Práctico 6: Dinámica del rígido en el plano

Repaso:

Rígidos
- → Masa
- → Centro de masas
- → Dist de velocidades y acel..
- → Cond de Mov

$$\overline{p} = M\overline{v}_G$$

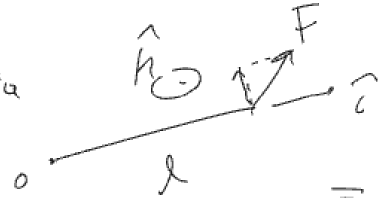
→ Ecuaciones cardinales: $\boxed{M\overline{a}_G = \overline{R}^{EXT}}$ (1a Cardinal)

$$\overline{L}_Q = M(\overline{r}_G - \overline{r}_Q) \times \overline{v}_Q + \mathbb{I}_Q \overline{\omega}$$

→ En la misma base

→ Tensor de inercia

$$\boxed{\frac{d\overline{L}_Q}{dt} = M\overline{v}_G \times \dot{\overline{r}}_Q + \overline{M}_Q^{EXT}}$$ (2a Cardinal)



$$\overline{M}_O^F = \overline{r} \times \overline{F}$$

$$\boxed{M(G-Q) \times \overline{a}_Q + \frac{d(\mathbb{I}_Q \overline{\omega})}{dt} = \overline{M}_Q^{EXT}}$$ (2a Cardinal)

$$\boxed{T = \frac{1}{2}Mv_p^2 + \frac{1}{2}\overline{\omega}\cdot\mathbb{I}_p\overline{\omega} + M\,\overline{v}_p\cdot[\overline{\omega} \times (\overline{r}_G - \overline{r}_p)]}$$

$\longrightarrow$ ¡¡Base de $\overline{\omega}$ y de $\mathbb{I}$ tiene que ser la misma!!

Matriz 3x3 simétrica.

Potencia: $\boxed{P = \overline{R^{EXT}} \cdot \overline{v}_Q + \overline{M^{EXT}_Q} \cdot \overline{\omega}}$

Caso plano:

$\overline{\omega} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$

$$\mathbb{I}_Q^{PLANO} = \begin{pmatrix} \boxed{\text{NO ME IMPORTA}} & 0 \\ & 0 \\ 0 \quad 0 & I_{33} \end{pmatrix} \qquad \mathbb{I}_Q \overline{\omega} = \begin{pmatrix} \boxed{\quad} & 0 \\ & 0 \\ 0 \quad 0 & I_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ I_{33}\,\omega \end{pmatrix}$$

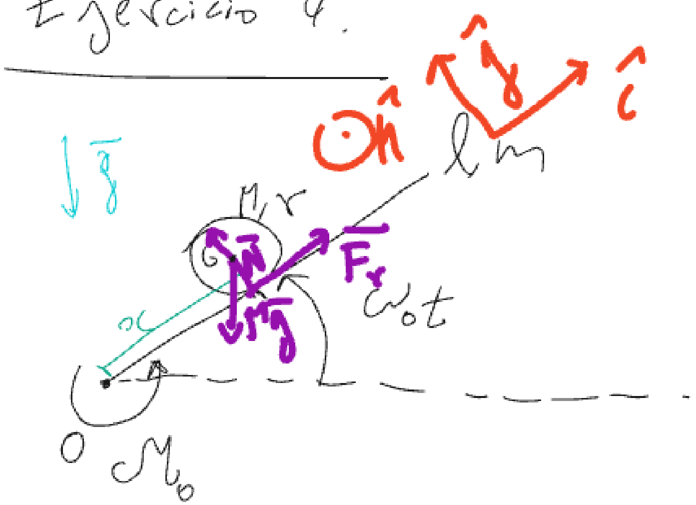
Sólo voy a calcular la entrada I_{33}.

En el plano tengo 3 ecs y 3 incógnitas.

- Cambio de punto de aplicación de momentos:

$$\begin{cases} \overline{L_{Q_1}} = \overline{L_{Q_2}} + \overline{P} \times (Q_1 - Q_2) \\ \overline{M_{Q_1}} = \overline{M_{Q_2}} + \overline{R^{EXT}} \times (Q_1 - Q_2) \end{cases}$$

Ejercicio 4:

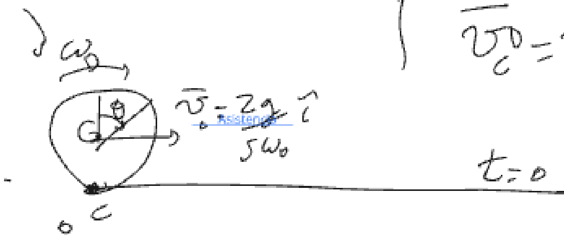


$$\boxed{M\vec a_G = \vec R^{EXT}}$$

$r\Delta\theta = \Delta x$

RSD: $\boxed{\omega = \frac{\dot x}{r}}$

$\vec v_C^D = \vec v_C^B$



$$\vec r_G = x\hat i + r\hat j \longrightarrow \vec v_G = \dot x\hat i + x\dot{\hat i} + r\dot{\hat j} = \dot x\hat i + x\omega\hat j - r\omega\hat i$$

$$\longrightarrow \vec a_G = \ddot x\hat i + \dot x\omega\hat j - x\omega^2\hat i - r\omega^2\hat j = \boxed{(\ddot x - x\omega^2)\hat i + (\dot x\omega - r\omega^2)\hat j}$$

