\left[3\cos\varphi - 2\right] + \frac{m v_0^2}{R}}$$

e) Hilo:

Desprendimiento $\iff \boxed{T = 0}$

→ Discutir validez de la ec.