GPS network design: Logistics solution using optimal and near-optimal methods

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2 authors:

Peter Dare
University of New Brunswick

Hussain Saleh
Ghent University

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GPS network design: logistics solution using optimal and near-optimal methods

P. Dare, H. Saleh

Applied Geodesy Research Group, School of Surveying, University of East London, Longbridge Road, Dagenham, Essex RM8 2AS, UK
e-mail: peter@uel.ac.uk; Tel.: +44 20 8223 2501; Fax: +44 20 8223 2918

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Abstract. Although permanently installed global positioning system (GPS) networks are necessary when investigating many geophysical descriptors, there is still a need for epoch surveys consisting of temporarily placing GPS receivers sequentially at pre-chosen points. It is shown how the cost of carrying out an epoch survey can be reduced by using techniques within the field of operational research. An epoch survey design consists partly of a list of sessions to be observed, with the length of observing time for each session also specified. The issue of how long to spend observing a session is not addressed, but cheap session schedules are identified given the required sessions and the cost of moving receivers between points. For small networks, optimal solutions are possible; however, for larger networks, non-optimal solutions based upon heuristics have to be accepted. This is necessary because for large networks the optimal solution could take days or weeks to determine, whereas heuristic methods can provide results within seconds.

Key words: GPS – Operational research – Optimal design – Heuristics – Travelling salesman problem

1 Introduction

Within the four orders of network design proposed by Grafarend (1974), the first-order problem relates to the design of the configuration of the network. Although designed originally for terrestrial networks, the first-order problem (as with the other three orders) is just as, if not more, relevant to global positioning system (GPS) surveys. Within the portfolio of the first-order design problem is the choice of the location of points within a network and the length of time to observe a session. However, the particular first-order design problem to be addressed in this paper is that relating to the *order* of the sessions to be adopted by the field survey crews. Here we define session as follows: a period of time over which two or more receivers are simultaneously recording satellite signals.

A great deal of scientific work using GPS is carried out using permanently installed GPS receivers (see e.g. Miller et al. 1998; Naito et al. 1998) which, by definition, do not require field surveyors to move GPS receivers between points to observe specific sessions. However, the concept of the epoch GPS survey is still valid; recent surveys of this type are reported in Harvey et al. (1998) and Chu et al. (1997). The possible benefit of using permanently installed GPS receivers in place of epoch surveys is discussed in Prescott and Savage (1996).

Operational research (OR) is the application of scientific methods to complex problems arising in the management of an organisation's operations in order to carry out those operations in a more efficient manner. Aspects of OR have been applied to geodetic problems in the past, but not to this GPS design problem. Cross and Thapa (1979) investigated the design of levelling networks using linear programming while Kuang (1993a) used quadratic programming. Linear programming was also used for the design of monitoring networks by Benzao and Shaorong (1995). Kuang (1993b) used quadratic programming to design 3-D engineering networks. Regarding horizontal networks, Kortesis and Dermanis (1987) developed an approach using graph theory relating to one surveyor visiting a number of points, although the method required all the work to be completed in one working day. For GPS networks, Wells et al. (1987) introduced many different aspects of GPS network design (e.g. redundancy, configuration) while Vanicek et al. (1985) presented algorithms that solved for the required number of receivers while differentiating between those making observations and those moving between points. Snay (1986) developed

Correspondence to: P. Dare

algorithms to generate schedules given different combinations of receivers and reoccupations, and Janes et al. (1986) continued this work. An approach to the solution of the second-order design problem has been developed (Evan-Tzur and Papo 1996) which includes an analysis of some geometrical aspects of the first-order design problem.

The purpose of the present paper is to show how techniques in OR can be used to solve for the GPS session schedule given a list of sessions to be observed and the cost to move receivers between points in the network. Here, a session schedule is defined as: a sequence of sessions to be observed consecutively.

The method produces guaranteed optimal solutions for small networks, and often optimal, or at least close to optimal, solutions for large networks, using the technique of Simulated Annealing (SA). Examples are given in this paper for both simulated and real networks. SA has been discussed before in the geodetic literature, by Johnson and Wyatt (1994), who used the technique to solve for the location of points in a monitoring network.

2 The logistics design problem

Once the sessions have been decided upon, the session schedule (i.e. the *order* in which the sessions will be measured) has to be determined. If S represents the number of sessions, then the number of possible session schedules is given by

$$S! \quad (1)$$

which is clearly a very large number for some networks. This paper addresses the problem of determining the optimal (according to some cost criteria) session schedule.

Let us consider the four points identified in Fig. 1, which also shows all the possible baselines (sessions) that can be measured (six in total) without repeating any observations. For two receivers, a possible schedule is as shown in Table 1. That schedule, of course, is only one schedule out of a possible 720 using Eq. (1). Another possible schedule is shown in Table 2 although, as can be seen, the schedule does not appear to be as efficient as that in Table 1 due to both receivers being moved between sessions. In each case the schedule has been arbitrarily selected.

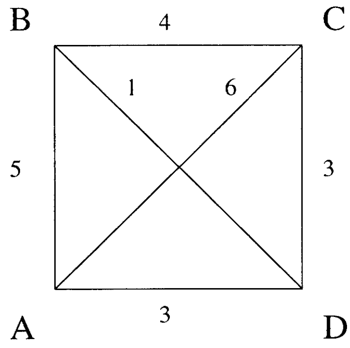

Fig. 1. Simple four point network producing a symmetric cost matrix

If the numbers in Fig. 1 represent the cost of moving a receiver in either direction between the two points, then the cost of the schedules is as shown in Table 3. For example, the cost of moving a receiver from point A to point B is 5 units; likewise for B to A. Whether the costs represent time or money is not important at this stage as the design process is being described in a general sense. Clearly, the first option (shown in Table 1) provides a cheaper solution than the second option. The challenge is to try to determine the session schedule giving the lowest cost from a specific cost matrix and a list of sessions to be observed. No attempt is made to remove

Table 1. Receiver schedule for two receivers

Session	Receiver 1	Receiver 2
1	A	B
2	A	C
3	A	D
4	B	D
5	C	D
6	C	B

Table 2. Apparently less efficient receiver schedule for two receivers

Session	Receiver 1	Receiver 2
1	A	B
2	C	D
3	A	C
4	B	D
5	D	A
6	C	B

Table 3. Comparison of schedule costs

Table	Session	Receiver 1	Receiver 2	Table total	Table	Session	Receiver 1	Receiver 2	Table total
1	1	–	–		2	1	–	–	
	2	0	4			2	6	1	
	3	0	3			3	6	3	
	4	5	0			4	5	3	
	5	4	0			5	1	3	
	6	0	1			6	3	5	
	Total	9	8	17		Total	21	15	36

(or add) sessions; it is the ordering of the sessions that is addressed here.

