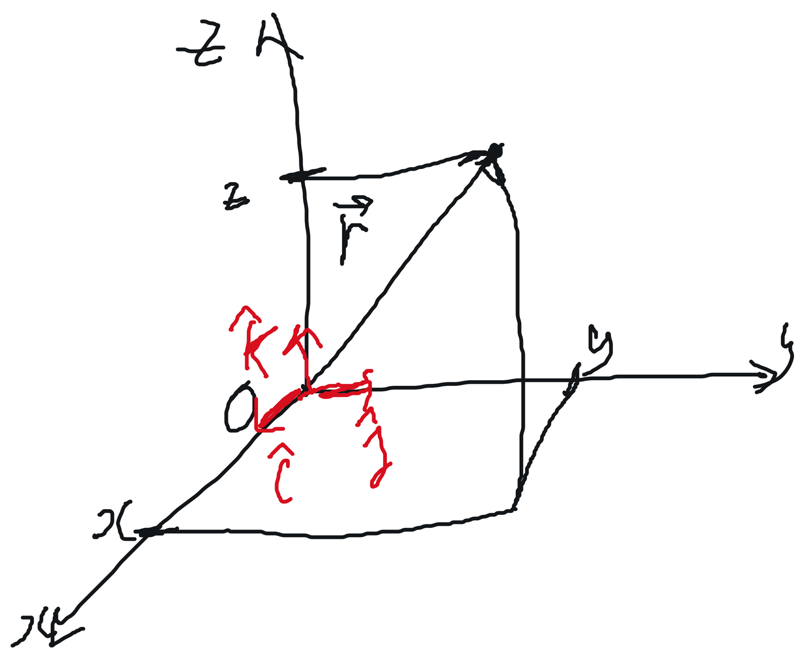
Práctico 1

Repaso:

Sist de coord:

$\frac{d\vec{r}}{dt} = \dot{\vec{r}}$

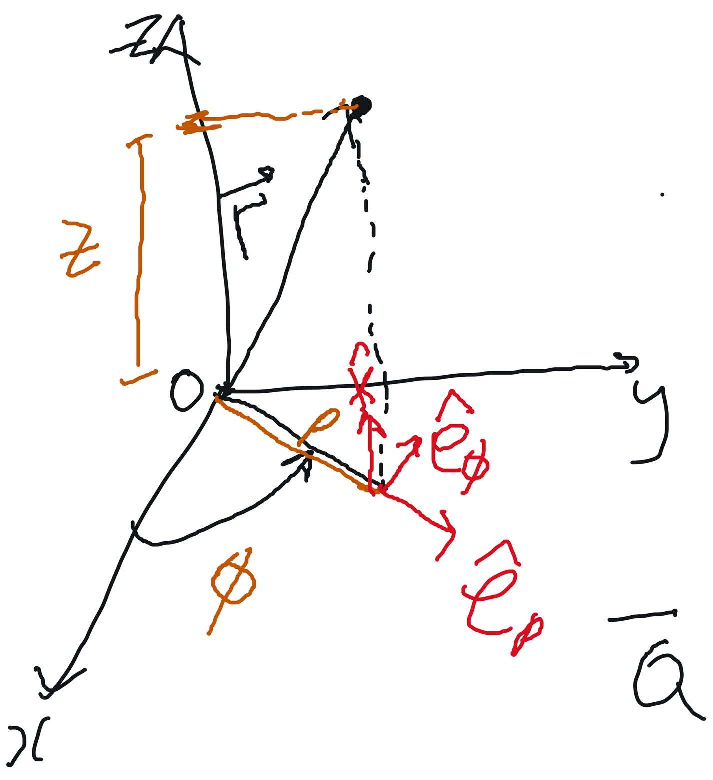
1 - Cartesianos: $\vec{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

