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Qualitative Indicators Used For the Optimization of 3D Geodetic Networks

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Abstract

Geodetic network optimization (or geodetic network design) is a notion first introduced by Erik Grafarend (b. 1939). The scope of the optimization is to improve the geodetic network configuration and the measurements plan, in order to achieve the desired accuracy for the final results, in a cost-effective way. In this paper are presented some of the qualitative indicators used for the optimization of the tridimensional geodetic networks, together with a case study which reveals the applicability of these quantities to the presented situation.

Keywords

geodetic network, optimization, measurement, redundancy, reliability.

1. Introduction and theoretical aspects

The reliability of a geodetic network is represented by a set of qualitative indicators which represents measures of network robustness (network behavior in the presence of observations affected by gross errors). The deduction of these quantities is based on the assumption that some measurement are blunders (affected by gross errors).

Redundancy numbers

The redundancy number it is a quantity proper to each measurement and its value is between 0 and 1. This indicator represents the contribution of the respective measurement to the total redundancy of the geodetic network.

$$r_i = q_{v_i v_i} \cdot p_i ,\ i = \overline{1,n} \qquad (1)$$

where:

r_i – the redundancy number of the measurement *i*;

$q_{v_i v_i}$ – the *i*-th element on the main diagonal of the cofactor matrix of the residuals, $\mathbf{Q_{vv}}$;

p_i – weight of the measurement *i*.

In matrix form Eq. (1) can be written as follows:

$$\mathbf{R} = \mathbf{Q_{vv}} \cdot \mathbf{P} \qquad (2)$$

and the vector which contains the redundancy numbers is:

$$\mathbf{r} = diag(\mathbf{R}) \qquad (3)$$

In the case of correlated measurements weight matrix, $\mathbf{P}$, will be a non-diagonal one, so Eq. (2) and (3) must be used to compute the redundancy numbers for the measurements in the geodetic network. This remark is particularly important in the case of GNSS measurements which are, par excellence, correlated observations.

The sum of the redundancy numbers for all measurements is equal with the total redundancy (the degrees of freedom) of the geodetic network:

$$\sum r_i = f \qquad (4)$$

The mean of the redundancy numbers in the network is:

$$\bar{r} = \frac{\sum r_i}{n} = \frac{f}{n} \qquad (5)$$

From the above Eq. it's obvious that the individual values of the redundancy numbers will be higher if the overall redundancy of the geodetic network will be higher.

Minimum detectable error (internal reliability)

The minimum detectable error is the minimum value of a blunder which can be revealed by the used statistical test [1]. This quantity is proper to each measurement in the geodetic network. The study of the minimum detectable error is a way of controlling type II errors, therefore its value will depend on the power of the statistical test.

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$$\nabla_{oi} = \frac{\delta_0}{\sqrt{r_i}}\sigma_i,\ i = \overline{1,n} \qquad (6)$$

δ_0 is called non-centrality parameter and it is determined by the risk factor α (the admitted probability of making a type I error) and by the self-imposed probability of making a type II error, β_0, (related to the power of the test in a complementary fashion [2]).

$$\delta_0 = \delta_0\left(\alpha, \beta_0\right) \qquad (7)$$

In literature is recommended a typical value for δ_0 [3], given by (8):

$$\delta_0 = 4 \qquad (8)$$

For a gross error to be revealed by the statistical test, then it must be higher than minimum detectable error:

$$|\nabla_i| \geq \nabla_{0i} \qquad (9)$$

where:

∇_{oi} – minimum detectable error for the measurement *i*;

∇_i – gross error that affects measurement *i*.

Corresponding to the measurements vector, another vector which contains the values of the minimum detectable errors for each observation, can be drawn:

$$\nabla = \begin{pmatrix}\nabla_{01} & \nabla_{02} & \dots & \nabla_{0n}\end{pmatrix}^T \qquad (10)$$

Observation: Minimum detectable error only depends on network configuration and stochastic model.

Absorption

The residuals vector of the measurements can be obtained from the discrepancies vector, weight matrix and the cofactor matrix of the residuals, with the Eq. below:

$$\mathbf{v} = \mathbf{Q_{vv}Pl} \qquad (11)$$

Assume further that the measurement *i* is affected by the ∇_i error. Then the residuals vector affected by this blunder from the measurement *i*, will be:

$$\mathbf{v} = \mathbf{Q_{vv}P}\left(\mathbf{l} - \mathbf{e_i}\nabla_i\right) \qquad (12)$$

where:

$\mathbf{e_i}$ – vector of nx1 size which contains 1 on the *i*-th position and 0 elsewhere:

$$\mathbf{e_i} = \begin{pmatrix}0 & \dots & 0 & 1 & 0 & \dots & 0\end{pmatrix}^T \qquad (13)$$

The effect of a gross error affecting an observation, on the residual of the respective measurement will be [4]:

$$\nabla v_i = -r_i \nabla_i \qquad (14)$$

An estimation for the blunder ∇_i can be obtained applying a calculation in Eq. (14), knowing that the effect of a gross error (∇v_i) in the residual of the measurement *i* is much higher than the effect of the random errors (v_i^*).

$$\nabla_i = -\frac{\nabla v_i}{r_i} \approx -\frac{v_i^* + \nabla v_i}{r_i} \approx -\frac{v_i}{r_i} \qquad (15)$$

The above Eq. shows that the least square method is not a robust estimator, because the redundancy number is strictly less than 1, only a part of the measurement error will be transferred to that measurement residual. The rest of the error will be distributed in the geodetic network.

Absorption is a measure of the quantity in which an error affecting a measurement is absorbed by the parameters of the model [3]:

$$A_i = (1 - r_i)\nabla_i \qquad (16)$$

or else

$$A_i = -\frac{1 - r_i}{r_i} v_i \qquad (17)$$

As it can be observed, absorption is directly related to the redundancy number of the measurement. The quantity $(1 - r_i)$ is called absorption number [3]. This value is also within the $(0,1)$ range and shows degree in which the gross error affecting a measurement will be distributed throughout the network.

