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Economics of Storage Systems

Interpretation of the dual solution in Energy Models using Linear Programming

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Abstract — Hydrogen or alternative synthetic energy carriers will become an integral part of a future non-fossil energy system. These energy carriers are mainly derived from electricity via electrolysis. This leads to a strong coupling between the hydrogen or synthetic fuel markets and the electricity market. Hydrogen and synthetic fuels offer unique characteristics when it comes to storage capabilities. The potential size of these storage systems can be much larger than conventional electricity storage systems including batteries and the costs are considerably lower. Storage systems smooth electricity prices. The paper investigates the impact of storage in a very simple coupled electricity-hydrogen system in a pure analytical form based on the dual solution of the primal problem as basis for the economic interpretation and on simple examples. The results indicate that the presence of new storage systems will have a rather strong impact on electricity prices and price variations. The smoothing might get so far that even the investment costs could be included in the electricity price. This would then create in an energy only market enough incentives for future investments. Still, this is an early hypothesis which needs much more detailed investigations.

1 Introduction

The massive implementation of renewable energy sources into the power system poses a series of problems. One is the determination of the electricity prices. This problem can well be described by the merit order effect. Renewable electricity sources with very low or even zero variable costs are dispatched even before base load plants. The result is a reduction in the electricity price [5]. But the overall cost of the system especially the investment costs increase. The reduced prices lead to reduced revenues of the power plant owner. Incentives for new investments especially in renewable sources are missing. The conventional energy only market fails.

The problem can certainly be phrased in another way. The liberalization of the power markets was driven by the idea to find price signals to dispatch the power plants in a cost optimal way and give in parallel with the produced rent enough incentives to plan and construct new power plants. Still the concept worked well, although there is no direct link between the investment cost and the produce rent. Only the most expensive power plant in the merit order, quite often oil-fired plants, could not be financed properly. A bonus for the investment costs was added to the variable and fuel costs to make also the operation and investment of these plants viable.

The two major renewable sources wind and PV supply electricity as quasi primary energy. In turn electricity has to deliver all kind of energy carriers like heat and chemical energy carriers like hydrogen. This concept is widely described as sector coupling [4]. In the following paper the coupling of electricity and hydrogen is studied more in detail, but still on a more general level. The very specific characteristic of hydrogen is that large scale storage options are available. The capacity costs are very low compared to all other storage options. The size of the storage facilities can be in the order of a few percent of the annual consumption.

The following analysis is based on a linear optimization approach. The dual value of the demand equation is interpreted as market price. The paper is divided in two major parts. In a first part the primal and dual problems of simple power and energy systems with and without major storage options are discussed on the basis of the equations only. In the second part the analysis is illustrated with simple examples.

The first part is organized as follows:

- The conventional problem is revisited with conventional power plants, with fuel and variable costs as only costs. Investment costs are not accounted, and capacities are fixed.
- The conventional part is extended and covers the investments
- A simple model with only renewable sources, storage options and a back-up source which has only variable costs is described, the storage has so called cyclic boundary conditions
- A simple model with only renewable sources, storage options and a back-up source which has only variable costs is described, the storage has an initial value

The second part follows the same scheme but develops simple system models. The results of these models are then interpreted in the light of the analytical solution. The last part discusses the impact of these findings for the energy system and especially possible price mechanism in future if large scale storage options are available and an integral part of the power system.

The central thesis of the work can be summarized as follows: A future energy system mainly based on renewable power sources with huge seasonal storage options like hydrogen sees a rather smooth distribution of electricity and other energy prices. Even investment costs could well be spread over the whole year. A bidding strategy for investment costs can be derived from the dual solution. The mechanism of a simple energy only market can then also organize an energy market based on renewable energies only. Still, this is just a working hypothesis and certainly much more detailed investigations are necessary. The work was inspired by discussions in the P2X-consortium to understand the dual solutions of the system model.

This paper contains most material which is well known, still we think the compilation is still valuable and can especially catalyze a discussion about the impact of seasonal storage capacities on the electricity market.

The paper is organized as follows:

Analytical Calculation	**Model Calculation**
The Conventional Electricity Market	Sceanrio 1
The Conventional Electricity Market with Investment Costs	Scenario 2
The first storage model	Scenario 3

2 Methodology and Abbreviations

The approach of the paper is rather simple and strait forward. The dual of the demand equation is used as proxy for the electricity price. This approach is certainly valid if a simple scheduling program is applied. The idea is here to do the same also in case of extension models meaning that also investment costs are reflected in the price. The interpretation of dual values in simple linear optimization problems has a long tradition [2]. All variables are positive in the primal models.

c	cost vector
x	vector of variables
y	vector of dual variables
A	matrix of constraints
b	resource vector

The dual model can easily be derived from writing the problem in the standard form.

$$\min c^T x$$
$$Ax = b$$
$$x \geq 0$$

The dual problem is then given by:

$$\max b^T y$$
$$y^T A \leq c^T$$

In the following we also use this form for the dual

$$\max b^T y$$
$$c^T - y^T A \geq 0$$

The further abbreviations are given in the following table:

pl	set of power plants
t	set of time with zeros time step
tm	set of time without zeros time step
$Inv_C(pl)$	Investment cost of plant pl
$C(pl)$	Capacity of plant pl (also variable)
$FC_{EB}(pl)$	fuel cost of plant pl
$D(tm)$	demand at time step tm
Inv_{st}	investment cost storage capacity
FC_{EB}	fuel cost back-up power plant
$Ts(pl, tm)$	capacity factor of renewable plant pl at time step tm
St_{ini}	state of charge of storage in the zeros time step
$C(pl)$	capacity of plant pl
$E(pl, tm)$	output energy of plant pl at time step tm
$Ca(pl, tm)$	capacity equation for plant pl at time step tm
$EB(tm)$	energy balance at time step tm
C_{st}	capacity of storage plant st
$Eb_{up}(tm)$	energy output of back-up plant at time step tm
$Est_{in}(tm)$	energy input to storage at time step tm
$Est_{out}(tm)$	energy output of storage at time step tm
$Est(t)$	energy content of storage at time step t
$EstB(t)$	energy balance of storage at time step t
$EstiL(tm)$	lower limit of storage content
$EstuL(tm)$	upper limit of storage content
$EstIni$	initialization of storage

3 The Conventional Electricity Market

The conventional electricity market can best be modelled by an optimization model which considers only variable and fuel costs and keeps capacities constant. In most models the demand is fixed, still a price elastic demand can also be modelled in the framework of a linear optimization model. Still for the time being we keep the demand fixed.

3.1 The Primal model

The following equations are the simplest approximation of a power market but reflect still the most important features. The dispatch is just made according to the fuel and variable costs. The energy output is just limited by the capacity. Certainly, more detailed picture need to include more details about the power plants like minimal power, ramping rates and start-up times.

$$\begin{aligned}
&\underset{E(pl,tm)}{\text{minimize}} && \sum_{pl,tm} FC_{EB}(pl)E(pl,tm) && \\
&\text{subject to} && && \\
&\forall\ pl,tm && E(pl,tm) \leq C(pl) && Ca(pl,tm) \\
&\forall\ tm && \sum_{pl} E(pl,tm) \geq D(tm) && EB(tm)
\end{aligned}$$

3.2 The Dual Model

The dual model gives now a much better understanding of the economic background of the system. The electricity price is given by $Eb(tm)$. The plant owners try to maximize the product of price and demand. The electricity price is now limited by the fuel price. Without capacity constraints the power plant with lowest fuel price is dispatched. In this case the variable Ca(pl) would be zero. In case the capacity limit of the cheapest power plant is reached the dual value of capacity constraints Ca(pl) has a negative value. The value will be exactly the difference between the fuel price of the plant limited by the constraint and the plant setting the electricity price.

$$\begin{aligned}
&\underset{Ca(pl,tm),EB(tm)}{\text{maximize}} && \sum_{pl,tm} Ca(pl,tm)\ C(pl) \ + \sum_{tm} EB(tm)\ D(tm) && \\
&\text{subject to} && && \\
&\forall\ pl,tm && FC_{EB}(pl) - Ca(pl,tm) - EB(tm) \geq 0 && E(pl,tm) \\
&\forall\ pl,tm && -Ca(pl,tm) \geq 0 && Slack1(pl,tm) \\
&\forall\ tm && EB(tm) \geq 0 && Slack2(tm)
\end{aligned}$$

The two terms in the objective function can though be interpreted: the second term is the revenue of the companies by selling the electricity to the market price, the first term is the producer profit which is given as negative value. The difference between revenue and producer profit are the costs of the companies. The value of the dual objective function is the same value as the primal problem. Figure 1 describes the result with a merit order curve.