The movement costs can be represented by a cost matrix where each element is the cost of moving a receiver between the two relevant points. If the cost of moving between two points is independent of the direction then a symmetric cost matrix will be produced. The network in Fig. 1 will produce the symmetric cost matrix shown in Table 4, but this can be modified (see e.g. Fig. 2) to produce a more realistic non-symmetric cost matrix. In Fig. 2, the arrowed arcs indicate the direction of movement along a line while the non-arrowed lines, as before, have costs independent of the direction of movement. Thus the cost of moving from point A to point B is 5 units while the cost of moving from B to A is 4 units. In surveying, a non-symmetric cost matrix is more realistic as movement between the points usually requires a combination of driving and walking and hence uphill journeys are usually slower than downhill journeys. If a helicopter is used to move the surveyors between the points then a symmetric cost matrix may be more appropriate. A reconnaissance is usually carried out before the actual survey and it is at this time that data to enable costs to be calculated can be collected. In addition, data can be interpreted from remote-sensing imagery of the ground and other sources.

The examples illustrated so far only include costs for moving between network points. However, in practice, the surveyors will be moving from some base, e.g. their office or hotel. This is included in the problem description by adding an additional point representing the office base (OB), as shown in Fig. 3. The costs radiating from OB represent the cost to move a receiver from OB to the relevant point in the network.

In the case of a surveyor who wants to find his/her optimal route through the points, it can be solved in an

Table 4. Symmetric cost matrix generated using data in Fig. 1

	A	B	C	D
A	0	5	6	3
B	5	0	4	1
C	6	4	0	3
D	3	1	3	0

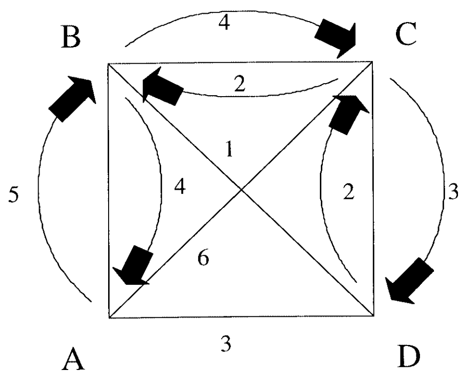

Fig. 2. Simple four point network producing a non-symmetric cost matrix

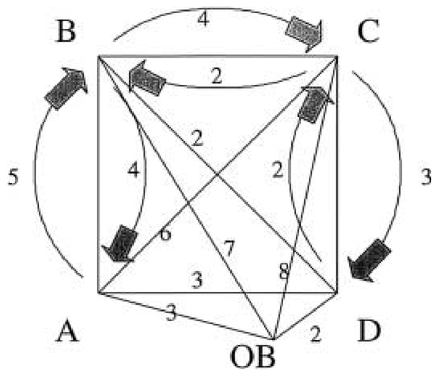

Fig. 3. Expanded simple four point network to include office base

optimal manner using many of the methods available to solve the classic Travelling Salesman Problem (TSP) in OR, e.g. the branch-and-bound process of Little et al. (1963). The TSP can be stated as: find a route that a salesman has to follow to visit *n* cities once and only once while minimising the distance travelled: the above GPS problem fits exactly into this definition where the cities represent points. In practice, however, the network to be observed is usually of sufficient size to require multiple returns to the base due to, for example, the work taking longer than one working day to complete.

In order to allow the survey to continue for more than one working period effectively means that the surveyors are going to return to and leave the office base on more than one occasion. This therefore invalidates the traditional TSP formulation which allows the salesman to visit each city once only.

In order to allow the TSP to become more generalised in terms of the problems it can model, we firstly have to redefine the problem using the more general OR terminology for points and costs. The points now become known as *nodes* and if there is a known or estimated cost for moving between two nodes in one or both directions this is known as an *arc*. This redefinition allows algorithms that solve the TSP to be used for problems that are of a more complex form.

This problem of allowing multiple returns to be base is similar to the problem addressed by Bellmore and Hong (1974). In their work, the more difficult solution of the multiple TSP (MTSP) was investigated. In the MTSP, there is more than one salesman and they have to share the visits to the cities is an optimal manner. Their solution is based upon an expansion of the TSP network to include additional arcs and additional $m - 1$ nodes, where m is the total number of salesmen available – more details are given later in this paper.

In the GPS application being described here, for each additional working period required a new node is introduced which represents the end of one working period and the start of another. The arcs are constructed as for the MTSP – the method of introduction depending upon the reason for the additional working period.

1. An additional working period is required to provide, for example, a break in the working day. In this case the arcs joining the nodes representing

the working periods should not be included. If they were, then this would allow the solution to join the nodes representing the working periods directly to each other, with the result that all the sessions are effectively observed in one working period.

2. An additional working period is required due to the amount of work to be carried out. In this case the arc joining the nodes representing the working periods again should not be included. Since this may represent staying in a hotel for an additional night, this will clearly affect the cost of the survey and so its costs need to be incorporated (i.e. the new node has a cost associated with it). This can be done by using the feature of a fixed cost for using a particular salesman that exists in the MTSP formulation. This simply involves adding half the fixed cost to all the costs of the arcs radiating from the relevant node.