Attention must be paid to the measurements with large absorption numbers. It is likely for a blunder that affect a measurement with low control (high absorption) to be distributed on other network measurements. These measurements (which have a low control), even if are affected by gross errors, can have low residuals, so they must be carefully analyzed before they are accepted as correct measurements.

Observation: The absorption value can be computed only by knowing the residuals, therefore only after the realization of the measurements and the adjustment of the network. On the other hand, the absorption number is the complement of the redundancy number and can be computed *a priori*.

External reliability

The study of external reliability deals with the determination of a set of quantities which emphasize the influence of the blunders on the model parameters.

The effect of a gross error that occurred in the measurement *i*, on the unknowns is:

$$\nabla\mathbf{x} = \mathbf{N}^{-1}\mathbf{A^T P e_i}\nabla_i \qquad (18)$$

The worst case intervenes if the measurement *i* is affected just by the minimum detectable error [5]:

$$\nabla\mathbf{x}_{0i} = \mathbf{N}^{-1}\mathbf{A^T P e_i}\nabla_{0i} \qquad (19)$$

Each observation generates a vector of size hx1 expressing the external reliability, corresponding to the respective measurement, for each parameter:

$$\nabla\mathbf{x}_{0i} = \begin{pmatrix}\nabla x_{0i}^1 & \nabla x_{0i}^2 & \dots & \nabla x_{0i}^h\end{pmatrix}^T \qquad (20)$$

If a diagonal matrix which contains minimum detectable errors for all measurements is considered, then:

$$\nabla \mathbf{x}_0 = \mathbf{N}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P} \cdot \begin{pmatrix} \nabla_{01} & & & 0 \\ & \nabla_{02} & & \\ & & \ddots & \\ 0 & & & \nabla_{0n} \end{pmatrix} \qquad (21)$$

The matrix from Eq. (21) is of hxn size and contains on line *i* the external reliability corresponding to parameter *i* for all measurements in the network and, on column *j*, the external reliability generated by the observation *j*.

To synthesize the value of the external reliability, for each measurement supposed to be affected by a gross error equal with the minimum detectable error, a global indicator for the external reliability will be defined:

$$\lambda_{0i}^2 = \frac{\nabla \mathbf{x}_{0i}^{\mathbf{T}} \cdot \mathbf{N} \cdot \nabla \mathbf{x}_{0i}}{\sigma_0^2} \qquad (22)$$

if we perform calculations in Eq. (22), then:

$$\lambda_{0i}^2 = \frac{1 - r_i}{r_i}\delta_0^2 \qquad (23)$$

Eq. (23) is valid only in the case of uncorrelated measurements. If in the network the measurements are correlated (which is the case for the geodetic networks realized by GNSS technology), then, for computing the global indicator λ_{0i}^2, Eq. (22) must be used.

For every measurement such global indicator, λ_{0i}^2, can be computed. If the values are of the same order of magnitude the network is considered homogeneous.

Observation: Both global indicators and local indicators for the external reliability are independent of the measured values, so they can be determined before realizing the measurements and processing the network.

2. Case study

In this section a case study regarding the analysis and the optimization of a combined geodetic network is presented. The adjustment of the measurements is carried out using local ellipsoidal coordinate system. To realize the adjustment and to compute the qualitative indicators, a MATLAB language application was developed.

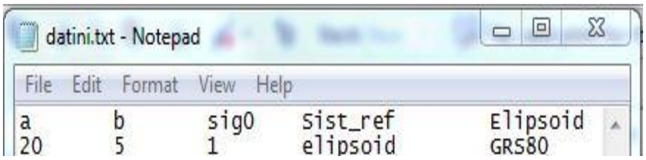
Initial data

The initial data (measurements and provisional coordinates) for this case study are taken from [6]. Some modifications were introduced in this data in order to make them suitable for the desired purpose (conversions between sexagesimal degrees and centesimal degrees, the introduction of different standard deviations for the measurements et al). The characteristics of the tridimensional geodetic network are shown in Table 1. Block processing of the terrestrial and spatial measurements from this network was presented as a case study [7].

Table 1 **Tridimensional geodetic network characteristics**

Reference system	WGS84
Coordinate system	Local ellipsoidal
Ellipsoid	GRS80
Network dimension	3D
Network type	Inner constrained
No. of points	5
No. of fixed points	0
No. of free points	5
Position unknowns	15
No. of stations	3
No. of measurements	31 (32)
No. of horizontal directions	9 (10)
No. of distances	5
No. of vertical directions	5
No. of height differences	3
No. of GNSS baselines	3x3=9
Redundancy	16 (17)
Rank Defect	3

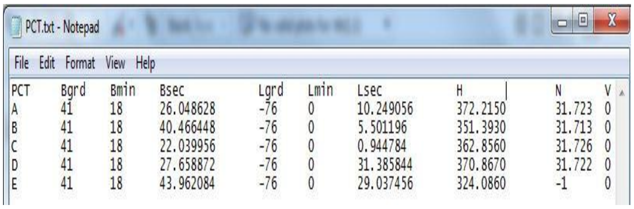
The initial data regarding the characteristics of the distances measurement instrument, the reference ellipsoid and the *a priori* standard deviation of the unit weight, are taken by the developed algorithm, from the file shown below.



```
a       b       sig0     sist_ref            Elipsoid
20      5       1        elipsoid            GRS80
```

Fig. 1 Initial data file

The file which contains the provisional coordinates (expressed in geodetic coordinate system) used for computing coefficients for the observations equations, is shown in Fig. 2.