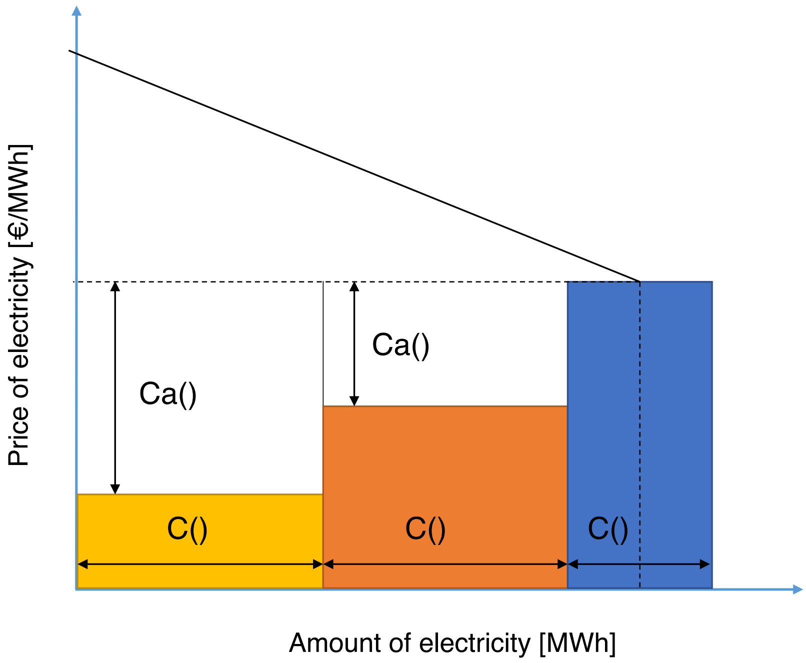


Figure 1 Merit order curve and electricity price

4 The Conventional Electricity Market with Investment Costs

4.1 The Primal Model

In the next step, the model is expanded to include the investment costs. This is the usual procedure in models describing the power plant expansion planning. The capacity of the power plants becomes a variable. The new variable multiplied with a cost factor forms a second term in the objective function.

$$
\begin{aligned}
&\underset{C(pl),E(pl,tm)}{\text{minimize}} \quad && \sum_{pl} Inv_C(pl)\; C(pl) \; + \sum_{pl,tm} FC_{EB}(pl)\; E(pl,tm) && \\
&\text{subject to} && && \\
&\forall\; pl, tm && C(pl) - E(pl,tm) \geq 0 && Ca(pl,tm) \\
&\forall\; tm && \sum_{tm} E(pl,tm) \geq D(tm) && EB(tm)
\end{aligned}
$$

4.2 The Dual Model

The dual model is now accordingly.

$$
\begin{aligned}
&\underset{EB(tm)}{\text{maximize}} \quad && \sum_{tm} EB(tm) D(tm) && \\
&\text{subject to} && && \\
&\forall\; pl && Inv(pl) - \sum_{tm} Ca(pl,tm) \geq 0 && C(pl) \\
&\forall\; pl,\; tm && FC_{EB}(pl) + Ca(pl,tm) - EB(tm) \geq 0 && E(pl,tm) \\
&\forall\; pl,\; tm && Ca(pl,tm) \geq 0 && Slack1(pl,tm) \\
&\forall\; tm && EB(tm) \geq 0 && Slack2(tm)
\end{aligned}
$$

The electricity price described by $EB(tm)$ is now limited by the fuel costs and a share of the investment costs. The investment costs are added in the hour of highest demand since this will maximize the objective function. The investment cost is only assign to one hour of the year (see figure 4). In case the demand peak is a plateau the investment cost can also be distributed over many time steps. But these are degenerated solutions which are equivalent. In case a plant is installed the capacity limit will be a binding constraint and the sum of all values $Ca(pl,tm)$ for one plant will add up to the investment costs. In case of a system with a conventional plant with fuel costs and a renewable plant with only investment costs the electricity price is limited for most time steps by the fuel costs. Only if the investment costs can stay below these costs the renewable plant is installed.

5 Three Simple Storage Models

The following models highlight the impact of storage options on the energy system. At this stage the focus is on long term storage options like hydrogen stores in salt domes.

Three different models are developed:

- a model with a cyclic closure of the storage and no costs and limits on in- and output capacities
- a model with a fixed storage content in the first time step and no costs and limits on in- and output capacities
- a model with a cyclic closure of the storage and costs on the input storage capacities and no limits and costs on output capacities

The models combine intermittent renewable production technologies with a simple back-up technology. The back-up technology is without any capacity limit and is only characterized by fuel costs. The storage technology is an ideal storage technology, without any in- or output efficiencies and losses. These characteristics are certainly very important when it comes to detailed evaluations but for the purpose of this paper the simplification is justified as will be discussed later.

5.1 The first primal storage model

The first model has a cyclic storage constraint, which means that the storage unit has the same state of charge at the beginning and the end of the investigated period. Power is mainly supplied by intermittent renewable sources, which are characterized by a time series of the capacity factors. A back-up power plant with infinite capacity and fixed energy costs is available. Only the storage capacity has costs, the input and output to the store is for free.

$$\underset{C(pl),C_{st},Eb_{up}}{\text{minimize}} \quad \sum_{pl} C(pl)\ Inv_C(pl) + C_{st}\ Inv_{St} + \sum_{tm} Eb_{up}(tm)\ FC_{EB}$$

subject to

$$\forall\ pl, tm \qquad Ts(pl,tm)\ C(pl) - E(pl,tm) \ = 0 \qquad Ca(pl,tm)$$

$$\forall\ tm \qquad \sum_{pl} E(pl,tm) + Eb_{up}(tm) - Est_{in}(tm) + Est_{out}(tm) = D(tm) \qquad EB(tm)$$

$$\forall\ t \qquad Est(t) - Est(t-1) - Est_{in}(t) + Est_{out}(t) = 0 \qquad EstB(t)$$

$$\forall\ tm \qquad Est(tm) \geq 0 \qquad EstiL(tm)$$

$$\forall\ tm \qquad Est(tm) \leq C_{st} \qquad EstuL(tm)$$

5.2 The first dual storage problem

The dual model is as follows:

$$
\begin{aligned}
&\underset{Eb(tm)}{\text{maximize}} && \sum_{tm} EB(tm)\; D(tm) && \\
&\text{subject to} && && \\
&\forall\, pl && Inv_C(pl) - \sum_{tm} Ts(pl,tm)\; Ca(pl,tm) \geq 0 && C(pl) \\
& && Inv_{St} + \sum_{tm} EstuL(tm) \geq 0 && C_{st} \\
&\forall\, pl,\; tm && Ca(pl,tm) - EB(tm) \geq 0 && E(pl,tm) \\
&\forall\, tm && EB(tm) + EstB(tm) = 0 && Est_{in}(tm), Est_{out}(tm) \\
&\forall\, tm && FC_{EB} - EB(tm) \geq 0 && Eb_{up} \\
&\forall\, tm && -EstB(tm) + EstB(tm+1) - EstlL(tm) - EstuL(tm) \geq 0 && Est(tm) \\
&\forall\, tm && EstlL(tm) \geq 0 && Slack1(tm) \\
&\forall\, tm && -EstuL(tm) \geq 0 && Slack2(tm)
\end{aligned}
$$

The price of electricity or more general of energy is given by $EB(tm)$. This quantity is confined by the price of the back-up technology and the investment costs of the power plants. The combination of equation Est_{in} and Est_{out} links $Eb(tm)$, the electricity or energy price with $EstB(tm)$. This again means that the volatility of electricity price is confined by the storage equation $Est(tm)$. The equation levels the electricity prices. Jumps in the electricity price occur only if the upper or lower limit of the storage capacity is reached. In a large more seasonal storage facility this will happen only at very few times in the year. In a real seasonal storage this would only happen twice depending on the power supply mix. In case the system is more dominated by wind, the storage is filled in winter and empties during the summertime. In case of solar based the system is exactly the other way around. This will be illustrated in scenario 3 (see figure 6).

The jump in the electricity price is given by the investment costs for the storage capacity. Since it is expected that the capacity costs of large hydrogen storage systems are rather low compared to many other storage options this means that the price difference is comparable small. In case of a storage facility, which is frequently filled and emptied, the price for the capacity will be distributed over many time steps, if in these time steps the storage boundaries are reached.