In practice, the best approach to deal with the arc joining the nodes representing the working periods would be to include it, but with a low cost. In this way, the solution would not be forced to be over two working periods. However, if the amount of work for one working period is prohibitive (e.g. the time required exceeds the maximum length of a working day), then the arc can be removed to force the solution to be over two working periods.

The network shown in Fig. 3 can be expanded to allow for this situation (see Fig. 4). Using the notation of Bellmore and Hong (1974), we have the original office base OB_0 and an additional node OB_{-1} (with a required fixed cost) representing the additional return to the office base. Whether or not this additional return to the office base is actually used in the solution depends on the relative cost of (1) moving the receivers from one session to another session (evaluated using the cost matrix) and (2) returning to the office base. If the cost of having an additional observing period is, relatively speaking, very high, then it is likely that the optimal solution will have OB_0 and OB_{-1} connected directly by an arc so that effectively there is only one working period. If it is essential to split the work over two shorter working periods (e.g. to reduce overtime payments; to ensure compliance with legal requirements over length of a working day), then the arc joining OB_0 and OB_{-1} should not be included, or else be included but with a very high relative cost so that the optimal solution would naturally ignore that arc.

For the network shown in Fig. 4, the fixed cost for OB_0 is set to be zero but the fixed cost for OB_{-1} is set to be 4 (the mean of all other arc costs in the network); this is implemented by modifying the cost of the arcs radiating from OB_{-1}. The costs of the arcs radiating from OB_{-1} are shown as the sum of two values, one value being the equivalent cost from OB_0 and the other being half the cost of using OB_{-1}.

Using Fig. 4 it becomes clearer why the OR term *node* is used. In Fig. 4, A, B, C and D represent physical points on the ground. OB_0 represents the office base for the start and end of a working period and OB_{-1} represents the office base for the start and end of a different working period – but they may be the same physical location. Thus the general term *node* is used to represent A–D and the office bases.

The concept of the node in place of the session now becomes more relevant. In this problem (i.e. the determination of the optimal session schedule), we are not trying to solve for the movement of one surveyor around a GPS network, but instead the order of the sessions. Thus the nodes now represent the sessions and the arcs now represent the cost of moving receivers between sessions. In other words, each node now represents two physical points and each arc the cost of moving from two points to up to two points. We can therefore construct the cost matrix for this network as shown in Table 5.

As an example of how the elements were evaluated, let us consider element (AB, CD). This represents moving the receivers from session AB to CD. In particular, the receiver at point A has moved to C and the receiver at point B has moved to D. The cost of moving from point A to C is 6 units (see Fig. 4) and the cost of moving from point B to D is 1 unit. For the evaluation of the cost matrix in Table 5, the rule used is that the cost to move between sessions is the maximum of the

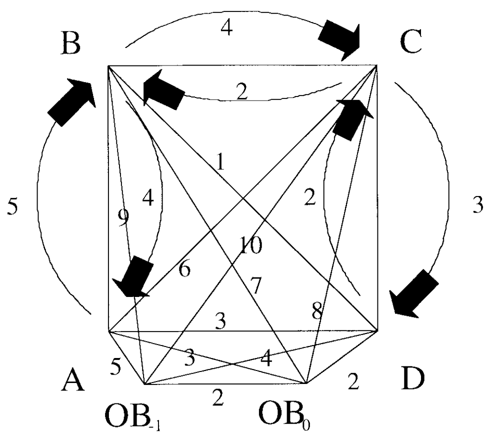

Fig. 4. Expanded simple four point network to include two working periods

Table 5. Initial cost matrix for two-receiver problem with two working periods

From/to	AB	BA	BC	CB	CD	DC	DA	AD	OB_0	BO_0	OB_{-1}	BO_{-1}
AB	∞	∞	5	6	6	4	4	1	7	7	9	9
BA	∞	∞	6	5	4	6	1	4	7	7	9	9
BC	4	6	∞	∞	4	1	6	4	8	8	10	10
CB	6	4	∞	∞	1	4	4	6	8	8	10	10
CD	6	3	2	1	∞	∞	3	6	8	8	10	10
DC	3	6	1	2	∞	∞	6	3	8	8	10	10
DA	5	1	6	5	3	6	∞	∞	3	3	5	5
AD	1	5	5	6	6	6	∞	∞	3	3	5	5
OB_0	7	7	8	8	8	8	3	3	∞	∞	2	2
BO_0	7	7	8	8	8	8	3	3	∞	∞	2	2
OB_{-1}	9	9	10	10	10	10	5	5	2	2	∞	∞
BO_{-1}	9	9	10	10	10	10	5	5	2	2	∞	∞

individual movements. This simulates the case of minimising the time taken for the survey. If you wished to minimise, for example, the total mileage covered, then the cost in element (AB, CD) would be the sum of the individual costs, giving 7 units in this case. In the cost matrix we have each session appearing twice to allow for all possible receiver movements. Thus for session move AB to BC we also allow for the possible move of AB to CB. In the second case the second receiver remains at point B while the first receiver moves from point A to C. This is beneficial as it allows for complete flexibility in terms of receiver movement between sessions. The block diagonal elements are set to ∞ (infinite cost) to prevent simple receiver swaps. When dealing with sessions of multiple baselines the size of the cost matrix can become prohibitive. In these cases a point-based cost matrix is created and the cost of moving between sessions is evaluated when necessary but not stored in a matrix.