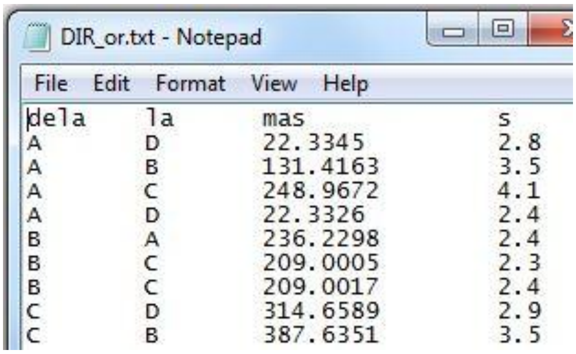
```
PCT   Bgrd   Bmin   Bsec        Lgrd   Lmin   Lsec        H          N        V
A     41     18     26.048628   -76    0      10.249056   372.2150   31.723   0
B     41     18     40.466448   -76    0      5.501196    351.3930   31.713   0
C     41     18     22.039956   -76    0      0.944784    362.8560   31.726   0
D     41     18     27.658872   -76    0      31.385844   370.8670   31.722   0
E     41     18     43.962084   -76    0      29.037456   324.0860   -1       0
```

Fig. 2 Provisional coordinates of points in the 3D geodetic network

Measurements realized in the geodetic network

The horizontal directions measurements performed in the studied geodetic network, are shown

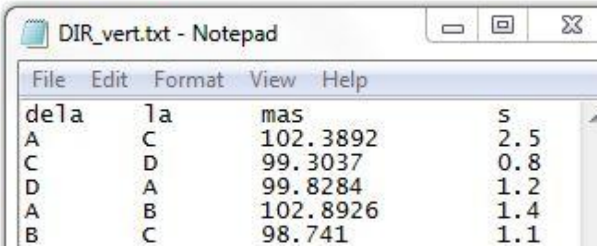
Fig. 3. On the last column are shown the standard deviations for each measurement. This quantities are used for weights computation.



```
dela    la      mas          s
A       D       22.3345      2.8
A       B       131.4163     3.5
A       C       248.9672     4.1
A       D       22.3326      2.4
B       A       236.2298     2.4
B       C       209.0005     2.3
B       C       209.0017     2.4
C       D       314.6589     2.9
C       B       387.6351     3.5
```

Fig. 3 Horizontal directions measurements and their standard deviations

In Fig. 4 horizontal directions measurements and their standard deviations are presented.



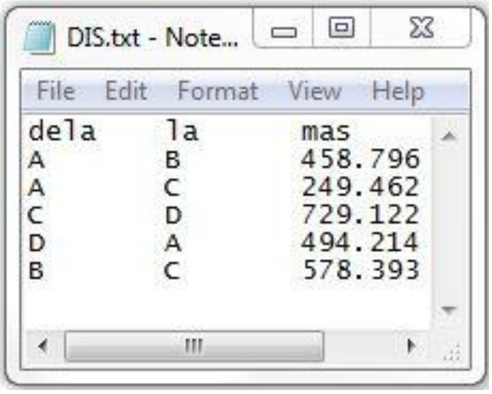
```
dela    la      mas          s
A       C       102.3892     2.5
C       D       99.3037      0.8
D       A       99.8284      1.2
A       B       102.8926     1.4
B       C       98.741       1.1
```

Fig. 4 Vertical angles measurements and their standard deviations

The distances measurements are shown in the file from Fig. 5. The standard deviations used for determining the weights are computed using the characteristics of the distances measurement instrument with the Eq.:

$$\sigma_D = a + b \cdot D[km] \qquad (24)$$

a, b - constants offered by the producing company or resulting from a calibration process.

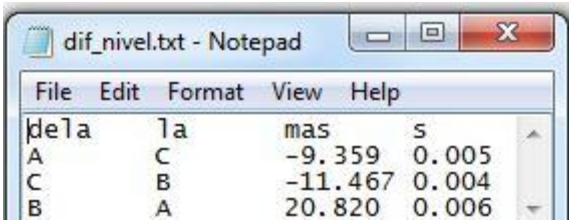


```
dela    la      mas
A       B       458.796
A       C       249.462
C       D       729.122
D       A       494.214
B       C       578.393
```

Fig. 5 Distances measurements

The file with the measured height differences is presented in Fig. 6.

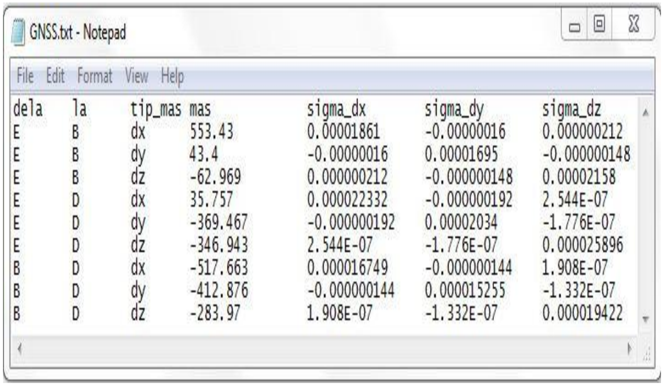
On the last column, the standard deviations for these observations can be found.



```
dela    la      mas       s
A       C       -9.359    0.005
C       B       -11.467   0.004
B       A       20.820    0.006
```

Fig. 6 Height differences measurements and their standard deviations

GNSS baselines measurements are taken by the algorithm from the file below. On the last three columns the variance-covariance matrices regarding each baseline, are shown. These matrices will be used for weight matrix computation.



```
dela    la      tip_mas mas          sigma_dx        sigma_dy        sigma_dz
E       B       dx      553.43       0.00001861      -0.00000016     0.000000212
E       B       dy      43.4         -0.00000016     0.00001695      -0.000000148
E       B       dz      -62.969      0.000000212     -0.000000148    0.00002158
E       D       dx      35.757       0.000022332     -0.000000192    2.544E-07
E       D       dy      -369.467     -0.000000192    0.00002034      -1.776E-07
E       D       dz      -346.943     2.544E-07       -1.776E-07      0.000025896
B       D       dx      -517.663     0.000016749     -0.000000144    1.908E-07
B       D       dy      -412.876     -0.000000144    0.000015255     -1.332E-07
B       D       dz      -283.97      1.908E-07       -1.332E-07      0.000019422
```

