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Network Adjustment Program using MATLAB

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ABSTRACT

The method of least squares, which arrives at a best solution by minimizing the sum of the weighted residuals, is one of the most frequently used methods to obtain unique estimates for a set of redundant measurements. Geodetic network analysis can be made in terms of 3D model or 2D model plus 1D. The concepts of least squares estimation (LSE) were introduced in early 1800s by Legendre and Gauss mainly for the purpose of reducing physical and astronomical data. There are a lot of commercial software like STARNET, GEOLAB and Move3 and educational software like Adjust 4.6 for least squares adjustment. STARNET is the most user friendly software but the problem is that STARNET does not provide the required information directly for further analysis after network adjustment such as covariance matrix. The others are not easy to use. The purpose of this work is to develop a program named ADJMAT in MATLAB7 to monitor and analyze least square network adjustment. This package is only limited to 2 dimensional network adjustment for distance, angle and azimuth observations and consists of three sections: least square adjustment, statistical analysis checking by Global test and Local test, quality of LSE results by error ellipse. The graphics for this program include error ellipse and station numbers for monitoring network. One data set is used to test and verify this program. Then the results from ADJMAT program are discussed and comparison is done between this program and STARNET.

Key words: Least Squares, LSE, Global test and Local test, *MATLAB*

1.0 INTRODUCTION

The LSE is a solution for sets of equations in which there are more equations than unknowns that leads to a unique answer. LSE falls in two categories: Linear and Non-linear. LSE arrives at a best solution by minimizing the sum of the weighted residuals, is one of the most frequently used methods to obtain unique estimates for a set of redundant measurements. Geodetic network analysis can be made in terms of 3D model or 2D model plus 1D.The concept of least squares estimation (LSE) were introduced in early 1800s by Legendre and Carl Friedrich Gauss mainly for the purpose of reducing physical and astronomical data. Gauss is developed the basics of least-squares analysis in 1795 and Legendre was the first to issue this technique (Bretscher, 1995).

This research focuses on Non-linear LSE for a network which consists of distance, angle and azimuth so they are needed to be linearized. Following the global test for outlier detection and Local test for localization of gross errors are performed. There are a lot of commercial and educational software like STARNET, GEOLAB and Move3 and Adjust 4.6 for LSE adjustment. STARNET is the most user-friendly and powerful software but the problem is that STARNET does not provide the required information directly for further analysis after network adjustment such as covariance matrix and it created a DMP file to extract covariance matrix. The others are not easy to use.
In this study, an application for LSE is implemented by using MATLAB 7 to monitor and analyze least square network adjustment. Then the results are compared with one of the most popular and dominant commercial LSE software called STAR*NET.

2.0 ADJUSTMENT PROGRAMS

There are a lot of network adjustment and surveying analysis software. Below some of them are discussed. The principle of this research is to extend a program for 2D network adjustments and as future work to add some other applications for 3D and 1D.

2.1 STAR*NET

STAR*NET is a powerful and popular windows-based commercial software for LSE. STAR*NET software suite handles general purpose, rigorous least squares analysis and adjustments of 2D, 3D and 1D land survey networks (Starplus, 2000). It employs a rigorous simultaneous least squares adjustment using the variation of coordinate method. Data input includes angles, bearings, azimuths, coordinate differences, directions, distances, elevation differences, zenith angles, etc. For LSE, all these data can be independently or globally weighted. Approximate coordinate can be entered manually or computed automatically. STARNET V6 performs LSE or adjustment of network, data checking, blunder detection and pre analysis.

2.2 GEO-LAB

Microsearch Corp has been developing a new application in the field of surveying called Geo-Lab. Geo-Lab performs least square adjustments of survey traverses and networks using survey measurements (Microsearch, 1985). Since 1985 GeoLab has been the world leading program for performing these types of calculations.

2.3 MOVE3

MOVE3 is a software package for the design, adjustment and quality control of 3D, 2D and 1D geodetic network in compliance with the procedures of the "Delft School" of geodesy. MOVE3 allows fully integrated processing of GPS and terrestrial observations (http://www.grontmij.nl/).

2.4 ADJ4.6

This software is for the educational purpose only. It is planned to help the instructor in an adjustment computation course (Wolf and GHilani, 1997).