Using the cost matrix shown in Table 5, the optimal receiver schedule obtained is as shown in Table 6. The full details of the branch-and-bound process used to determine the optimal solution (beyond the scope of this paper) are given in Dare (1995), but the outline process is as follows:

```
loop
 Compute a reduced cost matrix by subtracting
     the lowest value matrix element for each row
     and column from the same row and column
   Determine the penalties for not using a particular arc
   Select the arc with the largest penalty (e.g. i ⇒ j)
   Delete the row and column from the reduced cost
      matrix that contains the selected arc
   Set the reverse element (e.g. j ⇒ i) to ∞ to prevent
     the algorithm creating loops that do not pass
     through all the nodes
end loop when reduced cost matrix is of size 1 × 1.
backtrack to check other arcs
```

The penalty for not using a particular arc is easily computed. The reduced cost matrix contains elements of value zero where originally the lowest costs for each row and column existed. These zeros represent the cheapest arcs to and from each point (although they will not necessarily form a variable solution). The penalty for not using an arc with a zero value in the reduced cost matrix is the sum of the next lowest matrix element values in the same row and column. This penalty represents the increased cost in the solution for not using the relevant arc. Thus the selected arc is that with the highest penalty as this is the most important arc to use.

The row and column containing the selected element of the reduced cost matrix are then deleted, reducing the size of the reduced cost matrix by one. In order to prevent isolated loops being created, the value of the reverse element is set to infinity. Once the reduced cost matrix is of size 1×1, the procedure returns to check other possible solutions that may be cheaper than the current solution, e.g. elements of value zero in the reduced cost matrix with an equal highest penalty value.

It can be seen from Table 6 that OB_0 is followed immediately by OB_{-1}, showing that there is no need to return to the office base to provide an optimal solution. It is possible that a different conclusion would have been obtained had the office base been inside the network rather that outside it. In this way more returns to it could have been made without increasing the overall cost.

Table 6. Optimal solution with two working periods

Session	Receiver 1	Receiver 2	Cost of move
1	OB_0	OB_0	–
2	OB_{-1}	OB_{-1}	2
3	D	A	5
4	B	A	1
5	B	C	6
6	D	C	1
7	OB_0	OB_0	8

3 Example computation for optimal solution

The survey was to provide control for the positioning of a new road passing through Fundy Park, New Brunswick, Canada. Two receivers were used to position six points. The observational schedule adopted is shown in Table 7. The survey was carried out a number of years ago but the Fundy Park data is used as an example as the problem is compact and over two working periods.

There were no observations for session A on day 38 as the two points could not be located, thus this session was moved to day 39. Using the information provided, the cost of each move was determined by subtracting the start time of one session from the end time of the preceding session: summing the costs gives a total cost of the schedule of 180 min. For example, the cost of moving from point 3 to 4 is estimated using sessions B and C on day 38. The difference in the start and end times (15:51–15:30) gives 21 min. In the case where both receivers have moved (see sessions A and B on day 39), the costs for both moves have been set equal to each other, as points 1 and 2 are close and likewise points 3 and 4 (see Fig. 5).

Table 7. Fundy Park survey – observational schedule

Day of year	Session	Receiver 1	Receiver 2	Start time	End time
38	A	2	1		
	B	2	3	14:26	15:30
	C	2	4	15:51	17:20
	D	1	4	17:35	19:20
39	A	2	1	09:01	11:06
	B	3	4	12:23	14:26
	C	5	4	14:50	15:56
	D	6	5	16:17	17:56
	E	6	3	18:18	19:16

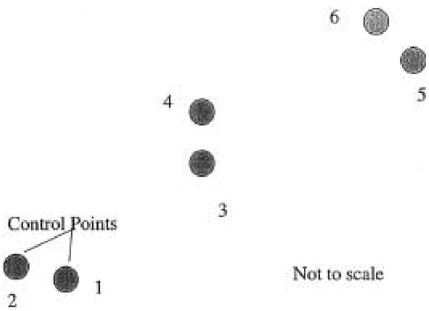

Fig. 5. Fundy GPS Network

Table 8. Fundy Park survey – point-based cost matrix. Bold elements represent estimated costs

From/to	1	2	3	4	5	6
1	∞	15	**77**	77	**98**	**100**
2	15	∞	77	**77**	**100**	**100**
3	**77**	77	∞	21	24	**24**
4	77	**77**	21	∞	21	**21**
5	**98**	**100**	22	21	∞	21
6	**100**	**100**	**24**	**21**	21	∞

The cost of a move includes equipment setup time. The costs calculated assume that the surveyors moved directly from one point to another. A point-based cost matrix has been constructed and is shown in Table 8. The elements shown in bold have been estimated, as there was no session change that involved the two relevant points. The estimated costs should be close to the truth due to the lack of choice of travel routes: access to the points was along a logging road. It has been assumed that the estimated costs are symmetrical for two reasons. First, as snowmobiles were being used, the surveyors could get very close to all the points. Second, the cost to change from point 3 to 5 (session B to C on day 39) was 24 min and the cost for the reverse of that move (session D to E on day 39) was only 2 min different at 22 min.

The list of sessions to be observed has already been specified (Table 7 shows the sessions present in the observational schedule). Thus it is now necessary to compute the cost of moving between the sessions. Allowance for observing over two days needs to be included, and so the model developed at the end of the previous section to allow for more than one working period will be used. In order to ensure that the solution did include two working periods, the arc joining the nodes representing the working periods is given a value of ∞. An arbitrary figure of 30 min is chosen as the cost for starting and ending the working periods (i.e. all arcs from the nodes that represent the working periods have a cost of 30, except those joining them which have a value of ∞ as previously mentioned). This will not affect the optimal solution of the order of the sessions to be observed as all arcs entering or leaving a node representing the start and end of a working period have the same cost – all that will change is the overall cost of the optimal schedule. The adoption of the same costs is justified because these costs were not included in the actual observed schedule – they

Table 9. Optimal solution to Fundy Park network

Working day	Session	Receiver 1	Receiver 2	Cost
1		Base	Base	
	A	2	3	30
	B	2	4	21
	C	1	4	15
		Base	Base	30
2		Base	Base	
	A	2	1	30
	B	3	4	77
	C	4	5	21
	D	6	5	21
	E	6	3	22
		Base	Base	30

are being included in the optimal solution to force the algorithm to produce a solution over two working periods. Considering all the above information, and realising that receiver reversals are to be included, the session-based cost matrix was constructed.