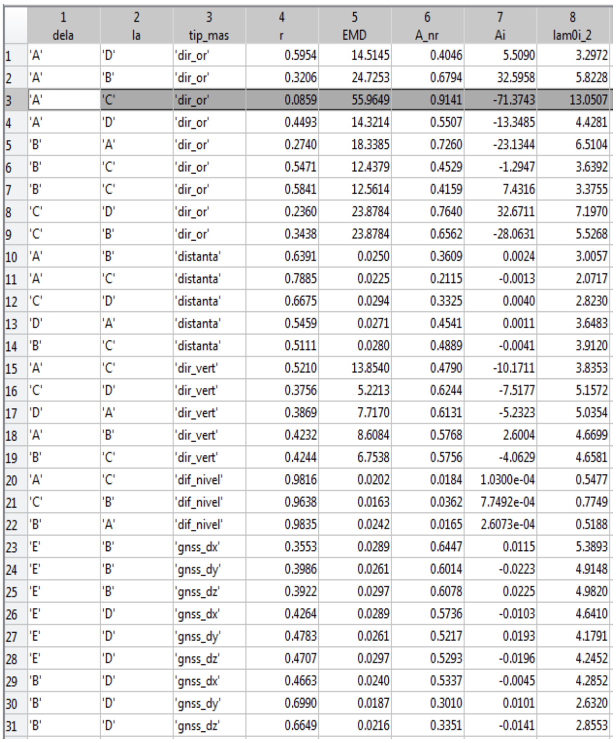
Fig. 7 GNSS baselines measurements and their variance-covariance matrices

The analysis and the optimization of the geodetic network

With the help of the developed MATLAB language algorithm, qualitative indicators regarding the reliability of the studied network, were computed. As it has been said for computing these indicators only the *a priori* functional-stochastic model is needed. This model is determined by the location of the network points (provisional coordinates) and by the measurements which are intended to be made between these points. These elements were presented so far.

In Fig. 8 are shown the qualitative indicators regarding the internal and external reliability for the studied network. The significance of the columns of the table presented in the figure is as follows:

Columns 1, 2 (dela, la) – the points between which the measurement is performed;
Column 3 (tip_mas) – measurement type;
Column 4 (r) – redundancy number;
Column 5 (EMD) – minimum detectable error;
Column 6 (A_nr) – absorption number;
Column 7 (Ai) - absorption;
Column 8 (lam0i_2) – global indicator regarding the external reliability.

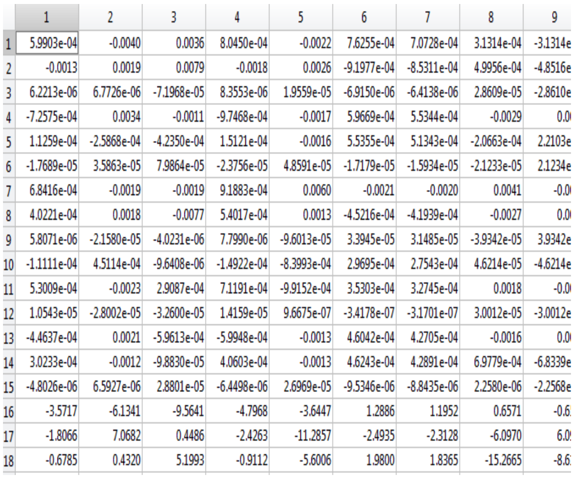


	1 dela	2 la	3 tip_mas	4 r	5 EMD	6 A_nr	7 Ai	8 lam0i_2
1	'A'	'D'	'dir_or'	0.5954	14.5145	0.4046	5.5090	3.2972
2	'A'	'B'	'dir_or'	0.3206	24.7253	0.6794	32.5958	5.8228
3	'A'	'C'	'dir_or'	0.0859	55.9649	0.9141	-71.3743	13.0507
4	'A'	'D'	'dir_or'	0.4493	14.3214	0.5507	-13.3485	4.4281
5	'B'	'A'	'dir_or'	0.2740	18.3385	0.7260	-23.1344	6.5104
6	'B'	'C'	'dir_or'	0.5471	12.4379	0.4529	-1.2947	3.6392
7	'B'	'C'	'dir_or'	0.5841	12.5614	0.4159	7.4316	3.3755
8	'C'	'D'	'dir_or'	0.2360	23.8784	0.7640	32.6711	7.1970
9	'C'	'B'	'dir_or'	0.3438	23.8784	0.6562	-28.0631	5.5268
10	'A'	'B'	'distanta'	0.6391	0.0250	0.3609	0.0024	3.0057
11	'A'	'C'	'distanta'	0.7885	0.0225	0.2115	-0.0013	2.0717
12	'C'	'D'	'distanta'	0.6675	0.0294	0.3325	0.0040	2.8230
13	'D'	'A'	'distanta'	0.5459	0.0271	0.4541	0.0011	3.6483
14	'B'	'C'	'distanta'	0.5111	0.0280	0.4889	-0.0041	3.9120
15	'A'	'C'	'dir_vert'	0.5210	13.8540	0.4790	-10.1711	3.8353
16	'C'	'D'	'dir_vert'	0.3756	5.2213	0.6244	-7.5177	5.1572
17	'D'	'A'	'dir_vert'	0.3869	7.7170	0.6131	-5.2323	5.0354
18	'A'	'B'	'dir_vert'	0.4232	8.6084	0.5768	2.6004	4.6699
19	'B'	'C'	'dir_vert'	0.4244	6.7538	0.5756	-4.0629	4.6581
20	'A'	'C'	'dif_nivel'	0.9816	0.0202	0.0184	1.0300e-04	0.5477
21	'C'	'B'	'dif_nivel'	0.9638	0.0163	0.0362	7.7492e-04	0.7749
22	'B'	'A'	'dif_nivel'	0.9835	0.0242	0.0165	2.6073e-04	0.5188
23	'E'	'B'	'gnss_dx'	0.3553	0.0289	0.6447	0.0115	5.3893
24	'E'	'B'	'gnss_dy'	0.3986	0.0261	0.6014	-0.0223	4.9148
25	'E'	'B'	'gnss_dz'	0.3922	0.0297	0.6078	0.0225	4.9820
26	'E'	'D'	'gnss_dx'	0.4264	0.0289	0.5736	-0.0103	4.6410
27	'E'	'D'	'gnss_dy'	0.4783	0.0261	0.5217	0.0193	4.1791
28	'E'	'D'	'gnss_dz'	0.4707	0.0297	0.5293	-0.0196	4.2452
29	'B'	'D'	'gnss_dx'	0.4663	0.0240	0.5337	-0.0045	4.2852
30	'B'	'D'	'gnss_dy'	0.6990	0.0187	0.3010	0.0101	2.6320
31	'B'	'D'	'gnss_dz'	0.6649	0.0216	0.3351	-0.0141	2.8553