3.0 METHOD FOR NETWORK ADJUSTMENT ANALYSIS

The method of network adjustment comprises of three sections: least square adjustment, statistical analysis checking by Global test and Local test, quality of LSE results by error ellipse. The flowchart of this work can be seen in Figure1.

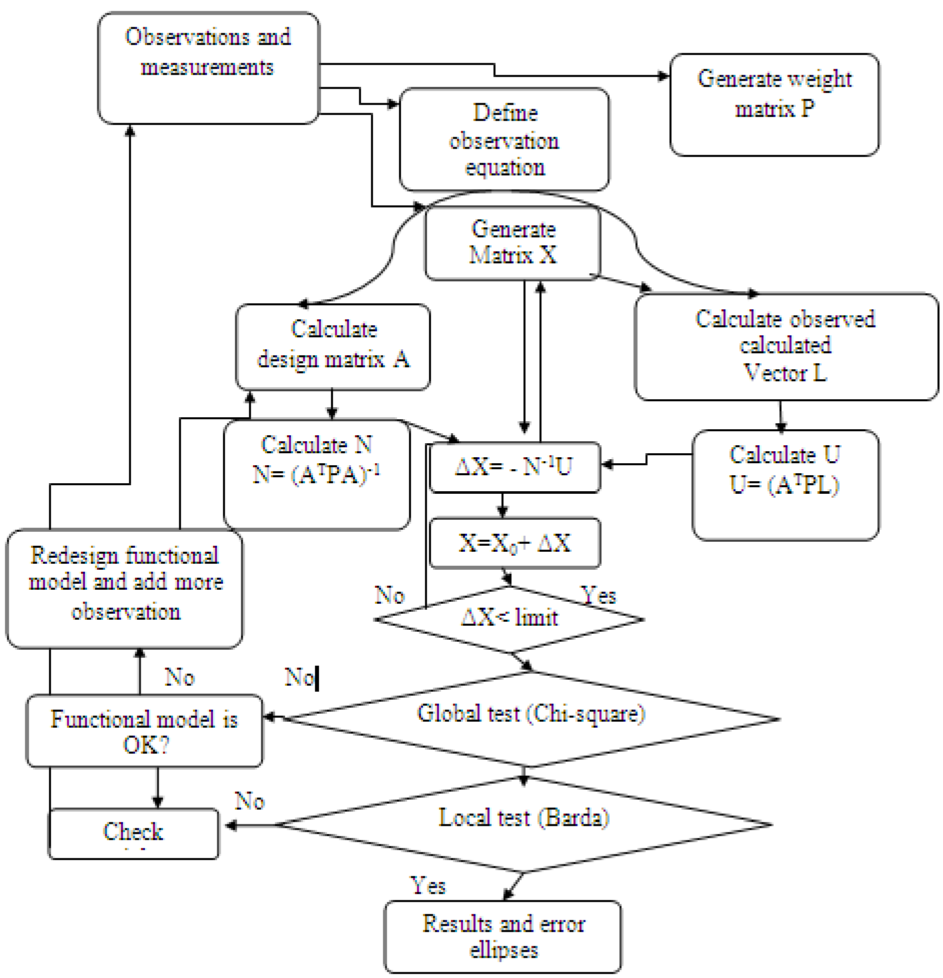


Figure 1: Flowchart of least square estimation with statistical testing

3.1 LSE

Geodetic network analysis can be made in terms of 3D model or 2D model plus 1D. In this study 2D network model is focused (Kuang, 1996).

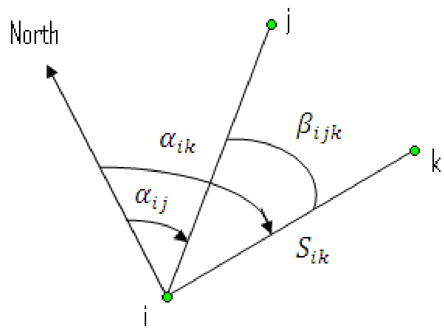


Figure 2: Observation of a horizontal angle, azimuth and horizontal distance

3.1.1 Nonlinear Observation Equations

For this 2D network adjustment, there are three type of observation equation (Wolf and Ghilani, 1997):

1. Horizontal Distance

Distance (Figure 2) between two points is related to the coordinates of the two points as follows:

$$S_{ik} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2} \tag{1}$$

Where S_{ij} is the measured raw distance and $(x_i, y_i) and \ (x_j, y_j)$ are the coordinates of the instrument station i and target station j, respectively.