Using the session-based cost matrix, the solution obtained over two working days is shown in Table 9 together with the costs.

The total cost for this schedule is 297 minutes. To compare this with the observed cost, four occurrences of 30 min (as there was a total of four journeys to and from the base point) need to be added to the 180 min previously given – thus the total observed cost was 300 min, which was 3 min more than the computed schedule.

Comparing the observed schedule with the computed schedule, the following difference becomes apparent: for session C of the second working day the points occupied by the receivers have been exchanged. This forces both receivers to move when changing between sessions B and C – in the observed schedule both receivers moved when changing between C and D.

4 Limitations

The method described previously does provide, for two receivers, optimal schedules which can span more than one working period. However, optimal solutions are only possible in relatively short time intervals for limited size problems. Once the number of receivers and sessions increases, the length of time to determine the optimal solution can quickly become days.

As an alternative, methods that do not guarantee optimal solutions have been investigated. Within OR, non-optimal solutions are generally obtained from methods known as heuristics that often produce optimal or near-optimal solutions – these are covered in the following section.

5 Heuristics

An heuristic, within the OR sense, is a technique to determine near-optimal solutions in a reasonable amount (a few hours) of computational time. They are used extensively within OR to solve large problems that cannot be solved optimally and reasonably quickly (see e.g. Rayward-Smith et al. 1996). Although an heuristic does not guarantee optimality, a good heuristic may provide the optimal solution (although this will not be known) or at least a solution close to it. The technique is based upon making a small number of changes to a current solution to obtain a new solution. The new solution is known to be in the neighbourhood of the previous solution as it was obtained by a small number of changes to that previous solution – hence it is close to that previous solution.

A basic strategy for an heuristic as applied to GPS session schedule design could be as follows.

1. Choose an initial session schedule.
2. Swap two of the sessions to make the session schedule of a lower cost.
3. Repeat step 2 until no improvements can be made.

The problem with the above simplistic approach is that the solution is often a local optimum rather than the global optimum; this is illustrated in Fig. 6. Figure 6 plots the iteration cost against the number of iterations carried out for a typical case. It can be seen that after a number of iterations the cost has reached a minimum (local optimum) – but not the global minimum. In order to try and obtain the global optimum, it becomes necessary to "climb out" of the local optimum to allow a later descent to a cheaper and possibly global optimum. One method which allows this is SA.

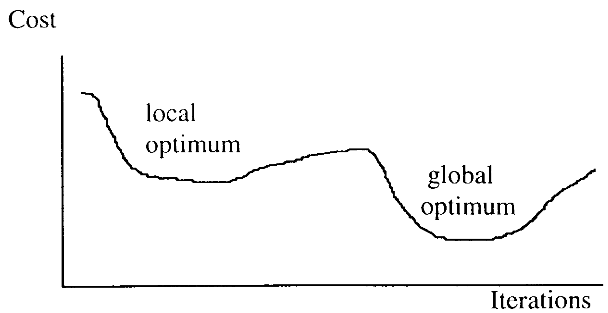

Fig. 6. Local and global optimum

6 Simulated Annealing

The cooling of a material in a heat bath is known as annealing. If a solid object is heated to make it melt (case I in Fig. 7), the rate of subsequent cooling, and the highest temperature reached, determine the structure of the reformed solid material. Thus, in Fig. 7, case II is created from slow cooling of the material while case III results from rapid cooling of the material. Although both case II and case III are solids formed at a temperature of zero ($T = 0$), the form of them is different due to the cooling process adopted. Case II has the optimum arrangement of the particles, while case III has a non-optimal arrangement.

SA is, obviously, a technique to simulate this process. SA is discussed in general in Rayward-Smith et al. (1996). However, the basic strategy for the SA heuristic as applied to GPS session schedule design is as follows:

i. Choose an initial schedule.
ii. Swap two of the sessions to make the schedule cheaper.
iii. Repeat step ii until no improvements can be made.
iv. Swap two of the sessions to make the schedule more expensive.
v. Repeat step iv for a small number of attempts.

