DESIGN OF GEODETIC NETWORKS BY USING GLOBAL OPTIMIZATION METHODS

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Summary: *Traditionally, in order to solve geodetic network optimization problems analytical methods such as linear and nonlinear programming or simulation methods are used. However, global optimization techniques such as genetic algorithm (GA) and particle swarm optimization (PSO) algorithm are increasingly used today to solve geodetic network optimization problems. In this paper, an application of these algorithms in second order design optimization of the geodetic network for the dam Liverovići is proposed. Since the subject of the analysis is the geodetic network for monitoring the dam, an optimality criterion is defined based on the sensitivity parameters of the geodetic network.*

Keywords: *optimization of geodetic networks, genetic algorithm, particle swarm optimization*

1. INTRODUCTION

Optimization can be defined as the process of achieving the best solution of the problem in relation to the given objective function while, in turn, some restrictions are satisfied. Another definition of optimization is to minimize or maximize the objective function. The main goals of the geodetic network optimization include designing an optimal network configuration and/or optimal observation plan in terms of satisfying the required

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network quality criteria. Different design criteria that represent the quality of the geodetic network can be used as objective functions [1]. In general, the quality of the geodetic network can be measured by precision and reliability.

Numerous intelligent optimization algorithms, such as neural network algorithms, *Genetic Algorithms* (GA), and *Particle Swarm Optimization* (PSO) algorithms, have attracted increasing attention due to the rapid development of computer technology over the past several years. PSO is a new evolutionary technique inspired by the simulation of social behavior [2], [3]. Due to global search capabilities, small number of adjustable parameters and high-speed calculation, the PSO algorithm has attracted considerable attention [4].

The paper analyzes the possibility of applying global optimization techniques in the process of optimizing the second order design of the geodetic network for the monitoring of the dam Liverovići.

2. NETWORK QUALITY CRITERIA

The success of the optimization is defined by the criteria of qualities of geodetic networks, which include the criteria of precision, homogeneity, isotropy, reliability, sensitivity and economy. The first three criteria usually appear integrated as accuracy, which represent statistical quality. First of all, they provide information about the presence and impact of random measurement errors. They can be divided into global, which relate to the quality of the network as an entirety, and local, which relate to particular points of the network [5].

The theory of reliability of geodetic networks refers to the possibility of identifying outliers using statistical tests, as well as determining their impact on the estimates of the required values if the outliers remain undetected. Reliability measures are based on the Gauss-Markov model and the corresponding assumptions which apply when it comes to the characteristics of the observation schedule. According to [6], the reliability analysis refers to the internal and external reliability of the geodetic network. Internal reliability represents the power or ability to control the measurement results in the adjustment process, ie. study the possibility of eliminating outliers from the results of the measured values in order to ensure their quality. External reliability deals with the consequences of undetected outliers. The criteria for internal and external reliability can be global, which serve to determine the impact of outliers on the whole network or parts of it, or local ones, which reveal outliers in individual observations. In both cases, the design of the network has a crucial role.

The sensitivity criteria for geodetic networks relate to the size of the deformations that can be identified for a particular geodetic network design and the planned measurement accuracy [5]. Sensitivity analysis is necessary to establish a measure for the quality of the monitoring network [7], [8]. From a separate adjustment, one obtains the parameter estimates $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ and their cofactor matrices, $\mathbf{Q}_{\hat{\mathbf{x}}_1}$ and $\mathbf{Q}_{\hat{\mathbf{x}}_2}$.

The global congruence test of the null hypothesis:

$$H_0: E(\mathbf{d}) = 0 \tag{1}$$

is given by the test statistics [7]:

$$T_{|H_0} = \frac{\mathbf{d}^T\mathbf{Q}_\mathbf{d}^+\mathbf{d}}{h\hat{\sigma}_0^2}|_{H_0} \sim F_{h,f} \tag{2}$$

where $\mathbf{d} = \hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2$ represents the displacement vector, $\mathbf{Q}_\mathbf{d}$=$\mathbf{Q}_{\hat{\mathbf{x}}_1} + \mathbf{Q}_{\hat{\mathbf{x}}_2}$ is the cofactor displacement matrix, $f = f_1 + f_2$ the unified number of degrees of freedom and h=rank($\mathbf{Q}_\mathbf{d}$). If H_0 holds, T follows the central F-distribution. For an alternative hypothesis:

$$H_0: E(\mathbf{d}) \neq 0 \tag{3}$$

the statistics T is distributed according to the non-central F-distribution with the non-centrality parameter:

$$\lambda = \frac{\mathbf{d}^T\mathbf{Q}_\mathbf{d}^+\mathbf{d}}{\sigma_0^2} \tag{4}$$