$\vec\tau = \vec r\times\vec F$

$$\vec R^{EXT} = -Mg(\cos(\omega t)\hat j + \sin(\omega t)\hat i) + N\hat j + F_r\hat i$$

$$\vec M_G^{EXT(disco)} = -r\hat j\times F_r\hat i = rF_r\hat k$$

- 1ª Cardinal: $\begin{cases} M(\ddot x - x\omega^2) = F_r - Mg\sin(\omega t) \\ M(\dot x\omega - r\omega^2) = N - Mg\cos(\omega t) \end{cases}$

- 2ª Cardinal $\underbrace{M(G-Q)\times\vec a_Q}_{=0} + \frac{d(\mathbb{I}_Q\vec\omega)}{dt} = \vec M_Q^{EXT}$

$$\mathbb{I}_G = \begin{pmatrix} \frac{Mr^2}{4} & 0 & 0 \\ 0 & \frac{Mr^2}{4} & 0 \\ 0 & 0 & \frac{Mr^2}{2} \end{pmatrix}$$

$$\frac{d\,\mathbb{I}_G\vec\omega}{dt} = \vec M_G^{EXT} = rF_r\hat k$$

$$\vec\omega_{Disco} = -\omega_D\hat k = \begin{pmatrix} 0 \\ 0 \\ -\omega_D \end{pmatrix}$$

$$\mathbb{I}_G\vec\omega = \frac{Mr^2}{2}\omega_D\hat k$$

$$\frac{d\,\mathbb{I}_G\vec\omega}{dt} = -\frac{Mr^2}{2}\dot\omega_D\hat k$$

$$-\frac{Mr^2}{2}\dot{\omega}_0 = rF_r$$

$\underline{RSD}$: $\omega_0 = \frac{\dot{x}}{r}$

$\dot{\omega}_0 = \frac{\ddot{x}}{r}$

$$-\frac{Mr}{2}\ddot{x} = rF_r \longrightarrow F_r = \frac{-M\ddot{x}}{2}$$

$$M(\ddot{x} - x\omega_0^2) = \frac{-M\ddot{x}}{2} - Mg\sin(\omega_0 t) \longrightarrow \boxed{\frac{3}{2}\ddot{x} - x\omega_0^2 = -g\sin(\omega_0 t)}$$

(H) $\frac{3}{2}\ddot{x} = x\omega_0^2 \Longleftrightarrow \ddot{x} = \frac{2x\omega_0^2}{3} \longrightarrow x_H = \underline{A}e^{\sqrt{\frac{2}{3}}\omega_0 t} + \underline{B}e^{-\sqrt{\frac{2}{3}}\omega_0 t}$

(P) $\begin{cases} x_p = \underline{C}\sin(\omega_0 t) \\ \ddot{x} = -\underline{C}\omega_0^2\sin(\omega_0 t) \end{cases}$

$$-\frac{3}{2}C\omega_0^2\sin(\omega_0 t) - \omega_0^2 C\sin(\omega_0 t) = -g\sin(\omega_0 t)$$

$$-C\omega_0^2\frac{5}{2} = -g \Longleftrightarrow C = \frac{2}{5}\frac{g}{\omega_0^2}$$

$$x = Ae^{\sqrt{\frac{2}{3}}\omega_0 t} + Be^{-\sqrt{\frac{2}{3}}\omega_0 t} + \frac{2}{5}\frac{g}{\omega_0^2}\sin(\omega_0 t)$$

$x(0) = 0$

$\dot{x}(0) = \frac{2g}{5\omega_0}$

$x(0) = A + B = 0 \qquad A = -B$

$\dot{x}(0) = \underbrace{\sqrt{\tfrac{2}{3}}\,\omega_0\,(\overset{2A}{\overline{A - B}})}_{=0} + \frac{2g}{5\omega_0} = \frac{2g}{5\omega_0} \Longrightarrow A = -B = 0$

$$\Longrightarrow \boxed{x(t) = \frac{2g}{5\omega_0^2}\sin(\omega_0 t)}$$

b) $\bar{F}_r = -\frac{M\ddot{x}}{2}\hat{i} = +\frac{Mg}{5}\sin(\omega_0 t)\,\hat{i}$

$\boxed{\dot{x} = \frac{2g}{5\omega_0}\cos(\omega_0 t)}$

$M(2\dot{x}\omega_0 - r\omega_0^2) = N - Mg\cos(\omega_0 t)$

$N(0) > 0.$

c) $\underline{M}$ $\quad \omega_0 t \quad O \quad M$

Hay que ponerse en el sist de la barra

OK

$\frac{d\,\bar{\bar{I}}_O^{\,BARRA}\,\omega_0}{dt} = \bar{M}_O^{Ext} \Longrightarrow \bar{M}^{Ext} = 0$

$\Longrightarrow \underline{M\,\hat{k} + \bar{M}_O^{Pes0} + \bar{M}_O^{N} + \cancel{\bar{M}_O^{Fr}} = 0}$