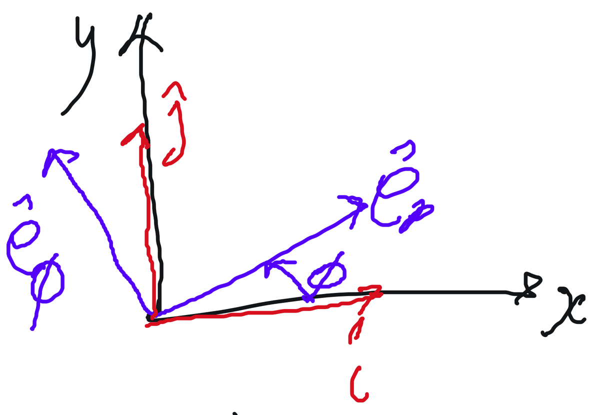


$\vec{v}(t) = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$

$\vec{a} = \frac{d^2\vec{r}}{dt} = \ddot{\vec{r}} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

2 - Cilíndricas: ρ, ϕ, z





$\dot{\hat{e}}_\phi = -\dot{\phi}\,\hat{e}_\rho$

$\vec{r} = \rho\hat{e}_\rho + z\hat{k}$

$\vec{v} = \dot{\vec{r}} = \dot{\rho}\hat{e}_\rho + \dot{z}\hat{k} + \rho\dot{\hat{e}}_\rho$

$= \dot{\rho}\hat{e}_\rho + \rho\dot{\phi}\hat{e}_\phi + \dot{z}\hat{k}$

$\vec{a} = \ddot{\rho}\hat{e}_\rho + \dot{\rho}\dot{\hat{e}}_\rho + \dot{\rho}\dot{\phi}\hat{e}_\phi + \rho\ddot{\phi}\hat{e}_\phi + \rho\dot{\phi}\dot{\hat{e}}_\phi + \ddot{z}\hat{k}$

$= (\ddot{\rho} - \rho\dot{\phi}^2)\hat{e}_\rho + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{e}_\phi + \ddot{z}\hat{k}$

$\hat{e}_\rho = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$

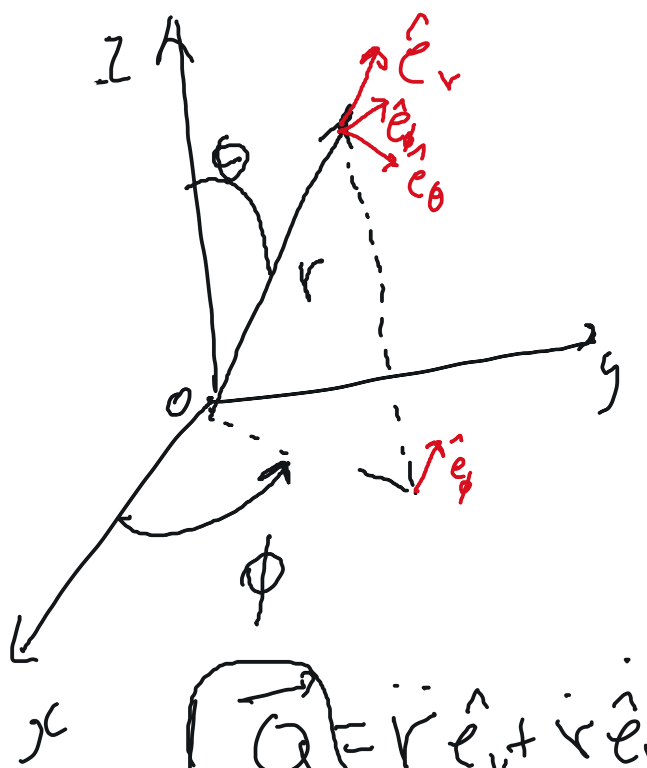
$\dot{\hat{e}}_\rho = \dot{\phi}\frac{d\hat{e}_\rho}{d\phi}$

$= -\dot{\phi}\sin\phi\,\hat{i} + \dot{\phi}\cos\phi\,\hat{j}$

$= \dot{\phi}\,\underbrace{[-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}]}_{e_\phi}$