2. Azimuth

An azimuth (Figure 2) can be measured either by an astronomical method or using a gyro instrument. A raw azimuth becomes a spatial azimuth referring to the normal section of the reference ellipsoid (Kuang, 1996). It is related to the horizontal coordinates of the terminal points as follows:

$$\alpha_{ij} = \arctan\left[(x_j - x_i)/(y_j - y_i)\right] \qquad (2)$$

Where, α_{ij} is the measured raw azimuth reduced to the local geodetic system and (x_i, y_i) and (x_j, y_j) are the horizontal coordinates of instrument station i and target station j, respectively.

3. Horizontal Angle

A horizontal angle (Figure 2) can be the difference between two azimuths or two directions. Thus, the observation equation for a horizontal angle can be obtained by differencing two direction or two azimuth observation equations referring to lines ij and ik ;

$$\beta_{ijk} = \alpha_{ik} - \alpha_{ij} = \arctan[(x_k - x_i)/(y_k - y_i)] - \arctan\left[(x_j - x_i)/(y_j - y_i)\right] \qquad (3)$$

Where, β_{ijk} is the measured raw angle reduced to the local geodetic system and (x_i, y_i) and (x_j, y_j) and (x_k, y_k) are the horizontal coordinates of instrument station i and target station j and k, respectively.

3.1.2 Linearized Observation Equations

For network adjustment computation, the above nonlinear observation equation has to be linearized, and an iterative solution approach is used. Assuming that the approximate coordinates of the network stations are available, the nonlinear observation equations can be approximated by the linear term of Taylor series as follows (Kuang, 1996):

Horizontal Distance

$$S_{ij} + v_{Sij} = S_{ij}^0 - \frac{\Delta x_{ij}^0}{S_{ij}^0}\delta x_i - \frac{\Delta y_{ij}^0}{S_{ij}^0}\delta y_i + \frac{\Delta x_{ij}^0}{S_{ij}^0}\delta x_j + \frac{\Delta y_{ij}^0}{S_{ij}^0}\delta y_j \qquad (4)$$

Azimuth

$$\alpha_{ij} + v_{\alpha ij} = \alpha_{ij}^{0} - \frac{\Delta y_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\delta x_i + \frac{\Delta x_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\delta y_i + \frac{\Delta y_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\delta x_j + \frac{\Delta x_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\delta y_j \qquad (5)$$

Horizontal Angle

$$\beta_{ij} + v_{\beta ij} = \beta_{ij}^{0} + \left(\frac{\Delta y_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}} - \frac{\Delta y_{ik}^{0}}{\left(L_{ik}^{0}\right)^{2}}\right)\delta x_i + \left(\frac{\Delta x_{ik}^{0}}{\left(L_{ik}^{0}\right)^{2}} - \frac{\Delta x_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\right)\delta y_i -$$

$$\frac{\Delta y_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\delta x_j + \frac{\Delta x_{ij}^{0}}{\left(L_{ij}^{0}\right)^{2}}\delta y_j + \frac{\Delta y_{ik}^{0}}{\left(L_{ik}^{0}\right)^{2}}\delta x_k - \frac{\Delta x_{ik}^{0}}{\left(L_{ik}^{0}\right)^{2}}\delta y_k \qquad (6)$$

The method of least square network adjustment tries to solve for an optimal estimate of both the coordinates and residuals by minimizing the sum of squares of the weighted residuals (v) (Setan, 2008). Leading to the following optimization problem (USACE, 2002):

Minimize: $v^T \mathrm{P} v$ (7)

Subject to observation equation: $l + v = Ax$

Where, *l* is observation matrix, *A* is design matrix and *x* is unknown matrix (Setan and Singh 2001).