5.3 The second primal storage model

The model is the same as the first, with the only exception that the storage is not operated in a cyclic fashion. The storage has a fixed content in the beginning and no constraint at the end. The state of charge in the beginning is for free.

$$
\begin{aligned}
&\underset{C(pl),C_{st},Eb_{up}}{\text{minimize}} && \sum_{pl} C(pl)\; Inv_C(pl) + C_{st}\; Inv_{St} + \sum_{tm} Eb_{up}(tm)\; FC_{EB} && \\
&\text{subject to} && && \\
&\forall\, pl,tm && Ts(pl,tm)\; C(pl) - E(pl,tm) \;= 0 && Ca(pl,tm) \\
&\forall\, tm && \sum_{pl} E(pl,tm) + Eb_{up}(tm) - Est_{in}(tm) + Est_{out}(tm) = D(tm) && EB(tm) \\
&\forall\, t && Est(t) - Est(t-1) - Est_{in}(t) + Est_{out}(t) = 0 && EstB(t) \\
&\forall\, tm && Est(tm) \geq 0 && EstiL(tm) \\
&\forall\, tm && Est(tm) \leq C_{st} && EstuL(tm) \\
& && Est(\text{"0"}) = St_{ini} && EstIni
\end{aligned}
$$

5.4 The second dual storage model

$$
\begin{array}{lll}
\underset{Eb(tm),EstIni}{\text{maximize}} & \sum_{tm} EB(tm)\ D(tm) + EstIni\ St_{ini} & \\
\text{subject to} & & \\
\forall\ pl & Inv_C(pl) - \sum_{tm} Ts(pl,tm)\ Ca(pl,tm) \geq 0 & C(pl) \\
 & Inv_{St} + \sum_{tm} EstuL(tm) \geq 0 & C_{st} \\
\forall\ pl\ tm & Ca(pl,tm) - EB(tm) \geq 0 & E(pl,tm) \\
\forall\ tm & EB(tm) + EstB(tm) = 0 & Est_{in}, Est_{out} \\
\forall\ tm & FC_{EB} - EB(tm) \geq 0 & Eb_{up} \\
 & EstB(1) - Estini \geq 0 & Est(\text{"0"}) \\
\forall tm/\{tmax\} & -EstB(tm) + EstB(tm+1) - EstlL(tm) - EstuL(tm) \geq 0 & Est(tm) \\
 & -EstB(tmax) - EstlL(tmax) - EstuL(tmax) \geq 0 & Est(tmax) \\
\forall\ tm & EstlL(tm) \geq 0 & slack1(tm) \\
\forall\ tm & -EstuL(tm) \geq 0 & slack2(tm)
\end{array}
$$

The model is in most of the aspects the same as the first with the only exception that the first and last time step needs special alterations. Still, it does not really change the argument given before. Storage's will of course neither be operated in a complete cyclic structure nor start from scratch in the beginning of the time period with an arbitrary state of charge. The reality will certainly be much closed to the cyclic solution.

5.5 The third primal storage model

The third model is very much the same as the first with just a price for the storage input. This leads to an extra entry in the objective function and new constraints on the installed capacity which limits the input to the storage unit.

$$
\begin{array}{lll}
\underset{C(pl),C_{st},Eb_{up}}{\text{minimize}} & \sum_{pl} C(pl)\ Inv_C(pl) + C_{st}\ Inv_{St} + C_{Stin}\ Inv_{Stin} + \sum_{tm} Eb_{up}(tm)\ FC_{EB} & \\
\text{subject to} & & \\
\forall\ pl,tm & Ts(pl,tm)\ C(pl) - E(pl,tm)\ = 0 & Ca(pl,tm) \\
\forall\ tm & \sum_{pl} E(pl,tm) + Eb_{up}(tm) - Est_{in}(tm) + Est_{out}(tm) = D(tm) & EB(tm) \\
\forall\ t & Est(t) - Est(t-1) - Est_{in}(t) + Est_{out}(t) = 0 & EstB(t) \\
\forall\ tm & Est(tm) \geq 0 & EstiL(tm) \\
\forall\ tm & Est(tm) \leq C_{st} & EstuL(tm) \\
\forall\ tm & Est_{in}(tm) \leq C_{Stin} & EstCin
\end{array}
$$

5.6 The third dual storage problem

The dual solution is as follows:

$$\underset{Eb(tm)}{\text{maximize}} \quad \sum_{tm} EB(tm)\; D(tm)$$

subject to

$$\forall\, pl \qquad Inv_C(pl) - \sum_{tm} Ts(pl,tm)\; Ca(pl,tm) \geq 0 \qquad C(pl)$$

$$Inv_{St} + \sum_{tm} EstuL(tm) \geq 0 \qquad C_{st}$$

$$Inv_{Stin} + \sum_{tm} EstCin(tm) \geq 0 \qquad C_{stin}$$

$$\forall\, pl\; tm \qquad Ca(pl,tm) - EB(tm) \geq 0 \qquad E(pl,tm)$$

$$\forall\, tm \qquad EB(tm) + EstB(tm) + EstCin(tm) \geq 0 \qquad Est_{in}(tm)$$

$$\forall tm \qquad - EB(tm) - EstB(tm) \geq 0 \qquad Est_{out}(tm)$$

$$\forall tm \qquad - EstB(tm) + EstB(tm+1) - EstlL(tm) - EstuL(tm) \geq 0 \qquad Est(tm)$$

$$\forall\, tm \qquad FC_{EB} - EB(tm) \geq 0 \qquad Eb_{up}(tm)$$

$$\forall\, tm \qquad EstlL(tm) \geq 0 \qquad slack1(tm)$$

$$\forall\, tm \qquad - EstuL(tm) \geq 0 \qquad slack2(tm)$$

$$\forall\, tm \qquad - EstCin(tm) \geq 0 \qquad slack3(tm)$$

The equation Est_{in} is altered compared with the first model. The electricity needs to recover the investment costs of the storage input capacity as indicated in equation C_{stin}. $EstCin(tm)$ is always non-zero when the capacity limit is reached. This leads of course to a change in the electricity price and an alternating electricity price. This needs more theoretical investigation in future.

6 Interpretation of Dual Problem as Electricity Price

The following chapter shows for some example scenarios the influence of different configurations of the generation system on the results of the dual solution of the demand constraint. With the help of a simple optimization model, the solutions of the scenarios are calculated and analyzed.

6.1 Description of energy model and input data

The simple model represents a power system for one year in hourly resolution. The relationships are formulated mathematically in terms of linear programming (LP). The solution of the LP optimization problem determines the cost-optimal deployment and expansion of available technologies to meet a given demand. In addition, the expansion of storage technologies is also available. The storage capacity is formulated as a cyclic constraint (compare section 5.1). The optimization can freely choose the state of charge at the beginning and end of the investigated period, but they must be the same. In addition to the cost assumptions, the different specifications of the scenarios can also be implemented by formulating constraints on the expansion capacities. The system is described by means of the following input variables:

- Investment costs and capacity limits for the generation technologies
- Investment costs and capacity limits for the storage technologies
  (charging, discharging and storage capacity)
- Operating costs for generation and storage technologies
- Generation characteristics for fluctuating renewable technologies
- Demand

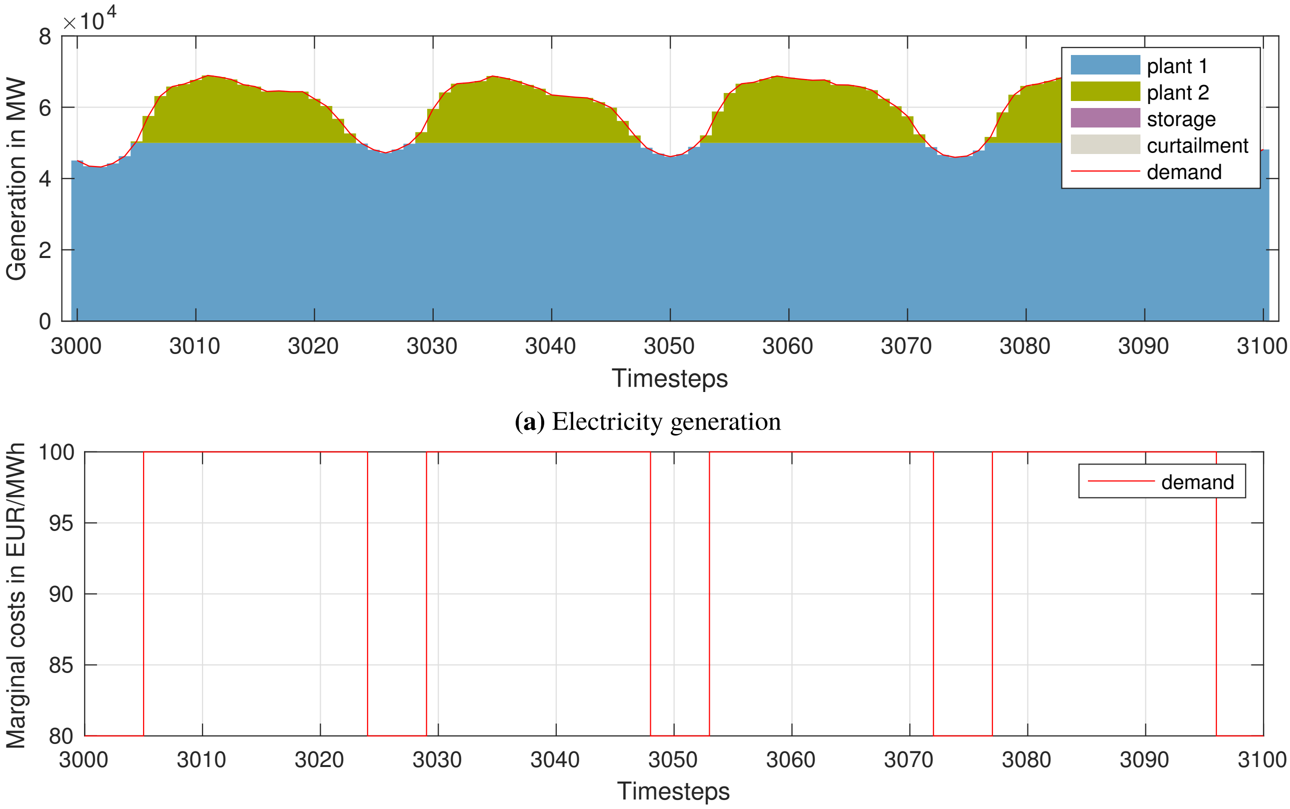
The result of the optimization calculation is then the cost-optimal configuration of expansion and deployment if all available technologies are used in compliance with the constraints.