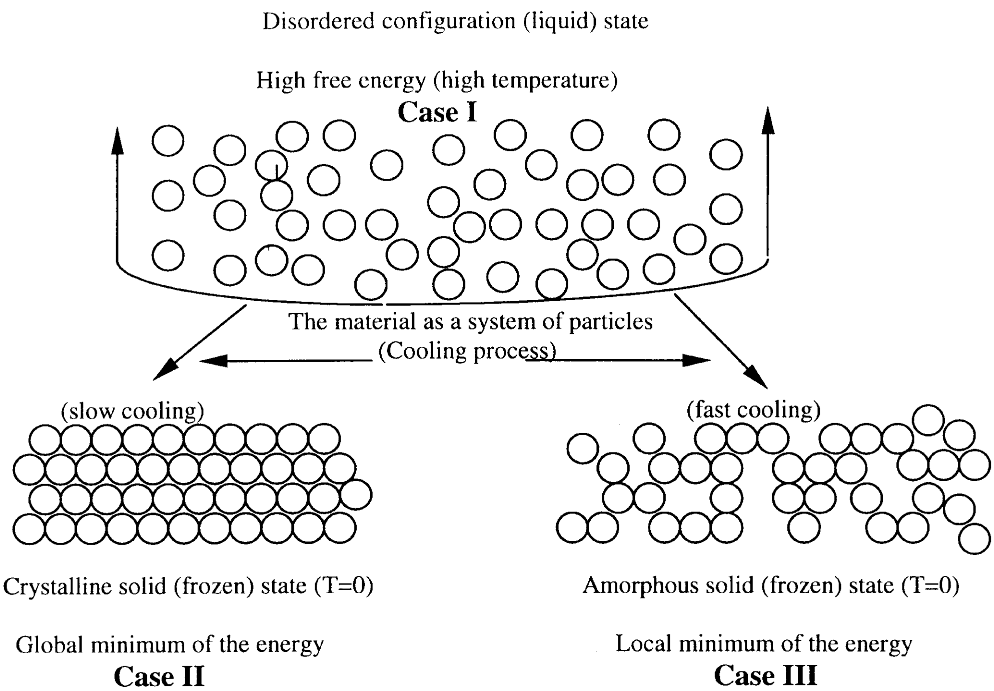

Fig. 7. Annealing

Step 1 Determine cost for each change of session
Create session cost matrix.
Set stopping criterion.
Step 2 Estimate a feasible schedule V with cost C(V) using the session cost matrix.
Step 3 Set the initial starting value of the temperature T_i.
Set the final temperature T_f.
Set the cooling ratio, F.
Set the number of iterations, K.
Step 4 Select a neighbour V' of V where V'$\in$ I(V).
Let C(V') = the cost of schedule V'.
Compute the move value Δ= C(V')-C(V).
Step 5 If $\Delta \leq 0$ accept V' as a new schedule and set V=V'.
else
IF $e^{-\Delta/T} > \theta$ set V=V', where θ is a uniform random number $0<\theta<1$.
otherwise retain the current schedule V.
Step 6 Update the annealing scale parameters using the geometric cooling rule $T_{(K+1)}=F\,T_K$ {K=1,2...}.
Step 7 If the stopping criterion is met then
Show the output.
Declare the cheapest schedule.
Declare the computation time.
otherwise go to Step 4
Step 8 If schedule is not acceptable go to Step 3.

End.

Fig. 8. The general framework for the GPS-SA procedure

vi. Go back to step ii.
vii. Stop when no improvements can be made.

This process is shown in more detail in Fig. 8. In step 4, $I(V)$ represents the neighbourhood of V.

The application of this method to GPS will enable optimal or near-optimal session schedules to be determined for much larger networks (i.e. more points, more receivers). This clearly will enable surveys to be carried out more efficiently. One critical stage is at steps 4 and 5 (expanded form of step iv); allowing too many expensive moves will cause the solution to move too far away from the optimal solution. Having too few, however, will not allow the solution to climb out of the local optimum. It is also important to make sure that once we have climbed out of the local optimum we move on to the next optimum and not simply roll back to where we came from. Another critical stage is step 7; stopping the iterative process too early may not provide a near-optimal solution, while allowing the process to iterate too many times wastes time with no guarantee of a lower-cost solution being found. Possible stopping criteria are:

1. Terminate after 100 iterations.
2. Terminate if solution remains unchanged for 10 iterations.

You can of course never be sure that you have obtained the most economic solution (except for trivial problems).

One of the difficulties of the SA approach is that there is no way, at present, to decide which stopping criteria for a particular problem should be used. In practice, a variety of stopping criteria are applied to a particular problem and the cheapest session schedule accepted. Similarly, a variety of T_i, T_f and F are used and the cheapest schedule accepted. Thus it can be seen that the SA method requires user intervention – it is not a totally automated procedure.

It is in step 5 that changes to the current session schedule causing it to become more expensive can take place. The acceptance of a more expensive session schedule is partly a random process, but it is also a function of the current temperature. The more expensive session schedule is accepted with probability $P(\Delta,T)$. During the course of the SA algorithm the temperature is adjusted from a high value (yielding a higher probability of an uphill move) and tending towards zero as the number of iterations increases, so reducing T (yielding a lower probability of an uphill move).

7 Example computation for heuristic solution

It is preferable in heuristics to evaluate the proposed algorithm by comparison with known optimal solutions. The optimal examples presented earlier in this paper were solved using the SA algorithm and identical results were obtained. However, in order to generalise the above SA procedure and to test it with larger networks, the data set for a network observed on the Republic of Malta with sessions comprising of two or three receivers has been used (see Fig. 9). The data and sessions were obtained from Dare (1994). The data set consisted of 37 sessions made over a period of 2 weeks during the summer of 1993.

Although no optimal schedule is available for this network, for reference purposes the actual operating schedule used during the survey of 1993 is available. This schedule, with a cost of 2264 min, was generated manually using intuition and experience on a day-to-day basis. At the end of one working day, a schedule for the following day was created. The cost of 2264 min was obtained from analysis of the starting and ending time for the observed sessions. However, in order to provide a more realistic scenario for the SA heuristic, some costs of session changes have been modified due to breaks being taken during the day due to the excessive heat. These breaks were not taken at an office base, but rather opportunistic stops en route between points were taken. The effect of the breaks was that some session changes which would only take 20 min or so, actually took about 90 min. Thus the total cost of 2264 min does not provide a reasonable test for the SA heuristic, as the costs used in its calculation are too high. Without modification of some of the costs, some cheap session changes would have become prohibitively expensive and so would be avoided by the SA heuristic. These "en route" breaks, although clearly needed in certain circumstances, should not be forced into a particular cost but added at the end of the design procedure when the schedule has been produced. The observed schedule with modified costs is shown in Table 10 – this was used as the initial schedule for the SA technique and had a cost of 2090 min over 10 days. Using the SA heuristic technique the overall cost was reduced to 2000 min over 8 days. For a survey company, the reduction in the number of days is probably far more important than the saving of 90 min.