Fig. 8 Qualitative indicators regarding the reliability of the geodetic network – before optimization

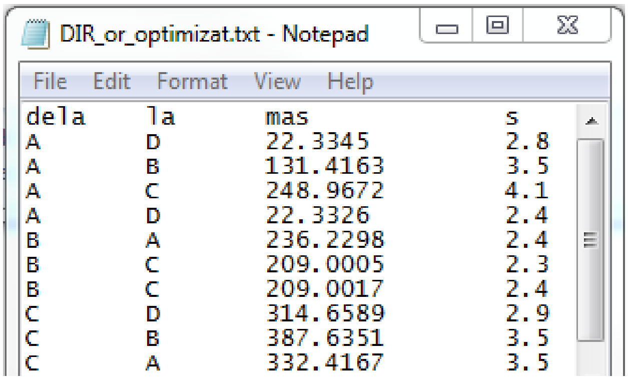
In Fig. 9 an excerpt from the matrix which contains the local external reliability indicators is presented. The meaning of the rows and columns of this matrix is explained in Section 1, Eq. (21) regarding the external reliability.

Studying the network reliability indicators it can be seen that the measurement no. (AC horizontal direction) shows very weak qualitative indicators (beginning with the redundancy number and ending with the external reliability global indicator). The consequence is that the reliability will be lower and the network will be inhomogeneous.



	1	2	3	4	5	6	7	8	9
1	5.9903e-04	-0.0040	0.0036	8.0450e-04	-0.0022	7.6255e-04	7.0728e-04	3.1314e-04	-3.1314e
2	-0.0013	0.0019	0.0079	-0.0018	0.0026	-9.1977e-04	-8.5311e-04	4.9956e-04	-4.8516e
3	6.2213e-06	6.7726e-06	-7.1968e-05	8.3553e-06	1.9559e-05	-6.9150e-06	-6.4138e-06	2.8609e-05	-2.8610e
4	-7.2575e-04	0.0034	-0.0011	-9.7468e-04	-0.0017	5.9669e-04	5.5344e-04	-0.0029	0.0
5	1.1259e-04	-2.5868e-04	-4.2350e-04	1.5121e-04	-0.0016	5.5355e-04	5.1343e-04	-2.0663e-04	2.2103e
6	-1.7689e-05	3.5863e-05	7.9864e-05	-2.3756e-05	4.8591e-05	-1.7179e-05	-1.5934e-05	-2.1233e-05	2.1234e
7	6.8416e-04	-0.0019	-0.0019	9.1883e-04	0.0060	-0.0021	-0.0020	0.0041	-0.0
8	4.0221e-04	0.0018	-0.0077	5.4017e-04	0.0013	-4.5216e-04	-4.1939e-04	-0.0027	0.0
9	5.8071e-06	-2.1580e-05	-4.0231e-06	7.7990e-06	-9.6013e-05	3.3945e-05	3.1485e-05	-3.9342e-05	3.9342e
10	-1.1111e-04	4.5114e-04	-9.6408e-06	-1.4922e-04	-8.3993e-04	2.9695e-04	2.7543e-04	4.6214e-05	-4.6214e
11	5.3009e-04	-0.0023	2.9087e-04	7.1191e-04	-9.9152e-04	3.5303e-04	3.2745e-04	0.0018	-0.0
12	1.0543e-05	-2.8002e-05	-3.2600e-05	1.4159e-05	9.6675e-07	-3.4178e-07	-3.1701e-07	3.0012e-05	-3.0012e
13	-4.4637e-04	0.0021	-5.9613e-04	-5.9948e-04	-0.0013	4.6042e-04	4.2705e-04	-0.0016	0.0
14	3.0233e-04	-0.0012	-9.8830e-05	4.0603e-04	-0.0013	4.6243e-04	4.2891e-04	6.9779e-04	-6.8339e
15	-4.8026e-06	6.5927e-06	2.8801e-05	-6.4498e-06	2.6969e-05	-9.5346e-06	-8.8435e-06	2.2580e-06	-2.2568e
16	-3.5717	-6.1341	-9.5641	-4.7968	-3.6447	1.2886	1.1952	0.6571	-0.6
17	-1.8066	7.0682	0.4486	-2.4263	-11.2857	-2.4935	-2.3128	-6.0970	6.0
18	-0.6785	0.4320	5.1993	-0.9112	-5.6006	1.9800	1.8365	-15.2665	-8.6

Fig. 9 Local indicators regarding the external reliability of the studied geodetic network – before optimization