6.2 Definition of example scenarios

One year is calculated for each of the example scenarios. The electricity demand in Germany from the year 2019 is used as demand [1]. The characteristics for the generation from wind and photovoltaics are also values for Germany from 2019 [1]. Both time series show seasonal behavior, with wind generating more electricity in winter and photovoltaics in summer.

6.3 Scenario 1 - Existing conventional power plants

Scenario 1 represents the situation in which renewable energies did not yet play a major role and the power supply was essentially dominated by existing, conventional power plants. The energy models were often operated as pure dispatch planning with predefined capacities. Thus, the investment costs of the power plants play no role in determining the cost-optimal operation (compare section 3.1). To make the example very simple, two generation options are available to meet the demand, each assumed to have a capacity of 50 GW. The variable costs of the options are 80 EUR/MWh for the type *plant 1* and 100 EUR/MWh for the type *plant 2*. The option storage is not available here. Figure 2a now shows the cost-optimal deployment of the two conventional generator types for hours 3,000 to 3,100. As was to be expected, the lower-cost generation option *(plant 1)* is deployed first, up to its capacity limit of 50 GW. If demand is higher, the second option *(plant 2)* is also used accordingly.



(a) Electricity generation

(b) Dual solution of the demand equation

Figure 2 Generation and dual solution of scenario 1 for the time steps 3,000 to 3,100

The marginal cost of demand in terms of the dual solution of the demand equation of the linear optimization problem is shown for the time steps considered in figure 2b. For the areas where only the option *plant 1* is used, it shows the value 80 EUR/MWh, which corresponds to the variable costs of this power plant. If *plant 2* is also required, the marginal costs are correspondingly 100 EUR/MWh. This scenario therefore simulates the functioning of the energy only market, whose pricing is described via the so-called merit order. In the respective hour, the most expensive power plant still required for covering the demand determines the marginal costs, which can be interpreted as the current electricity price.

If the marginal costs of demand are now multiplied by the respective demand for each hour and then summed over the entire year, the total revenues earned by the two generation types in the electricity market simulated here are obtained. The type *plant 2* always takes in 100 EUR/MWh when it is deployed. The revenues are therefore exactly equal to the expenses since the variable costs are also equal to this value. With *plant 1*, the situation is different. Whenever the demand is higher than 50 GW and thus *plant 2* sets the price, the revenues amount to 100 EUR/MWh, whereas the variable costs are only 80 EUR/MWh. Thus, there are additional revenues or contribution margins for *plant 1*, which are needed to cover the fixed costs.

In this example, the annual sum of the revenues does not correspond to the expenses but differs by the additional revenues of *plant 1*. The expenses in turn are the costs, which the value of the objective function of the optimization shows.

6.4 Scenario 2 - New built conventional power plants

The second example now examines the effects of the expansion option. As in scenario 1, only the two conventional generation options are available. A storage facility cannot be used. In contrast to the first example, however, the installed capacity of the individual technology options is no longer specified but is itself part of the optimization (compare section 4.1). Investment costs for the individual options are now added to the variable costs. For the type *plant 1* these are assumed to be 40,000 EUR/MW/a and 25,000 EUR/MW/a for *plant 2*. The optimization now determines the most cost-effective combination of expansion and operation of the individual technologies.

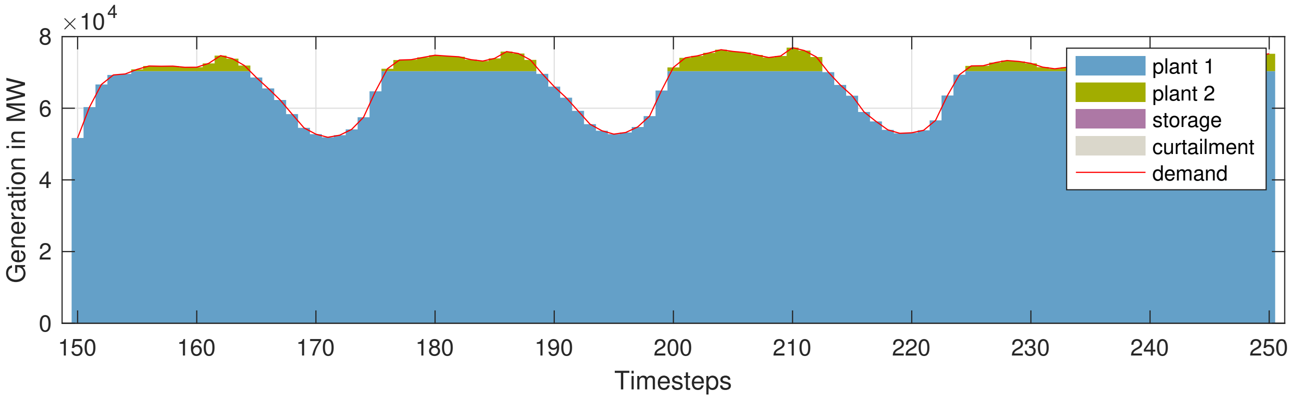
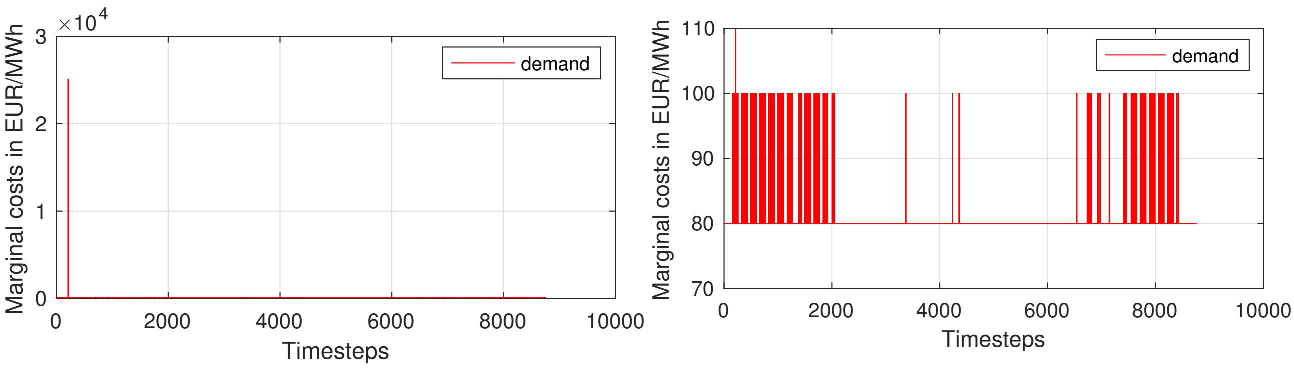


Figure 3 Electricity generation in scenario 2 for the time steps 150 to 250

The result initially shows a total installed capacity that corresponds exactly to the maximum demand. Since in this simple example no further effects such as outages or maintenance are taken into account and thus the power plants are always available, this is a logical result. Additional capacity would only add further costs. The share of installed capacity of type *plant 1* is 91.5% and that of type *plant 2* only 8.5%. Nevertheless, both technologies are used and thus represent the most cost-effective option in each case. Two conditions are necessary for this kind of result, both are fulfilled in this example. First, both technologies must prove to be more favorable in one cost area each, investment costs or operating costs. If there were one technology with favorable investment costs and favorable variable costs, only this technology would be selected. On the other hand, this only occurs if demand is not constant and thus power plants are used for different lengths of time during the year according to the merit order model. If the utilization period is short, the favorable operating costs then dominate, and if the utilization period is long, the favorable variable costs dominate. Thus, each type of power plant is cost-optimal for a specific utilization range. Figure 3 now shows the generation in analogy to scenario 1, this time for the hours 150 to 250. Again, the dispatch takes place according to merit order. The more expensive option is only used if the less expensive option is used at the capacity limit and the demand cannot yet be met.