Calculate:

$$\hat{v} = A\hat{x} + b \qquad (8)$$

$$\hat{x} = -N^{-1}U \qquad (9)$$

Where, *v* is residual matrix and $N = A^T \mathrm{P}\,\mathrm{A}$, coefficient matrix

$$U = A^T \mathrm{P}\,\mathrm{b} \quad \text{And} \quad b = l - l_0 \qquad (10) \text{ and } (11)$$

l is vector of actual observations and l_0 is vector of computed observations

$\widehat{x_a} = \hat{x} + x_0$ (12) , The updated parameters

$\widehat{Qx_a} = (A^T \mathrm{P}\,\mathrm{A})^{-1}$ (13) , cofactor matrix

After initial stage of LSE, iteration must be done. At the end of step one, approximate coordinates must be updated and *lo* and **A** should be calculated again. Limit for iteration is (Setan, 2008):

1. $\widehat{x_i}$ close to zero

2. $v_i - v_{i-1}$ close to zero

3. $v^T \mathrm{Pv}$ Stable

In this study first limit is used. In each computation step, matrices, A and L are changed, matrices *P* and l_0 unchanged (Olyazadeh and Setan, 2010).

And the posteriori variance factor can be estimated by

$$\widehat{\sigma_0^2} = \frac{\hat{v}^T P \hat{v}}{r}$$

Where, *r* is degree of freedom.

3.2 Global Test

Global test and data snooping are the most frequently used post-adjustment data screening techniques. Barda (1968) proposed the global test for outlier detection and data snooping for localization of gross errors (Kuang, 1996). After network adjustment, the global test is applied first, which tests the compatibility of the estimated a posteriori variance factor with a priori selected variance factor which in this study is assumed 1. If global test failed, it means that something wrong with the null hypothesis.

The estimated a posteriori variance factor from a least squares network adjustment is as follows (Caspary, 1987):

$$\widehat{\sigma_0^2} = \frac{\hat{v}^T P \hat{v}}{r} \qquad (14)$$

Global test is to examine the compatibility of the posterior estimated variance factor with the priori given variance factor. It uses in test statistic as follows:

$$y = \frac{r.\widehat{\delta_0^2}}{\delta_0^2} \qquad (15)$$

Where, r is degree of freedom or number of redundant observations.

Global test can be performed in two different ways, one-tailed and two-tailed test (Setan, 2008). According to the principle of two-tailed test, and significant level of $\alpha = 0.05$, the test will be accepted if:

$$\chi^2_{\frac{\alpha}{2}}(r) \le y \le \chi^2_{1-\frac{\alpha}{2}}(r) \quad (16)$$

χ^2_k , The Chi-Square distribution is a special case of the gamma distribution where b = 2 in the equation for gamma distribution (Kuang, 1996).

$$f(x;\, k) = \frac{1}{2^{k/2}\Gamma(k/2)}\, x^{k/2-1} e^{-x/2}\, \mathbf{1}_{\{x\ge 0\}}, \quad (17)$$

If the global test is failed, there will be an infinite choice of alternative hypotheses, and two special cases can be concentrated on are:

1. H_{a1} , incorrect observation weighting
2. H_{a2} , gross errors exist in observation data

3.3 Local Test

Local test is performed after global test. Application of the data snooping technique is a combined process of *outlier detection* and *gross error localization* and *elimination*. In this research, local test via Barda method is used which assumes only one measurement contains gross error (Caspary, 1987). For statistical testing, requires normalizing adjusted residuals. If local test pass, it will be assumed no gross error in measurements.

Test statistic:

$$\widehat{v_i^0} = \frac{\hat{v_i}}{\delta_{vi}} \sim N(0, \delta_{vi}) \quad (18)$$

Where, $\widehat{v_i^0}$ are normalized adjusted residuals and *N* is Normal distribution. So the test is accepted if:

$$\left|\widehat{v_i^0}\right| < N_{\alpha/2} \quad (19)$$

Typical values of α and β are 0.1 and 0.2. So it leads to typical critical value 3.29, and Barda test is accepted if (Setan, 2008):

$$\left|\widehat{v_i^0}\right| < 3.29 \tag{20}$$

3.4 Error Ellipse

Error ellipses are derived from the covariance matrix, and provide a graphical means of viewing the results of a network adjustment (USACE, 2002): Error ellipses can show:

Orientation weakness (minor axes pointing to the datum)

Scale weakness (major axes pointing to the datum)