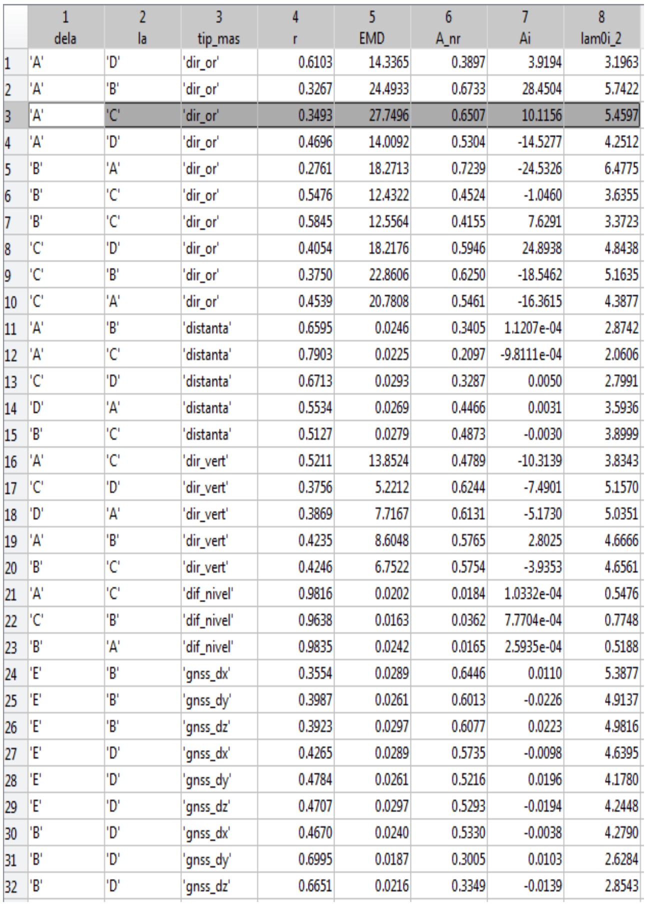


DIR_or_optimizat.txt - Notepad

```
dela     la       mas             s
A        D        22.3345         2.8
A        B        131.4163        3.5
A        C        248.9672        4.1
A        D        22.3326         2.4
B        A        236.2298        2.4
B        C        209.0005        2.3
B        C        209.0017        2.4
C        D        314.6589        2.9
C        B        387.6351        3.5
C        A        332.4167        3.5
```

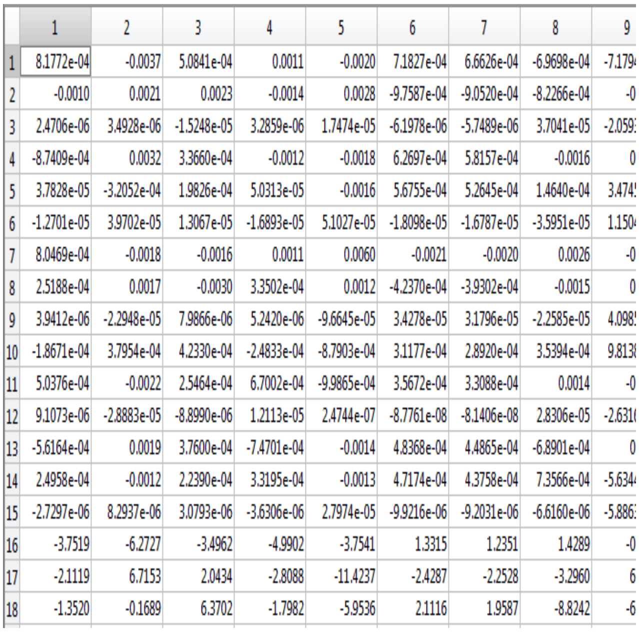
Fig. 10. Horizontal directions provided to be measured - after optimization

The intention is to optimize the measurements plan in order to improve the reliability of this measurement. At a closer look, a potential cause for which that measurement is unreliable can be identified. In the measurements plan the “backward” direction, CA, was not provided. The measurements plan will be improved by adding the CA horizontal direction. The new file with the horizontal directions provided to be measured is shown in Fig. 10.



	1 dela	2 la	3 tip_mas	4 r	5 EMD	6 A_nr	7 Ai	8 lam0i_2
1	'A'	'D'	'dir_or'	0.6103	14.3365	0.3897	3.9194	3.1963
2	'A'	'B'	'dir_or'	0.3267	24.4933	0.6733	28.4504	5.7422
3	'A'	'C'	'dir_or'	0.3493	27.7496	0.6507	10.1156	5.4597
4	'A'	'D'	'dir_or'	0.4696	14.0092	0.5304	-14.5277	4.2512
5	'B'	'A'	'dir_or'	0.2761	18.2713	0.7239	-24.5326	6.4775
6	'B'	'C'	'dir_or'	0.5476	12.4322	0.4524	-1.0460	3.6355
7	'B'	'C'	'dir_or'	0.5845	12.5564	0.4155	7.6291	3.3723
8	'C'	'D'	'dir_or'	0.4054	18.2176	0.5946	24.8938	4.8438
9	'C'	'B'	'dir_or'	0.3750	22.8606	0.6250	-18.5462	5.1635
10	'C'	'A'	'dir_or'	0.4539	20.7808	0.5461	-16.3615	4.3877
11	'A'	'B'	'distanta'	0.6595	0.0246	0.3405	1.1207e-04	2.8742
12	'A'	'C'	'distanta'	0.7903	0.0225	0.2097	-9.8111e-04	2.0606
13	'C'	'D'	'distanta'	0.6713	0.0293	0.3287	0.0050	2.7991
14	'D'	'A'	'distanta'	0.5534	0.0269	0.4466	0.0031	3.5936
15	'B'	'C'	'distanta'	0.5127	0.0279	0.4873	-0.0030	3.8999
16	'A'	'C'	'dir_vert'	0.5211	13.8524	0.4789	-10.3139	3.8343
17	'C'	'D'	'dir_vert'	0.3756	5.2212	0.6244	-7.4901	5.1570
18	'D'	'A'	'dir_vert'	0.3869	7.7167	0.6131	-5.1730	5.0351
19	'A'	'B'	'dir_vert'	0.4235	8.6048	0.5765	2.8025	4.6666
20	'B'	'C'	'dir_vert'	0.4246	6.7522	0.5754	-3.9353	4.6561
21	'A'	'C'	'dif_nivel'	0.9816	0.0202	0.0184	1.0332e-04	0.5476
22	'C'	'B'	'dif_nivel'	0.9638	0.0163	0.0362	7.7704e-04	0.7748
23	'B'	'A'	'dif_nivel'	0.9835	0.0242	0.0165	2.5935e-04	0.5188
24	'E'	'B'	'gnss_dx'	0.3554	0.0289	0.6446	0.0110	5.3877
25	'E'	'B'	'gnss_dy'	0.3987	0.0261	0.6013	-0.0226	4.9137
26	'E'	'B'	'gnss_dz'	0.3923	0.0297	0.6077	0.0223	4.9816
27	'E'	'D'	'gnss_dx'	0.4265	0.0289	0.5735	-0.0098	4.6395
28	'E'	'D'	'gnss_dy'	0.4784	0.0261	0.5216	0.0196	4.1780
29	'E'	'D'	'gnss_dz'	0.4707	0.0297	0.5293	-0.0194	4.2448
30	'B'	'D'	'gnss_dx'	0.4670	0.0240	0.5330	-0.0038	4.2790
31	'B'	'D'	'gnss_dy'	0.6995	0.0187	0.3005	0.0103	2.6284
32	'B'	'D'	'gnss_dz'	0.6651	0.0216	0.3349	-0.0139	2.8543