Now the test power is defined as the probability $1 - \beta$ which will lead d to the rejection of H_0 at a level of significance α. If a 1-dimensional test is performed and the size is chosen as $\alpha = 0.05$ and the test power $1 - \beta = 0.80$, f=2, a value of non- centrality parameter in $\chi^2(f)$ is λ_0=32 [8]. If the non-centrality parameter λ should be equal to the theoretical value $\lambda_0 = f(h, \alpha_0, \beta_0)$ in the previous term [6], then one can write:

$$\mathbf{d}^T\mathbf{Q}_\mathbf{d}^+\mathbf{d} = \sigma_0^2 \lambda_0 \tag{5}$$

Thenceforth the displacement vector can be expressed as:

$$\mathbf{d} = a\mathbf{g} \tag{6}$$

where a is a scalar to be determined and **g** is the form vector, i.e. the relative magnitude of movements which is known. If expression in Eq. (6) is substituted in Eq. (5), the minimum value of the displacement which can be detected in the direction of a given vector **g** can be obtained [9]:

$$a_{min} = \sigma_0 \sqrt{\frac{\lambda_0}{\mathbf{g}^T\mathbf{Q}_\mathbf{d}^+\mathbf{g}}} \tag{7}$$

Further, based on expression in Eq. (6) one can determine the minimum deformation which can be detected in the direction of a given vector **g**:

$$\mathbf{d}_{\mathbf{min}} = a_{min}\mathbf{g} \tag{8}$$

It is very important to determine the directions in which one will obtain the weakest estimate of unknown parameters. These directions are actually the directions of the largest axis of the confidence ellipsoid (ellipse) [9].

3. OPTIMIZATION OF GEODETIC NETWORKS

Optimization is a scientific discipline whose purpose is to determine the "best" solutions for certain mathematically defined problems, which often represent a physical reality. Optimization of geodetic networks gives answers to the questions: where are the optimal position of the points in the network, how to measure the geodetic network in order to achieve the results of the required quality, how to design and materialize the geodetic network with the lowest possible costs, and similarly. It is most often classified in different orders, in relation to the constant, that is, the free parameters of the functional and stochastic model of indirect adjustment by the least squares method [5]: *Zero-Order Design* (ZOD) is the selection of the optimal coordinate system for the geodetic network parameters;

- *First-Order Design* (FOD) leads to the solution of the optimal design of the geodetic network, where the problem is reduced to determining the optimal positions of the points of the network, as well as the optimal plan of observation;
- *Second-Order Design* (SOD) provides solutions of optimal weights or accuracy of planned measurements;
- *Third-Order Design* (THOD) enables the optimum improvement of existing networks in terms of design and accuracy. This is most commonly related to increase the density of the network with additional observations or points in parts of the network where the accuracy or reliability is poor [5], [10].

In practice, two methods of designing geodetic networks are most often applied: Method of trial and error and analytical method. In the first procedure, the objective function is calculated using the proposed solution. If it does not satisfy the objective function, the solution changes slightly and the objective function is recalculated. This process is repeated until the request is completed [11]. On the other hand, the analytical approach uses a mathematical algorithm and designs the network so that the quality requirements is satisfied at the same time, and the network is optimal from the mathematical point of view. Over the past few decades, thanks to computer support, many global optimization methods such as genetic algorithms have been developed. This method does not require neither the linearization of an objective function nor a difference in the solution process, which makes it very attractive to solve the optimization problem.

4. GLOBAL OPTIMIZATION METHODS

The *Genetic Algorithm* (GA) represents an evolutionary optimization technique inspired by Darwin's theory of the natural evolution of species. It first appeared in the '70s of the previous century on the proposal of John Holland [12], and over the years it has been modified and improved by numerous researchers. As in the theory of evolution, the candidate population for solutions to the optimization problem obtained by applying this algorithm evolves towards better solutions. GA investigates the space unconditionally, serching for a better solution, where the only condition is the objective function with the defined relationship between the parameters that are evaluated by this function. Traditional genetic algorithms are represented by a series of fixed-length bits. Each position in a series expresses a particular characteristic of the individual, and each gene represents a structurally independent entity. Two approaches have been developed to solve a specific problem: adapting the problem to a genetic algorithm or adapting the genetic algorithm to the specificities of the problem.