A **standard** error ellipse shows the region, if centered at the **true** point position, where the least squares estimate falls with a confidence of 39.4%.To obtain different confidence regions the length of the ellipse axis is just multiplied by an appropriate factor (Kuang, 1996). These factors are determined from the distribution. Generally the 95% confidence region ellipses (Table 1) are plotted:

Table 1: Confidence region ellipses

Confidence region	39.4%	86.5%	95.0%	98.9 %
Factor	1	2	2.447	3

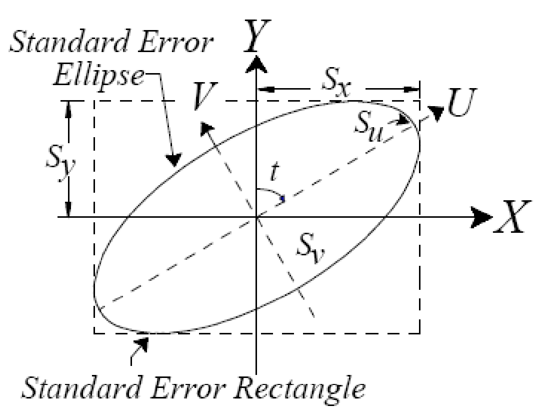


Figure 3: Error ellipse

Figure 3 illustrates a standard error ellipse where (Wolf and Ghilani, 1997):

1. *t* is rotation angle from Y axis to axis of largest error.
2. *Su* is the semi-major axis of ellipse (Largest error).
3. *Sv* is the semi-minor axis of ellipse (Least error).
4. *Sx* is the standard deviation in X coordinate
5. *Sy* is the standard deviation in Y coordinate

The covariance matrix of a 2D point based on the local [x, y] coordinate system:

$$\mathbf{C}_{xy} = \begin{pmatrix} \sigma^2_x & \sigma_{xy} \\ & \sigma^2_y \end{pmatrix} \qquad (21)$$

There is another coordinate system [u, v], that can be rotated into using:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{pmatrix} \sin(\theta) & \cos(\theta) \\ -\cos(\theta) & \sin(\theta) \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad (22)$$

The corresponding covariance matrix for the [u, v] system is gained via propagation of variance:

$$\mathbf{C}_{uv} = \begin{pmatrix} \sigma^2_u & \sigma_{uv} \\ & \sigma^2_v \end{pmatrix} = \begin{pmatrix} \sin(\theta) & \cos(\theta) \\ -\cos(\theta) & \sin(\theta) \end{pmatrix} \mathbf{C}_{xy} \begin{pmatrix} \sin(\theta) & -\cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{pmatrix} \qquad (23)$$

The interested parameters are s_u and s_v which evaluate to:

$$\sigma^2_u = \sigma^2_x \sin^2(\theta) + 2\sigma_{xy} \sin(\theta)\cos(\theta) + \sigma^2_y \cos^2(\theta) \qquad (24)$$

$$\sigma^2_v = \sigma^2_x \cos^2(\theta) + 2\sigma_{xy} \sin(\theta)\cos(\theta) + \sigma^2_y \sin^2(\theta) \qquad (25)$$

So semi major axis and semi minor axis by the 95% confidence region can be calculated as follows (Setan, 2008):

$$Su = 2.447\ \sigma_u \qquad Sv = 2.447\ \sigma_v$$

4.0 RESULTS AND ANALYSIS

This package has two modules as follows:
1. ADJMAT Program which can be done LSE.
2. STARNETFILE Program which converts observation data format to STARNET format that it is possible to check LSE results with STARNET results.

First module includes least square adjustment; statistical analysis checking by Global test and Local test and error ellipse .Then comparison is done between this program and STARNET. For checking LSE results with STARNET, firstly data must be converted to STARNET format. In such case, STARNETFILE program can be useful. The graphics for this program include error ellipse and station numbers for monitoring network.

4.1 Input Data

These programs can read data in text format and station numbers must be in numeric format. Data format for adjustment should be similar to STARNET format which they can also be handled by STARNET. In this project, '-'is removed from STARNET format to make programming and converting observation simple (Olyazadeh and Setan, 2010). So after running the adjustment program if it is needed to use STARNET to check the results, it is recommended to read data file in STARNETFILE program to convert them to STARNET format.