$= \dot{\phi}\,\hat{e}_\phi$

3 - Esféricas: (r, ϕ, θ)



$\vec{r} = r\hat{e}_r$

$\vec{v} = \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + r\dot{\phi}\sin\theta\,\hat{e}_\phi$

$\frac{d\hat{e}_r}{dt} = \dot{\theta}\frac{d\hat{e}_r}{d\theta} + \dot{\phi}\frac{d\hat{e}_r}{d\phi} = \dot{\theta}\hat{e}_\theta + \dot{\phi}\sin\theta\,\hat{e}_\phi$

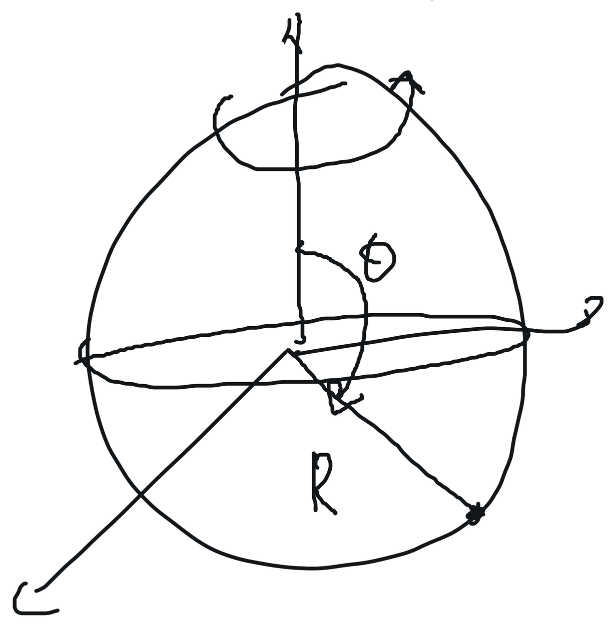
$\dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r + \dot{\phi}\cos\theta\,\hat{e}_\phi$ | $\dot{\hat{e}}_\phi = -\dot{\phi}\cos\theta\,\hat{e}_\theta - \dot{\phi}\sin\theta\,\hat{e}_r$

$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta + \dot{r}\dot{\phi}\sin\theta\,\hat{e}_\phi + r\ddot{\phi}\sin\theta\,\hat{e}_\phi + r\dot{\phi}\dot{\theta}\cos\theta\,\hat{e}_\phi + r\dot{\phi}\sin\theta\,\dot{\hat{e}}_\phi$

$= (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{e}_r$

$+ (2\dot{r}\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{e}_\theta$

$+ (2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\ddot{\phi}\sin\theta)\hat{e}_\phi$



$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + r\dot{\phi}\sin\theta\,\hat{e}_\phi$

$= 6400\cdot10^3\cdot 7{,}27\cdot10^{-5}\sin(125^\circ)\,\hat{e}_\phi$

$= 381$ m/s $\hat{e}_\phi$

$\dot{r} = 0$

$\ddot{r} = 0$

$\dot{\theta} = 0$

$\ddot{\theta} = 0$

$\ddot{\phi} = 0$

$R = 6400$ km $= 6400\cdot10^3$ m

$\theta = 125^\circ$

$\dot{\phi} = \frac{2\pi}{24\cdot60\cdot60} \simeq 7{,}27\cdot10^{-5}$ rad/s

$\vec{a} = -r\dot{\phi}^2\sin^2\theta\,\hat{e}_r - r\dot{\phi}^2\sin\theta\cos\theta\,\hat{e}_\theta$

$= -r\dot{\phi}^2\sin\theta\,\underbrace{(\sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta)}_{=\hat{e}_\rho}$

$= -R\dot{\phi}^2\sin\theta\,\hat{e}_\rho$

$\| \; (-2{,}3\cdot10^{-2}\hat{e}_r + 1{,}6\cdot10^{-2}\hat{e}_\theta)$ m/s²