(a) Dual solution of the demand equation for the entire year

(b) Dual solution of the demand equation - zoomed axis

Figure 4 Generation and dual solution of scenario 1

Figure 4 now again shows the dual solution of the demand constraint and thus the marginal cost of electricity generation. On the left side (4a) we see a very high peak relative to the beginning of the period under consideration. This value occurs in the hour of peak load and is 25,100 EUR/MWh. Also in this hour, the marginal costs describe how the objective function value would change with an increase of one unit (1 MWh). Since this is the hour of peak load, an additional unit of capacity would need to be built. The less expensive option in this case would be *plant 2* at 25,000 EUR/MW/a. Likewise, the electricity for this unit would have to be generated in a one-hour time frame, which would incur further costs of 100 EUR/MWh. In total, this results in the value in the diagram. In order to better recognize the lower values in the diagram, the y-axis on the right side (4b) is scaled accordingly. Here, the change between 80 and 100 EUR/MWh already known from scenario 1 is shown, which depends on the level of demand at this point in time and the associated price-setting power plant. If the revenues of the two generation options on the electricity market are also determined in scenario 2 by multiplying the demand by the dual solution and then summing them, these correspond exactly to the expenses, i.e., the objective function value of the optimization. Thus, there is no additional revenue here. If the calculation is made separately for the two types of generation, both technologies take exactly the amount on the electricity market that they each need for the investment costs for the capacity of the plants and the variable costs for electricity generation. To analyze this in more detail, we first focus on *plant 2*. Over the course of the year, this generation option does not receive any additional revenue from the electricity market beyond its variable costs (compare scenario 1) and thus has no way to recover investment costs. The technology must therefore earn all capacity costs in the hour of greatest scarcity, i.e., at load maximum. In this hour, the option is fully utilized and earns all investment costs in this one hour due to the marginal costs of 25,000 EUR. Overall, the balance of expenditures and revenues for *plant 2* is thus balanced. The situation is somewhat different for *plant 1*. This type also earns a lot in the hour of maximum load, but exactly 15,000 EUR are missing per MW, the difference of the investment costs of both options. Thus, *plant 2* must collect the missing amount through additional revenues over the year. Since the difference in variable costs is 20 EUR per MWh, the additional revenue is the therefore 20 EUR per MW and hour. So, in order to compensate for the 15,000 EUR additional costs in the area of investments, *plant 1* has to generate additional revenues in 750 hours. The distribution of the outputs of both options adjusts exactly to the value which *plant 2* sets the price in 750 hours. The generation costs of both options are identical at this utilization period.

6.5 Scenario 3 - New built renewable power plants with storage

Scenario 3 now examines a future energy system in which generation essentially consists of fluctuating renewable energies. For the seasonal balancing of the generation profiles, a storage technology with low costs for the storage capacity is available as an option, which is intended to reflect hydrogen storage (generation, underground storage and subsequent re-conversion to electricity). In addition, a back-up power plant is available.
In order to be able to analyze the influence of the various parameters of the storage technology, the scenario is divided into three sub-scenarios (3a to 3c).

6.5.1 Assumptions and preliminary considerations

Instead of the conventional power plants from scenarios 1 and 2, the renewable options wind and photovoltaics (PV) are now available. The potential is assumed to be unlimited. The technologies are assumed to incur annuity investment costs of 50,000 EUR/MW/a (wind) and 30,000 EUR/MW/a (PV). For the storage option, the cost per year for storage capacity is assumed to be 10 EUR/MWh/a [3]. The cost for the charging and discharging power is treated differently in scenarios 3a to 3c and described in the sub-scenarios. For simplicity, variable costs are not incurred for both renewable technologies and the storage. The back-up power plant is assumed to already exist and thus no investment costs are applied. The variable costs amount to 100 EUR/MWh.

In principle, renewables only play a role in scenario 3 if their levelized cost of electricity (LCOE) are lower than the variable costs of the back-up power plant, in this scenario below 100 EUR/MWh. With the assumed costs for wind and PV, this would occur with a number of full load hours of 500 h (wind) and 300 h (PV), which is true in both cases for the assumed site conditions. However, this consideration assumes the ability to fully utilize renewable generation. Beyond a certain installed capacity, however, this is no longer possible because generation exceeds demand at many points in time. This excess generation is then either not used (curtailment), which successively reduces the number of full load hours of the respective technology, or it can be used later with the help of storage, which, however, causes further investment costs. The combination of renewable generation plus storage is therefore only economical up to the point where the LCOE of this combination are still below the variable costs of the backup power plant.

6.5.2 Scenario 3a - No storage losses and no investment costs for charging and discharging

In scenario 3a, the costs for the storage capacity for charging and discharging energy are not taken into account (compare section 5.1). Accordingly, they are assumed to be available indefinitely. Likewise, no storage losses are assumed to occur in this sub-scenario, so the storage efficiency is 100%.
As the preliminary considerations suggest, renewable energy is used in this scenario. The optimization determines a combination of wind and storage as the cost-optimal generation. The backup power plant is not used. The total amount of energy is therefore provided by wind and thus has no variable costs.

The curve of the storage level is shown in figure 5. It shows the expected balancing of the seasonal behavior of the wind. In winter, the higher wind production leads to a charging of the storage, whereas in summer the storage is discharged again. At about time step 1,800 the storage is full, at about time step 6,500 the storage is empty.

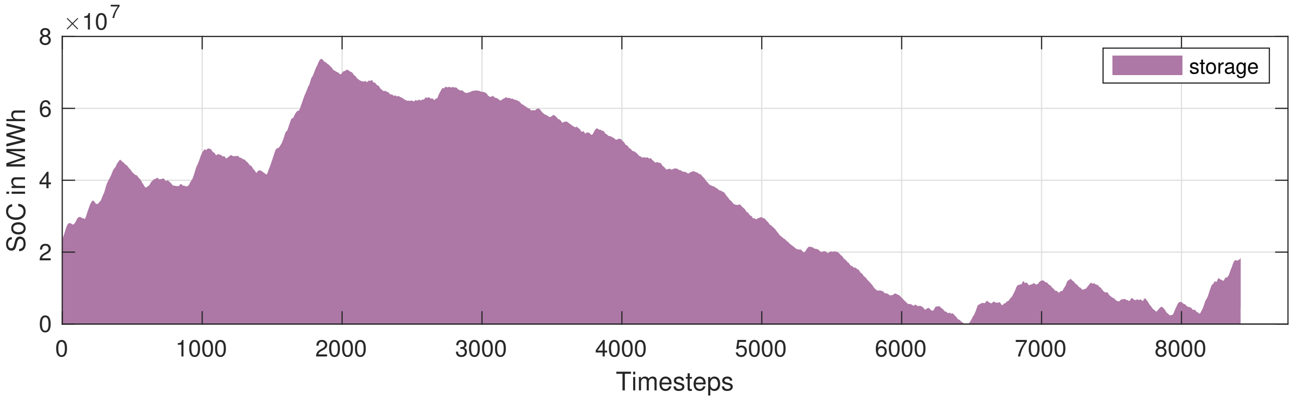


Figure 5 State of charge of the storage unit

Figure 6 now shows the dual solution of the demand equation. Despite the fluctuations in wind and demand, there are no short-term fluctuations in the dual solution, but only two cost levels. The levels change exactly at the times when the storage is either full or empty. The higher marginal cost level is present when the storage is discharged, which in this example is the case in summer. The difference between the two levels corresponds to 10 EUR/MWh, which is accurately the cost of the storage capacity (compare section 5.2).

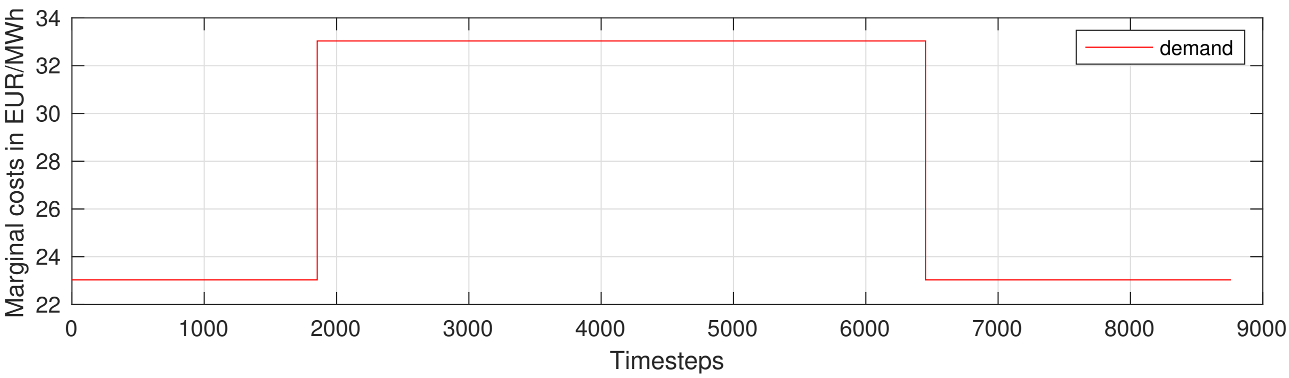


Figure 6 Dual solution of the demand equation

In scenario 3a, the revenues on the electricity market are also determined. Analogous to the first two scenarios, the demand is multiplied by the dual solution and then summed. This value again corresponds exactly to the expenditure, i.e., the objective function value of the optimization. Thus, there is no additional revenue in this case either. In this case, too, the two types of generation can be considered separately. For wind, the respective output in the time step is multiplied by the demand and then all time steps are summed up. The net revenues of the storage are the difference between the expenses for charging and the revenues for discharging. Both technologies again take exactly the amount on the electricity market that they each need for the investment costs of the plants. For the interpretation of the hourly values, we first consider a point in time in winter. Here, the price level is lower than in summer and amounts to about 23 EUR/MWh. If the demand were increased by one unit at this time, more wind would have to be installed to produce this additional unit. This would cause additional costs of about 26.5 EUR/MWh, which is above the marginal costs occurring here. However, since the additional load occurs in winter, this fits better with the characteristics of wind generation, so to speak, and reduces the specific need for storage capacity per unit of wind generation. Therefore, the marginal cost here is below the cost of wind generation. At some point in the summer, the opposite would occur. In addition to the cost of generation, there would also be a proportionally higher cost of storage, which would increase the imbalance of generation and consumption and thus increase the need for storage.