Table 2: Example of input data for ADJMAT and STARNET

ADJMAT:	*STARNET:*
C 14 1556.8000 260.7970 1 1	*C 1 100.011000 100.103000*
C 16 809.0000 777.0000 0 0	*C 2 109.003000 111.601000*
C 17 1168.0000 820.0000 0 0	*C 3 144.013000 122.181000*
D 14 15 176.9124 0.0109	*D 1-2 14.596300 0.000300*
D 15 16 736.8567 0.0087	*D 1-3 49.229200 0.000300*

A 17 14 11 303 52 2.10 3.0	*A 1-2-3 25-19-38.780000 1.000000*
A 17 14 12 308 34 52.70 3.0	*A 1-2-3 25-19-38.780000 1.000000*
Z 14 16 304 40 41.00 5.0	*B 1-3 304 40 41.00 5.0*

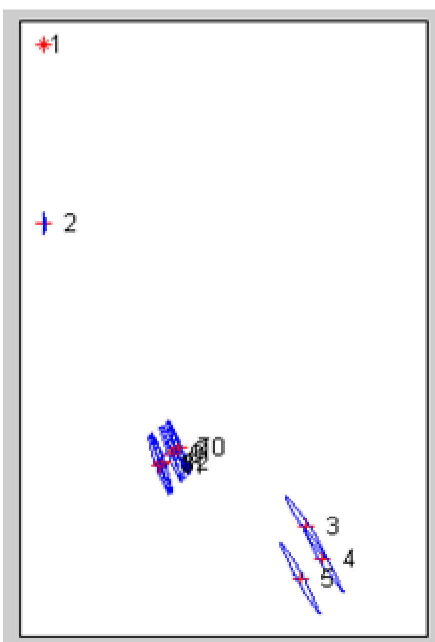
Where, C: coordinate, D: Distance, A: Angle, Z: Azimuth and 1 is for fixed station and 0 for no fixed station. In Star*net difference is just for Azimuth that can be shown by B instead of Z (Olyazadeh and Setan, 2010).

4.2 Output Data

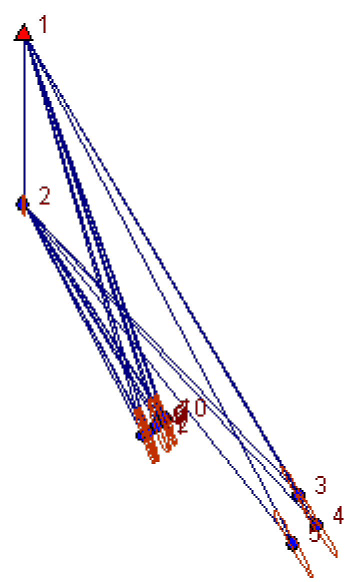
In this study, one data set is used to check and verify this program. This data set includes of 12 stations and station 1 as fixed, with 22 azimuth and 15 distance observations (M Idris et al, 2004).

4.3 Comparison between Results and Graphics

For examination LSE results with STARNET, firstly data must be transformed to STARNET format. In such case, STARNETFILE program can be helpful. The graphics for this program comprise error ellipse and station numbers for monitoring network.



A



B

Figure 4: ADJMAT Program (A) and STARNET (B) plots

In Table 2 LSE results between two programs are compared. The Global Test, Upper/Lower Bounds (1.0437 and 0.2378) passed in AdjMat program. As seen from the table STARNET also passed Global Test. Both programs performed LSE with 2 iterations. Figure 5 shows coordinate differences between STARNET and AdjMat which the biggest values can be found in station 11 with 0.00005 meter. Subsequently Figure 6 exhibits differences in Error propagation in both programs.

Table 3: Comparison results of LSE between ADJMAT and STARNET

ADJMAT Program	STARNET
Number Of Stations : 12	Number of Stations = 12
Number Of Observations : 37	
Number Of Distances : 15	Number of Observations = 37
Number Of Angles : 0	
Number Of Azimuth : 22	Number of Unknowns = 22
Number Of Unknowns : 22	
Degree Of Freedom : 15	Number of Redundant Obs = 15
Factor variance : 0.4357365	
The Chi-Square Test at 5.00% Level Passed	The Chi-Square Test at 5.00% Level Passed
Upper/Lower Bounds (1.0437 and 0.2378)	Lower/Upper Bounds (0.646/1.354)
Local Test (BARDA) Passed	Convergence Iterations = 2
Number of iterations : 2	

5.0 CONCLUSIONS

A program for 2D network adjustment using MATLAB7, called ADJMAT has been described. Therefore LSE results from one data set are compared with STARNET and they are concluded same results as STARNET with minor difference (e^{-5}).