A computer process that imitates a process of evolution in nature and applies it to abstract individuals is called a *Simple Genetic Algorithm* (SGA). Each evolutionary program reflects the population of individuals in a particular generation, while each individual represents a potential solution to the problem being processed. Each individual is represented by the same data structure (number, string, matrix, tree, etc.) [13]. Performance and the structure of simple genetic algorithms is reduced to copying and replacing parts of chromosomes by means of reproduction operations: crossover and mutations. Crossing and mutations in genetic algorithms are genetic operators, and the process of allocating the most capable individuals within each generation is called selection. The second category includes *Adaptive Genetic Algorithm* (AGA) representing a GA group whose parameters, such as the size of the population, and the probability of crossing and mutation, are variable during the implementation process [14]. In order to change the probability of crossing and mutations in a timely manner, it is necessary in some way to determine the current state of the GA population, that is, to determine whether it converges to an optimum. The simplest case can be implemented by changing the rate of mutation in line with changes in the population. If it is noticed that for a long period of time the population does not improve, the rate of mutation increases. Genetic algorithms have over the course of time found enormous application in various areas of optimization, and so optimization of geodetic networks is no exception. The best results of this method have been shown when it comes to first and second order design, that is, when it is necessary to define the optimal locations of the network points, the plan of observation and their optimal weight.

PSO is a stochastic optimization technique based on the population and inspired by the social behavior of a bird flock or school of fish [15]. Implicit rules adhered to by members of the bird flock and fish school allow their synchronized movement without collision and with excellent choreography. When the swarm searches for food, its individuals will spread around the environment and move on their own. Every individual has a degree of freedom or coincidence in its movements, which allows it to find concentrated sources of food. In the end, one individual will find something digestible and publish it to its neighbors. Other individuals can then access the food source. It is precisely this social exchange of information among individuals that provides an evolutionary advantage that served as a basic idea of the development of the PSO algorithm [16].

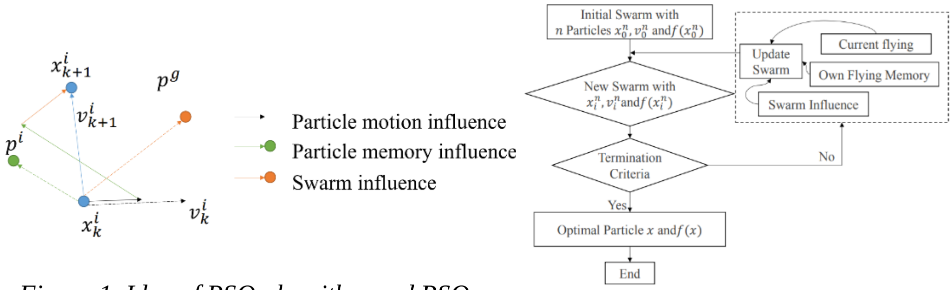

Figure 1. Idea of PSO algorithm and PSO flowchart [17]

PSO has many similarities with evolutionary computational techniques such as GA. The system is initialized by a random population and searches for optimum by updates over generations. However, unlike GA, PSO has no evolutionary operators such as crossover and mutations. In PSO, potential solutions, called particles, fly through a problem space following current optimal particles [15]. Each particle flies in a defined search area to find the best solution and adjusts the movement based on the experience of its own flight and flight of other particles. The PSO algorithm has four basic steps (Figure 1) [17]:

- *Initial particles.* Generation of *i* particles with random positions x^i_0 within the search domain *D*, and random velocities v^i_0 and objective function $f(x^i_0)$ is performed;
- *Speed Update.* The velocity of each particle v^i_k is updated based on the local optimum position p^i of this particle over time, that is, based on the current and previous optimal motion, as well as the optimal movement of all particles;
- *Position Update.* The positions are updated based on updated velocities;
- *Termination criteria.* Steps 2 and 3 are repeated until the stopping criterion is met. Finally, a position with a minimal value of the objective function will be saved as an optimal solution.