Table 10. Observed schedule for the Malta network

Day	Session	Receiver 1	Receiver 2	Receiver 3	Cost
1		Base	Base		
	1	Luqa Barracks	Tas-Silg		45
		Base	Base		45
2		Base	Base		
	1	Ii-Mara	Tas-Silg		45
	2	Ii-Mara	Nigret		35
	3	Luqa Barracks	Nigret		25
		Base	Base		35
3		Base	Base		
	1	Nigret	Tal-Lunzjata		35
	2	Nigret	Handaq		40
	3	Luqa Barracks	Handaq		35
		Base	Base		30
4		Base	Base		
	1	Handaq	Tal-Lunzjata		35
	2	Handaq	Dwejra		40
	3	Tal-Lunzjata	Dwejra		30
		Base	Base		35
5		Base	Base		
	1	Gordon Lighthouse	Xaghra School		115
	2	Gordon Lighthouse	Ghain Damma		30
	3	Gordon Lighthouse	Wardija Racal		30
	4	Gordon Lighthouse	Wardija		5
		Base	Base		115
6		Base	Base		
	1	Dare point	Gordon Lighthouse		115
	2	Dare point	Wardija		20
	3	Ta' Cenc	Wardija		25
	4	Xaghra School	Wardija		20
	5	Xaghra School	Ta' Cenc		25
		Base	Base		115
7		Base	Base		
	1	Dwejra	Pellegrin		40
	2	Bahrija	Pellegrin		25
	3	Bahrija	Dwejra		20
		Base	Base		40
8		Base	Base		
	1	Tal-Lunzjata	Dingli Reservoir	Bahrija	50
	2	Pellegrin	Dwejra	Mellieha Fort	35
	3	Qawra Point	Dwejra	Gharghur	30
	4	Qawra Point	Tal-Madonna	Mellieha Fort	40
	5	Ras il-Qammieh	Tal-Madonna	Mellieha Fort	45
	6	Ras il-Qammieh	Pellegrin		45
		Base	Base	Base	80
9		Base	Base	Base	
	1	Comino Tower	Ras il-Qammieh	Ta' Cenc	75
	2	Comino Tower	Tal-Madonna	Ii-Qasam	25
	3	Ta' Cenc		Ii-Qasam	35
	4	Ii-Qortin Isopo	Xaghra School	Ii-Qasam	30
	5	Ii-Qortin Isopo	Xaghra School	Ghain Damma	30
		Base	Base	Base	115
10		Base	Base	Base	
	1	Gharghur	Palace	Handaq	30
	2	Dwejra	Palace	Dingli Reservoir	30
	3	Zonqor Reservoir	Palace	Luqa Barracks	40
	4	Zonqor Reservoir	Tas-Silg	Luqa Barracks	45
	5		Ii-Mara	Luqa Barracks	25
		Base	Base	Base	35

For this network, the initial temperature (T_i) was set at 50, the final temperature (T_f) at 2, the cooling ratio (F) at 0.9 and the number of iterations (K) at 17000; the software execution time was just a few seconds.

Table 11 displays the results of applying the SA technique. Figure 10 shows a graphic depiction of the rapid convergence of the SA heuristic for this solution. In Fig. 10, "Current Cost" refers to the cost of the

Table 11. Computed schedule for the Malta network

Day	Session	Receiver 1	Receiver 2	Receiver 3	Cost
1		Base	Base		
	1	Luqa Barracks	Tas-Slig		45
	2	Ii-Mara	Tas-Slig		35
	3	Ii-Mara	Nigret		35
	4	Luqa Barracks	Nigret		25
		Base	Base		35
2		Base	Base		
	1	Nigret	Tal-Lunzjata		35
	2	Nigret	Handaq		40
	3	Luqa Barracks	Handaq		35
	4	Handaq	Tal-Lunzjata		30
	5	Handaq	Dwejra		40
	6	Bahrija	Dwejra		15
		Base	Base		40
3		Base	Base		
	1	Gordon Lighthouse	Xaghra School		115
	2	Gordon Lighthouse	Ghain Damma		30
	3	Gordon Lighthouse	Wardija Racal		30
	4	Gordon Lighthouse	Wardija		5
		Base	Base		115
4		Base	Base		
	1	Dare point	Gordon Lighthouse		115
	2	Dare point	Wardija		20
	3	Ta' Cenc	Wardija		25
	4	Xaghra School	Wardija		20
	5	Xaghra School	Ta'Cenc		25
		Base	Base		115
5		Base	Base		
	1	Dwejra	Pellegrin		40
	2	Bahrija	Pellegrin		25
	3	Tal-Lunzjata	Dwejra		20
		Base	Base		35
6		Base	Base		
	1	Zonqor Reservoir	Ii-Mara	Luqa Barracks	35
	2	Pellegrin	Dwejra	Mellieha Fort	45
	3	Qawra point	Dwejra	Gharghur	30
	4	Qawra point	Tal-Madonna	Mellieha Fort	40
	5	Ras il-Qammieh	Tal-Madonna	Mellieha Fort	45
	6	Ras il-Qammieh	Pellegrin	Mellieha Fort	45
		Base	Base		80
7		Base	Base		
	1	Comino Tower	Ras il-Qammieh	Ta' Cenc	75
	2	Comino Tower	Tal-Madonna	Ii-Qasam	25
	3	Ta' Cenc	Tal-Madonna	Ii-Qasam	35
	4	Ii-Qortin Isopo	Xaghra School	Ii-Qasam	30
	5	Ii-Qortin Isopo	Xaghra School	Ghain Damma	30
		Base	Base		115
8		Base	Base		
	1	Gharghur	Palace	Handaq	30
	2	Dwejra	Palace	Dingli Reservoir	30
	3	Zonqor Reservoir	Palace	Luqa Barracks	40
	4	Zonqor Reservoir	Tas-Silg	Luqa Barracks	45
	5	Tal-Lunzjata	Dingli Reservoir	Bahrija	30
		Base	Base	Base	50

session schedule for a particular iteration – thus at iteration 4 the current cost has a value of 2100. The value of the current cost rises and falls as the heuristic allows (in step 5 in Fig. 8) some changes to the session schedule to increase the overall cost in the hope of finding a cheaper session schedule at a later iteration. "Best Cost" in Fig. 10 portrays the lowest current cost at a particular iteration; it is, therefore, always of equal or lower value than the current cost. At iteration 6 the best cost has a value of 2030 (equal to the current cost), and also at iteration 7, while the current cost increases its value to 2090 at iteration 7.