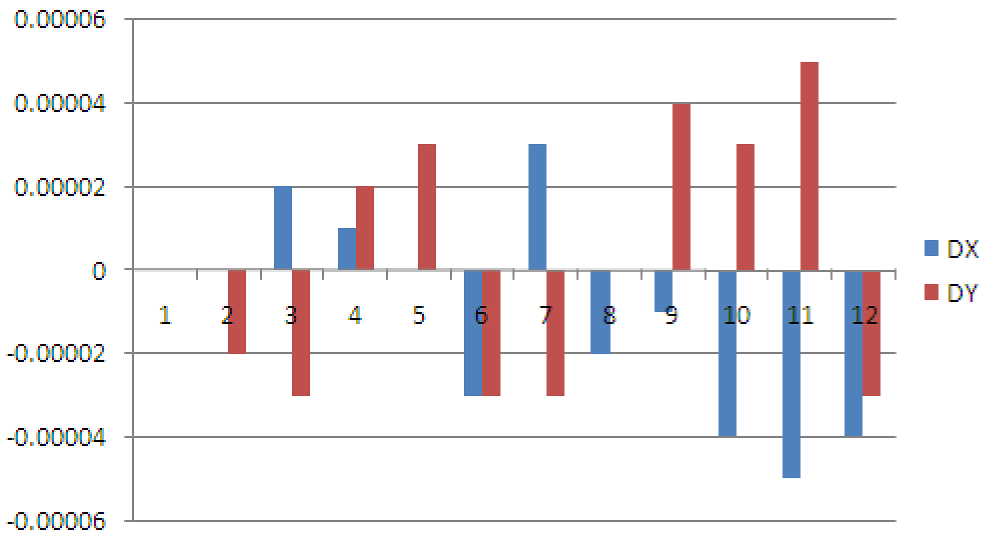


Figure 5: Coordinates comparison between ADJMAT and STARNET (meter)

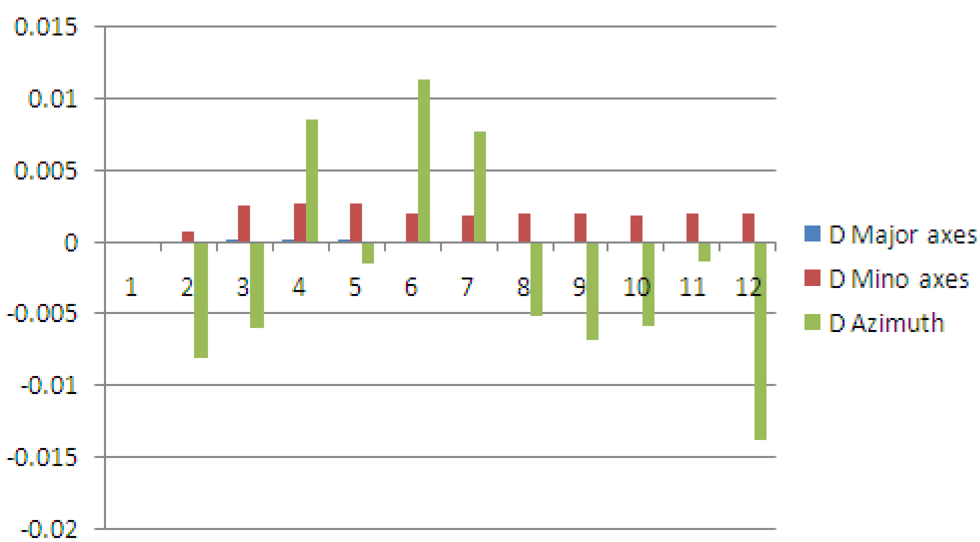


Figure 6: Error Ellipse comparison between ADJMAT and STARNET (Meter for Major and Minor axes and Decimal Degree for Azimuth)

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