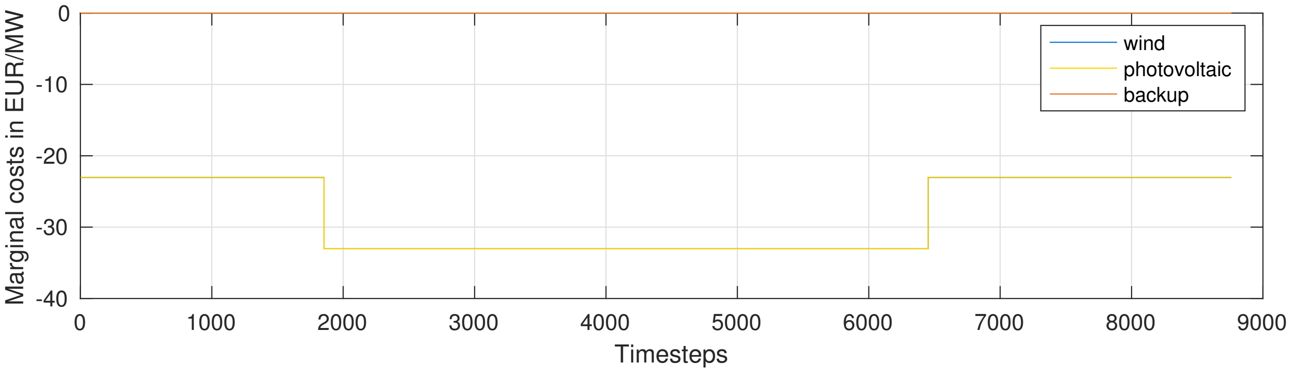


Figure 7 Dual solution of the capacity equation - generation units

For further elaboration and understanding, figure 7 shows the dual solution of the constraints for the generators' capacity limits. Here, the values for wind correspond exactly to the values of the dual solution for demand, but with a negative sign. If the values are now multiplied by the wind capacity at each time step and then summed, the result is exactly the investment cost for the wind turbines. Now, in this example, the wind generation is exactly equal to the demand. Both are weighted with a quasi-identical price curve, but the total cost calculated is the cost of wind and storage in the first case and only the cost of wind in the second case. The difference, which means the storage cost, must therefore result from the different characteristics of load and wind. In winter, when marginal costs are low, comparatively much energy is generated, and in summer, when costs are high, rather little. The weighted sum is therefore lower than that of the dual solution of the load condition, which has hardly any seasonality in demand.

An interpretation of the hourly values is also possible for figure 7. The values show how the objective function would change if the wind turbine could produce one unit more here (1 MWh). As a result, this means exactly the opposite case, as hourly interpretation of the dual solution of the load constraint, which finally leads to the identical numerical values, but reversed signs. This also applies to the dual solution of photovoltaics. It is not built in this scenario, but an increase of one unit in the respective hour would have the same effect as with wind energy and thus also the same pattern of marginal costs. Since the curves of wind and photovoltaics lie on top of each other in figure 7, the result here is the mixed color green.

6.5.3 Scenario 3b - No storage losses and investment costs for charging and discharging

In contrast to the previous scenario, in 3b investment costs of 25,000 EUR/MW/a each are now applied for the charging and discharging power of the storage (compare section 5.5). In general, this makes the combination of renewable generation and storage more expensive and especially the integration of high peaks is very cost intensive. Since the expansion options are still not limited in this example, no additional revenues occur for certain technology options.

The optimization result shows some changes compared to scenario 3a. On the one hand, photovoltaics also finds a place in the portfolio, and on the other hand, the back-up power plant is also used. In addition, the cost of charging and discharging power makes full integration uneconomical, so generation is curtailed at some points in time. Figure 8 shows the coverage of demand by each option for time steps 1,350 to 1,500.

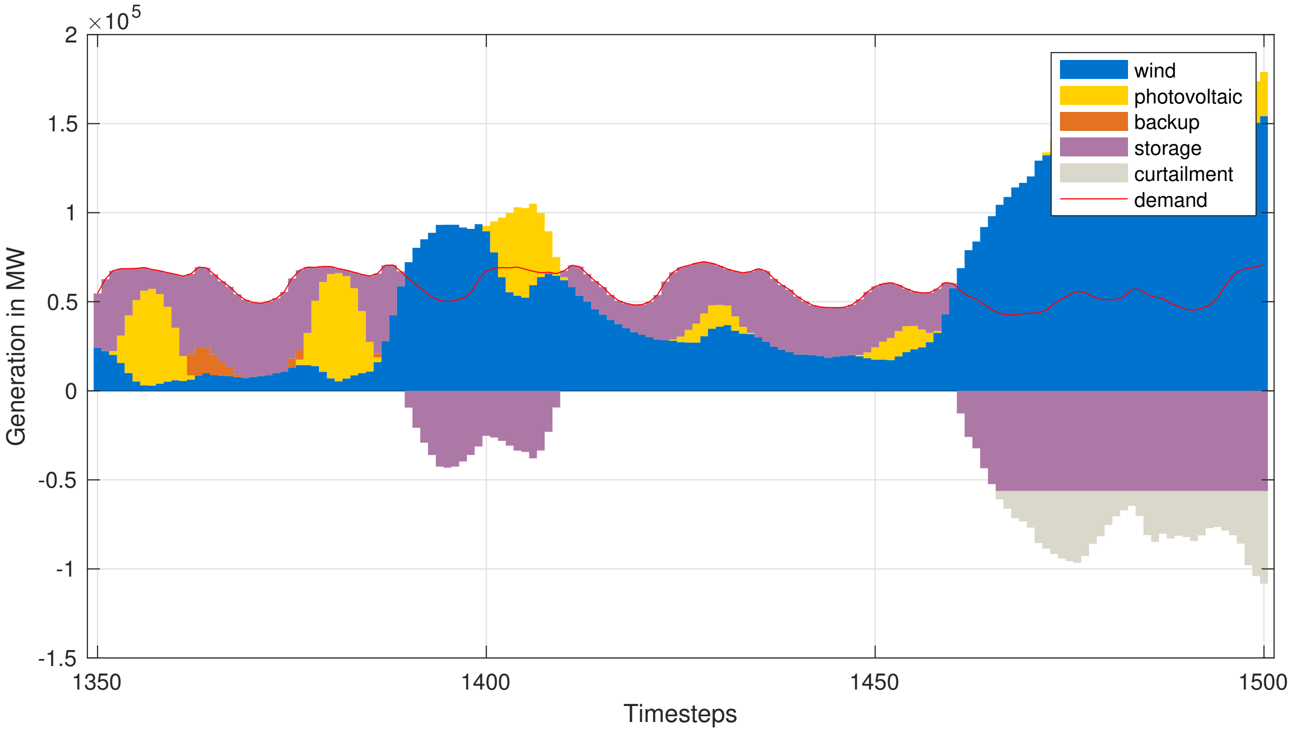


Figure 8 Electricity generation - time steps 1,350 to 1,500

For the same time range, Figure 9 shows the dual solution to the demand constraint. In principle, the dual solution shows the same behavior with two seasonal prices. However, the use of the backup power plant and the curtailment of surplus leads to differences. In the time steps from 1,350 to 1,500, four time periods occur in which the backup power plant is used. At these times, the marginal costs jump to 100 EUR/MWh, which is the variable cost of this power plant. From the end of the section, a high wind generation occurs, which is not fully integrated and thus is curtailed. At these times, the marginal costs fall to 0 EUR/MWh.