Fig. 11 Qualitative indicators regarding the reliability of the geodetic network – after optimization



	1	2	3	4	5	6	7	8	9
1	8.1772e-04	-0.0037	5.0841e-04	0.0011	-0.0020	7.1827e-04	6.6626e-04	-6.9698e-04	-7.179
2	-0.0010	0.0021	0.0023	-0.0014	0.0028	-9.7587e-04	-9.0520e-04	-8.2266e-04	-0
3	2.4706e-06	3.4928e-06	-1.5248e-05	3.2859e-06	1.7474e-05	-6.1978e-06	-5.7489e-06	3.7041e-05	-2.059
4	-8.7409e-04	0.0032	3.3660e-04	-0.0012	-0.0018	6.2697e-04	5.8157e-04	-0.0016	0
5	3.7828e-05	-3.2052e-04	1.9826e-04	5.0313e-05	-0.0016	5.6755e-04	5.2645e-04	1.4640e-04	3.474
6	-1.2701e-05	3.9702e-05	1.3067e-05	-1.6893e-05	5.1027e-05	-1.8098e-05	-1.6787e-05	-3.5951e-05	1.150
7	8.0469e-04	-0.0018	-0.0016	0.0011	0.0060	-0.0021	-0.0020	0.0026	-0
8	2.5188e-04	0.0017	-0.0030	3.3502e-04	0.0012	-4.2370e-04	-3.9302e-04	-0.0015	0
9	3.9412e-06	-2.2948e-05	7.9866e-06	5.2420e-06	-9.6645e-05	3.4278e-05	3.1796e-05	-2.2585e-05	4.098
10	-1.8671e-04	3.7954e-04	4.2330e-04	-2.4833e-04	-8.7903e-04	3.1177e-04	2.8920e-04	3.5394e-04	9.813
11	5.0376e-04	-0.0022	2.5464e-04	6.7002e-04	-9.9865e-04	3.5672e-04	3.3088e-04	0.0014	-0
12	9.1073e-06	-2.8883e-05	-8.8990e-06	1.2113e-05	2.4744e-07	-8.7761e-08	-8.1406e-08	2.8306e-05	-2.631
13	-5.6164e-04	0.0019	3.7600e-04	-7.4701e-04	-0.0014	4.8368e-04	4.4865e-04	-6.8901e-04	0
14	2.4958e-04	-0.0012	2.2390e-04	3.3195e-04	-0.0013	4.7174e-04	4.3758e-04	7.3566e-04	-5.634
15	-2.7297e-06	8.2937e-06	3.0793e-06	-3.6306e-06	2.7974e-05	-9.9216e-06	-9.2031e-06	-6.6160e-06	-5.886
16	-3.7519	-6.2727	-3.4962	-4.9902	-3.7541	1.3315	1.2351	1.4289	-0
17	-2.1119	6.7153	2.0434	-2.8088	-11.4237	-2.4287	-2.2528	-3.2960	6
18	-1.3520	-0.1689	6.3702	-1.7982	-5.9536	2.1116	1.9587	-8.8242	-6

Fig. 12 Local indicators regarding the external reliability of the studied geodetic network – after optimization

The qualitative indicators regarding the reliability of the network will be computed again after the optimization of the measurements plan. These new indicators are presented in Fig. 11.

Fig. 11It can be seen that there is a significant improvement of the indicators characterizing the reliability of the measurement no. 3 (AC horizontal direction).

Also an improvement of the network in terms of homogeneity can be observed. The values of the qualitative indicators are now of the same order of magnitude.

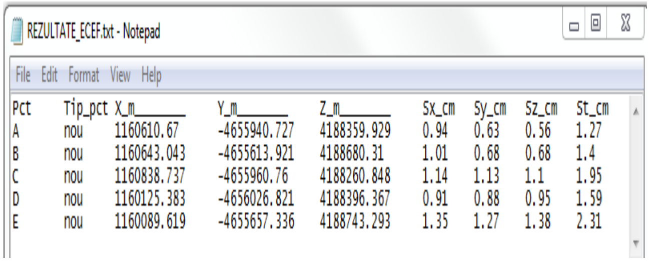
It can be seen that in the matrix that contains the local indicators regarding the external reliability there is a significant improvement of the values regarding AC direction (column 3).

In practice an iterative method can be considered: first the lowest redundancy measurement must be identified and extra measurements will be added in that area until the qualitative indicators will meet the requirements. Also, to not unnecessary load the measurements plan, the highest redundancy measurement can be eliminated. After each change in the measurements plan the application will be run in order to estimate again the values of the reliability indicators. In this way the external reliability indicators will become of the same order of magnitude for all measurements, so the network will become homogenous.