The session schedule solution for the Malta network obtained by the SA algorithm is cheaper than the actually observed schedule. The SA heuristic is flexible enough to cope with the constraint that the third

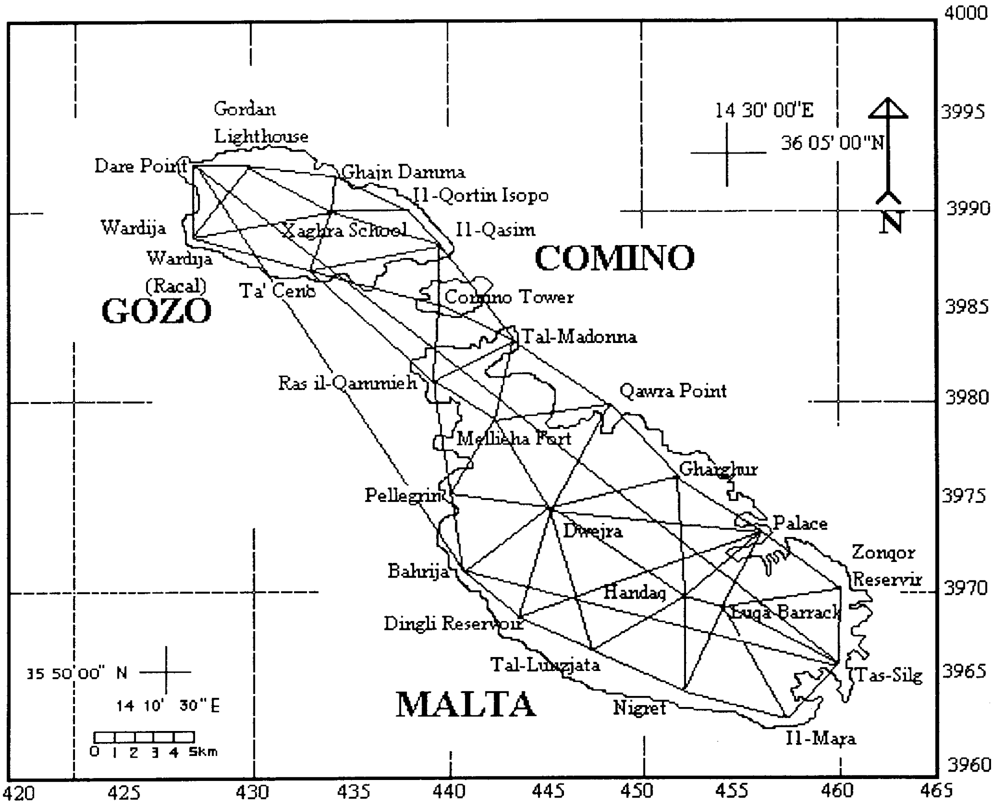

Fig. 9. Network observed for the Republic of Malta (from Dare, 1994)

receiver was only available for a few days and so those sessions have been grouped together in both the observed and the computed schedule.

8 Results and conclusions

The presented method has been tested on several problems and appears to be a useful technique in terms of the reduction of the cost of a provided session schedule. It can be seen that the developed SA technique is able to generate rapidly solutions for GPS session schedules, given a starting session schedule. It is a proven technique within OR to eliminate the computational difficulties of solving large problems with algorithms guaranteeing optimal solutions. Further tests are to be made using data collected from GPS surveys carried out in Mauritius and the Seychelles.

The developed algorithm reduces the cost of the survey, where cost can be defined by time or distance. Other aspects that may need to be considered in the design of a schedule (for example, the selection of points to occupy at each session) are not at present modelled. In addition, it may be best in some circumstances to accept a more expensive session change to reduce the effort required by the surveyors. For example, in Table 11, the change from session 3 to session 4 on day 2 requires receiver 1 to move from Luqa Barracks to Handaq while receiver 2 moves from Handaq to Tal-Lunzjata. From Fig. 9 it can be seen that, to minimise the time, this is correct as Tal-Lunzjata is closer to Handaq than Luqa Barracks; in addition the road network requires a vehicle from Luqa Barracks to Tal-Lunzjata to go through Handaq. However, it may be better to allow the surveyors with receiver 2 to stay and rest at Handaq and so receiver 1 would move from Luqa Barrack to Tal-Lunzjata. In order to accomplish this, the heuristic would have to develop a memory of moves and build in periods of no movement. This would enable rest periods to be taken during the day by each group of surveyors.

What is disappointing is the inability of the SA algorithm to reduce the cost of the provided session schedule for the Malta network by a larger amount. It is expected that further reductions can be made without simply reducing the number of observing days by working longer hours on the remaining days; further investigation is being carried out to try and reduce this cost. However, it was pleasing to achieve a reduction in the number of observing days. For both the test network and the Fundy network, the SA solution had the same

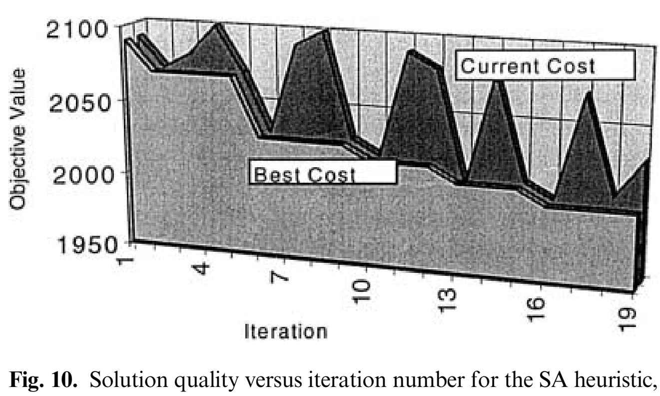

Fig. 10. Solution quality versus iteration number for the SA heuristic, as it visits five local optima

cost as the known optimal solutions, which is encouraging.

A solution of the logistics (often the most difficult part in observing a GPS network) can be obtained using the above SA heuristic procedure. As no optimal solution will be available for large networks, the general applicability of other heuristic techniques such as Tabu Search and genetic algorithms to GPS-surveying logistics design is a matter for further research. These alternative heuristic methods will also enable a check on the SA solution to be made. These other heuristic techniques are of a far more complex nature and it is hoped that they will produce cheaper session schedules than the SA approach.

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