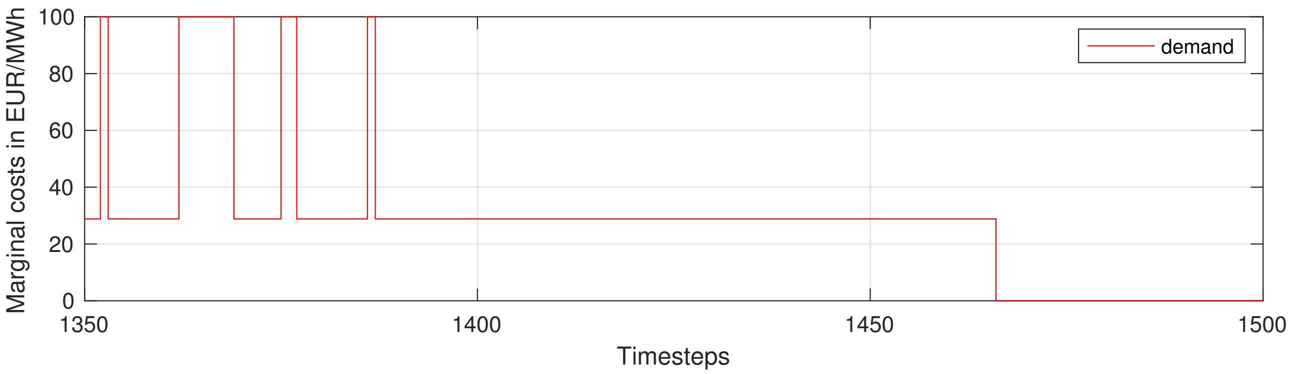
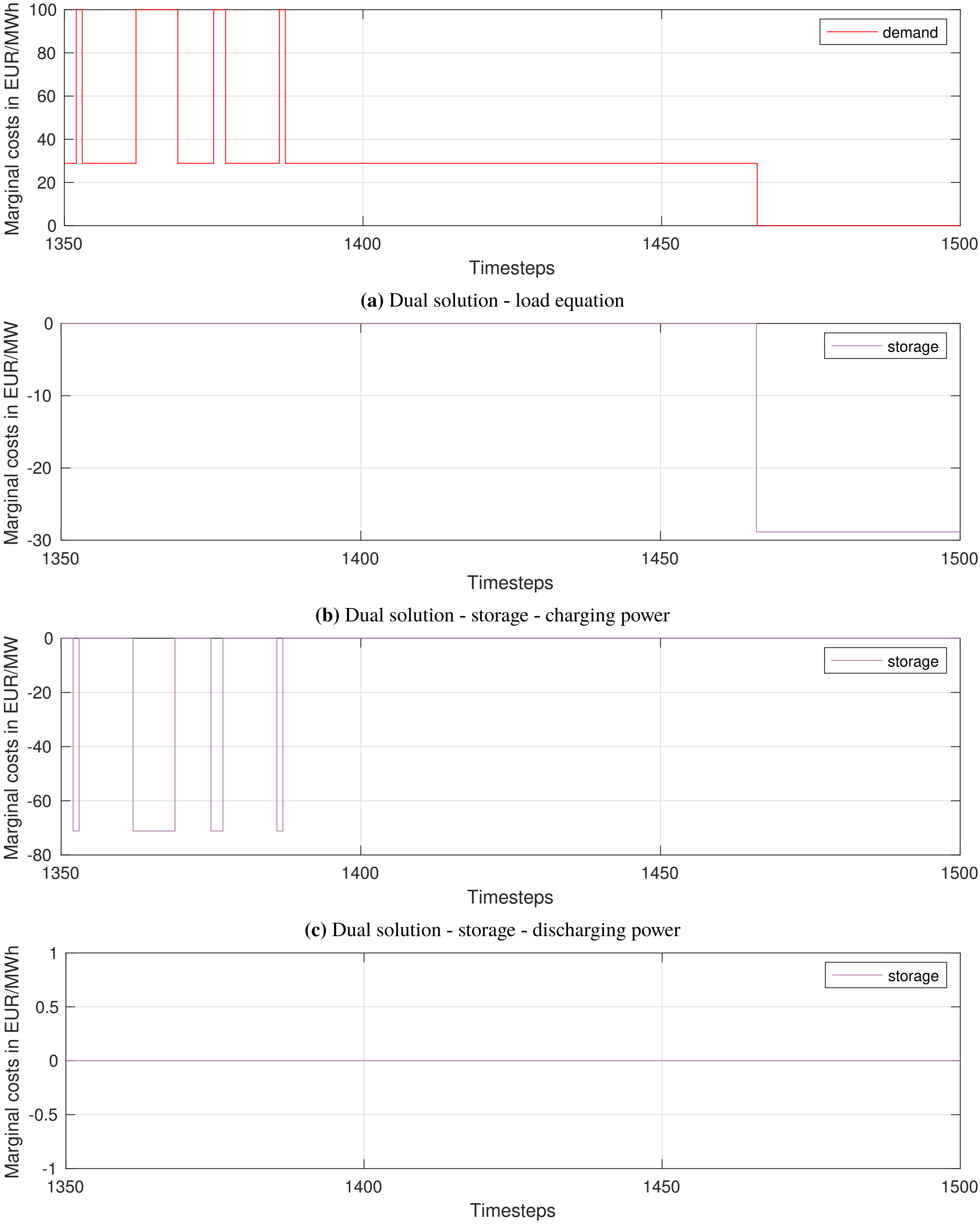


Figure 9 Dual solution of the load equation - time steps 1,350 to 1,500

The hourly interpretation of these changes is comparatively simple. If the backup power plant is used, another unit of load would also be covered by it and generate corresponding costs. If renewables are curtailed, one unit of additional load would lead to less curtailment. This would not generate any additional costs. The marginal costs are therefore 0 EUR/MWh in this case.



(a) Dual solution - load equation

(b) Dual solution - storage - charging power

(c) Dual solution - storage - discharging power

(d) Dual solution - storage - storage capacity

Figure 10 Comparison - dual solutions of demand and storage unit

The relationship between the dual solution of the demand constraint and the dual solutions to all storage limits is illustrated in figure 10. The time range for consideration is again the steps from 1,350 to 1,500. In sub-figure 10b, the dual solution for the charging power limit is plotted. The value is different from zero only

when the marginal cost of demand is 0 EUR/MWh (see figure 10a), which is equivalent to renewable curtailment occurring. At these times, the installed charging capacity is fully utilized and thus reaches the limit. In figure 10c the dual solution of the limit of the discharge power is entered. This reaches the limit when the backup power plant is used. First, the discharge of the storage is fully utilized. If the demand still cannot be met, the backup power plant is used. In the dual solution of the demand constraint, marginal costs of 100 EUR/MWh occur at these times (compare section 5.6).
The dual solution of the storage capacity limit is shown in sub-figure 10d. The value is always zero except for one time step in the period under consideration. Here, the storage only reaches the limit at the time with maximum charge, which in this scenario takes place at about time step 3,000 and is therefore not visible in this diagram. If all dual solutions of the storage limits are multiplied by the respective limit capacities, the investment costs for charging capacity, discharging capacity and storage capacity result.

6.5.4 Scenario 3c - With storage losses and with investment costs for charging and discharging

In scenario 3c, the efficiency of the storage is now reduced to 50%. Consequently, only half of the charging energy can be fed back into the system. The result of the optimization is similar to that from scenario 3b. Figure 11 again shows for the time steps 1,350 to 1,500 the coverage of the load with the different technology options.