Final results and geodetic network plot

Measurement (after optimization) were adjusted together in the geodetic network. The adjustment was realized using a mathematical model based on the local ellipsoidal coordinate system. To express the results in the Cartesian geocentric coordinate system or in the geodetic coordinate system, a conversion from the local ellipsoidal system was performed.

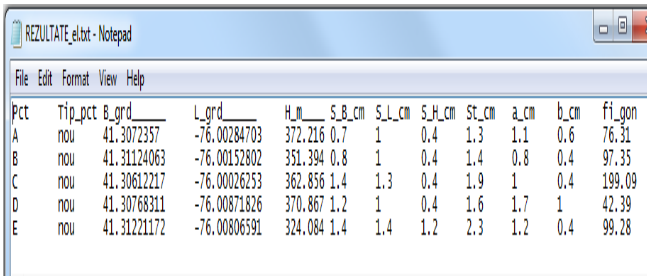
In Fig. 13 are shown the results in the Cartesian geocentric system, obtained following the adjustment performed using the model mentioned above.



```
Pct     Tip_pct X_m_______    Y_m_______    Z_m_______    Sx_cm  Sy_cm  Sz_cm  St_cm
A       nou     1160610.67    -4655940.727  4188359.929   0.94   0.63   0.56   1.27
B       nou     1160643.043   -4655613.921  4188680.31    1.01   0.68   0.68   1.4
C       nou     1160838.737   -4655960.76   4188260.848   1.14   1.13   1.1    1.95
D       nou     1160125.383   -4656026.821  4188396.367   0.91   0.88   0.95   1.59
E       nou     1160089.619   -4655657.336  4188743.293   1.35   1.27   1.38   2.31
```

Fig. 13 Results in Cartesian geocentric coordinate system

In Fig. 14 are presented the results in the geodetic coordinate system, obtained following the adjustment performed using the model based on the local ellipsoidal coordinate system.



```
Pct     Tip_pct B_grd_____    L_grd_____    H_m___ S_B_cm S_L_cm S_H_cm St_cm  a_cm   b_cm   fi_gon
A       nou     41.3072357    -76.00284703  372.216 0.7   1      0.4    1.3    1.1    0.6    76.31
B       nou     41.31124063   -76.00152802  351.394 0.8   1      0.4    1.4    0.8    0.4    97.35
C       nou     41.30612217   -76.00026253  362.856 1.4   1.3    0.4    1.9    1      0.4    199.09
D       nou     41.30768311   -76.00871826  370.867 1.2   1      0.4    1.6    1.7    1      42.39
E       nou     41.31221172   -76.00806591  324.084 1.4   1.4    1.2    2.3    1.2    0.4    99.28
```

Fig. 14 Results in geodetic coordinate system

3. Conclusions

In this paper the qualitative indicators which characterize the reliability of a geodetic network were presented. Those indicators can be global or local and they are a measure of network robustness in the sense of its resistance when blunders are present in the measurements. Those indicators can be computed if the *a priori* functional and stochastic models are known, without the necessity of effectively realizing the measurements in the network. So this indicators must be used to analyze and to optimize the geodetic network in design phase.

More specifically said, the qualitative indicators characterizing the reliability of the geodetic networks, can be determined as soon as the network configuration, the measurements plan and the instruments to be used (which determines the *a priori* stochastic model), are set. So the study of the geodetic networks in terms of reliability helps to improve their configuration and the measurements plan (stages called by E. Grafarend First Order Design – FOD and Second Order Design – SOD). The indicators determined in these stages, as it has been said, can be used to analyze and propose solutions to improve the quality of the respective geodetic network.

Although the focus is on 3D geodetic networks, the qualitative indicators presented throughout this paper characterize the reliability of geodetic networks of any kind.

Of all network reliability indicators one stands out as important. It is network redundancy. This should not be seen only as global redundancy (as the number of additional measurements carried out in the network) but especially as local redundancy (given by the redundancy number). With the help of this quantity, week points of the network can be identified and also network areas where measurements are more than necessary. This indicator is used to compute the other geodetic network reliability indicators.

Basically the particular importance of the redundancy and the redundancy numbers, together with their influence on all other network reliability indicators, means that nothing can replace the contribution that the additional measurements have for the realization of a qualitative geodetic network.

In the case study presented was shown that the developed algorithms for processing and analyzing geodetic networks can be useful for the optimization of the measurements plan.

As it has been seen, computing the qualitative indicators revealed a lower confidence measurement that was also leading to a network inhomogeneity.

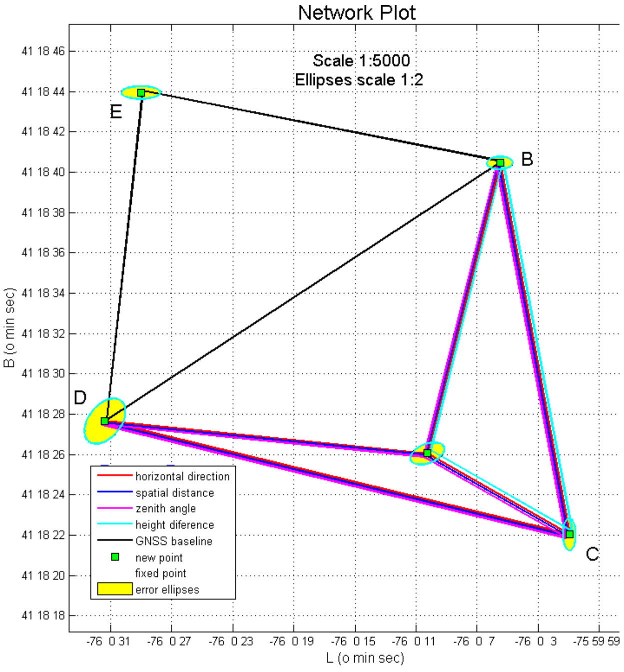


Fig. 15 Geodetic network

Then the problem was addressed precisely so, by adding a single measurement, the indicators which characterized the reliability and the homogeneity of the network, were improved.

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