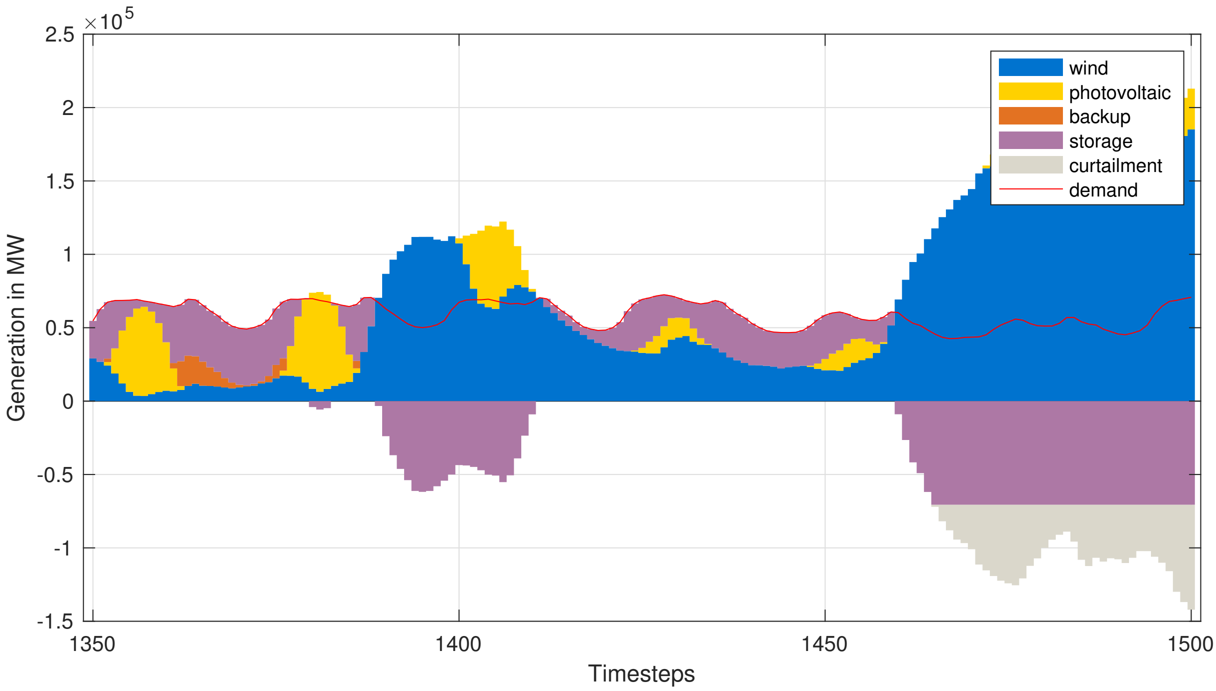
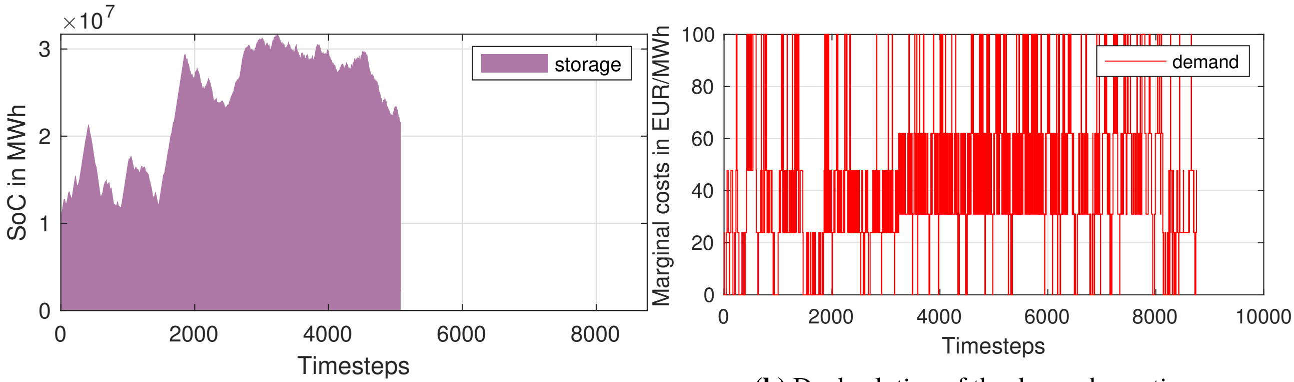


Figure 11 Electricity generation - time steps 1,350 to 1,500

Figure 12b shows the dual solution of the demand constraint over the entire period under consideration. First, cost peaks are again shown here with the use of the backup power plant and the fall to 0 EUR/MWh with renewable curtailment. Likewise, there are again two seasonal price levels, which, however, no longer have a constant value, but move between an upper limit and a lower limit. The seasonal limits are again characterized by the maximum and minimum state of charge of the storage (compare figure 12a). Also in this case, multiplying the time series by the demand and then summing up results in the total system cost.

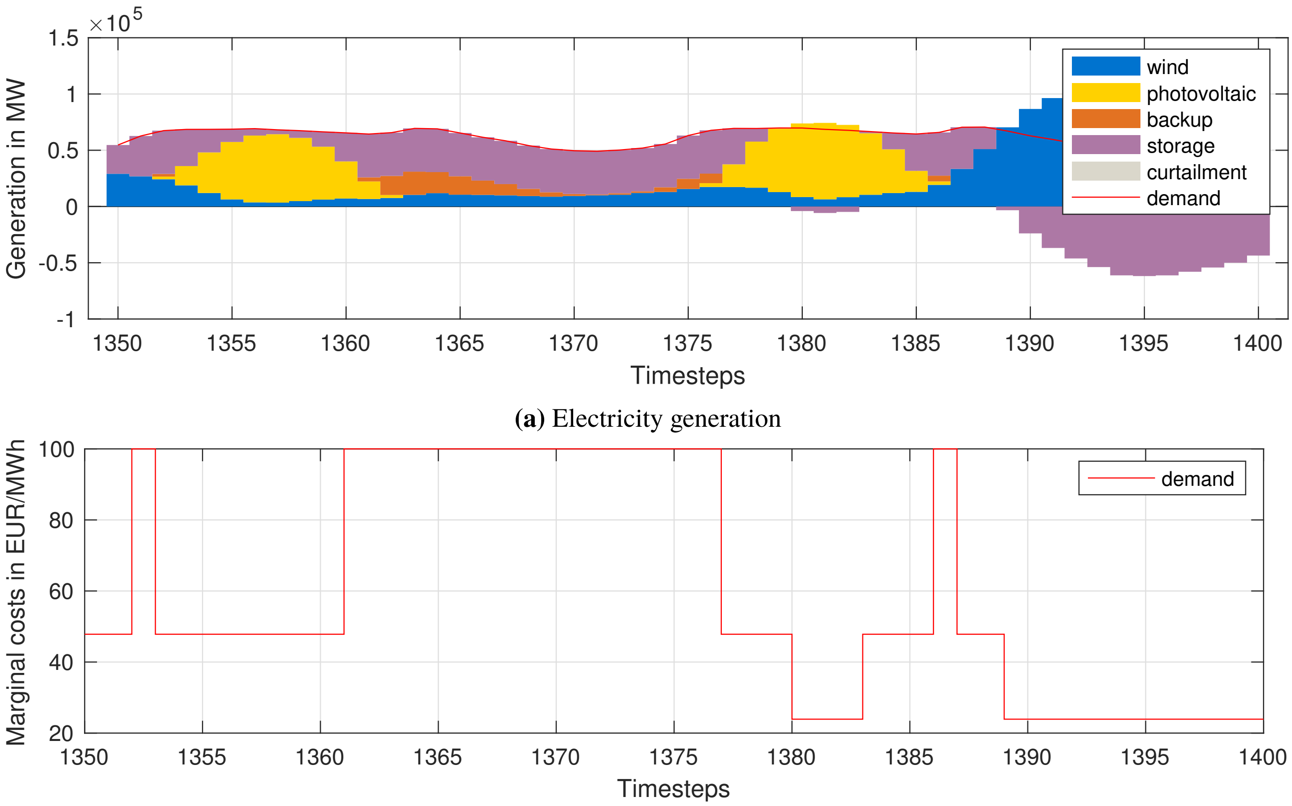


(a) State of charge of the storage unit

(b) Dual solution of the demand equation

Figure 12 Storage state of charge and dual solution of demand - scenario 3c

This behavior is shown in more detail in figure 13 for the time steps 1,350 to 1,400. In connection with the generation also shown in this figure (13a), the relationship becomes clear. If the storage is discharged, the upper cost level occurs for the marginal costs, if it is charged, the lower level occurs. The upper cost level is exactly twice as high as the lower one, which corresponds to the efficiency of the storage (50%). If the demand were increased by one unit in the discharging mode, two units would have to be charged beforehand. If the increase occurred in charge mode, one unit would be charged, which explains the discrepancy by a factor of 2, which in turn reflects efficiency. Thus, the higher the efficiency of the storage, the closer the upper and lower limits of the cost level. In the limiting case at 100% efficiency, both levels converge together, as in scenario 3b.



(a) Electricity generation

(b) Dual solution of the demand equation

Figure 13 Generation and dual solution of scenario 3c for the timesteps 1350 to 1400

7 Conclusion, summary and outlook

Major transformations are expected in the energy system. Electricity will become more important and synthetic fuels like hydrogen are necessary to supply all kind of energy services and to store and transport energy properly. Electricity is the quasi "primary energy" and will be the source of all other energy carriers and will certainly also determine the price of energy. The paper presents very simple electricity models with hydrogen as seasonal storage option. The models are simple linear programming models. The models are presented in primal and dual form. The dual form is used to analyze and interpret the economic parameters, especially the electricity price. A major finding - although expected - is, that the electricity price is smoothed by the storage technology. In a very simple model, it can be shown that only in the few moments when the storage capacity is reached or the storage capacity is emptied the price will change. In case of a seasonal storage system this would lead then to a summer and a winter price. This is of course altered when the storage system is modelled more in detailed and more economic considerations are considered. Still the strong smoothing mechanism remains. The analytical investigations are complemented by simple real models which show exactly the behaviour expected. Especially the dual value of the electricity demand equation can fully be reconstructed by the analysis. This needs in a next step be done for more complex models but the results are promising that a full reconstruction is possible which opens than a much better understanding of the systems with seasonal storage options. This will be especially important to compare different options to make the system more flexible. This requires of course highly integrated models, which combine many different flexibility options.

The new storage options might also make even more far-reaching changes in the design of the electricity market possible and especially the inclusion of investment costs in the bidding strategies. This is of course only an indication which needs much further investigation.

8 Acknowledgements

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