5. PROJECT OF GEODETIC NETWORK OF DAM LIVEROVIĆI

The concrete dam Liverovići was built on the river Gračanica in the period from 1953. to 1957. The dam consists of an arched section with spillway, a gravity abutment and bottom outlet with a canal. The length of the upper arch of the dam is 110.96 m and the total height is 45.50 m. For the purpose of monitoring, the control geodetic network has been established consisting of 8 reference points and 32 points on the object (Figure 2). Within the project, the geometry of the network with the observation plan, as well as the accuracy of planned measurements are defined ($\sigma_\alpha = 1''$ i $\sigma_d = 1\text{mm} + 1\text{ppm}$). The geodetic network quality parameters are available in [18].

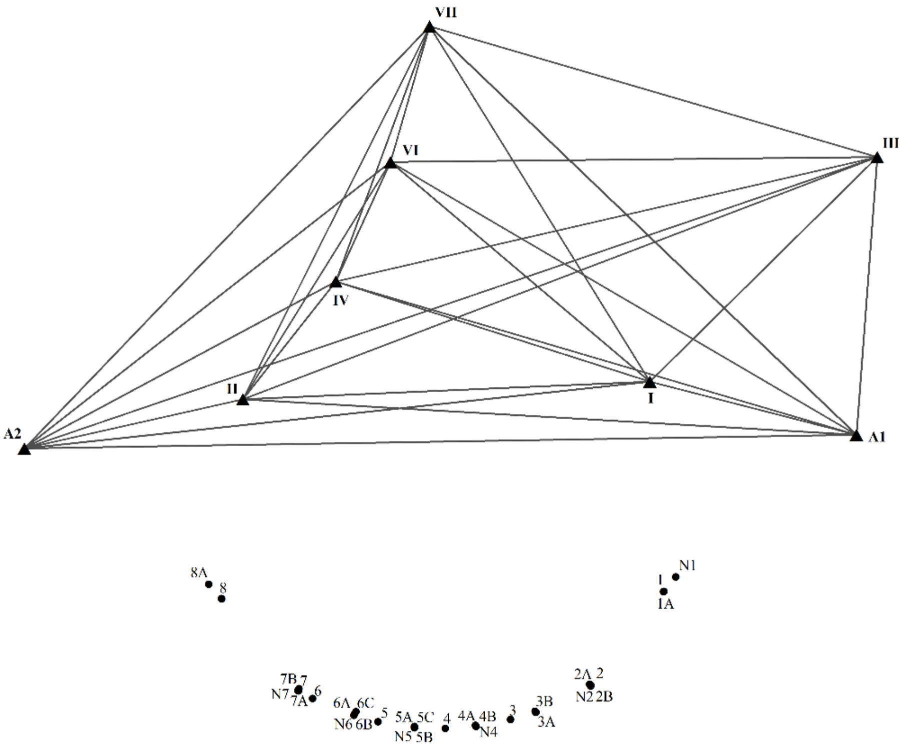

Figure 2. Control geodetic network of the Liverovići dam

Below is a methodology for optimization of second order geodetic network design using global optimization methods, on the example of the monitoring geodetic network of the Liverovići dam. For the purposes of applying global optimization methods (GA and PSO algorithm), it is necessary to define the variable sizes, the permissive search area, and the objective function. The variable sizes are actually the weights of the planned observations, because optimum values of the weight are determined in the second order optimization process. The permissive area of search is directly conditioned by the adopted accuracy of planned observations. Therefore, based on the accuracy, the limits for the weight of the observation are defined:

$$0 \leq p_{\alpha_i} \leq \left(\frac{\sigma_0^2}{\sigma_\alpha^2}\right) \quad \text{and} \quad 0 \leq p_{d_i} \leq \left(\frac{\sigma_0^2}{\sigma_{d_i}^2}\right) \tag{9}$$

In the deformation analysis of geodetic networks, the most unfavorable case of testing stability of point refers to the direction of a large semi-axis of a confidence ellipse, since the biggest error of the determination the position of the point is in that direction. For this reason, the objective function is defined based on the smallest intensities of the displacement vector that can be detected in the direction of a large half-axis of the confidence ellipse, as follows:

$$\mathrm{f} = \|\mathbf{d}_{\min} - \mathbf{d}^{\mathrm{c}}_{\min}\|_1 = \min \tag{10}$$

where the vector ${\mathbf{d}_{\min}}^{\mathrm{T}} = \left[\ldots, \mathrm{d}_{\min_{\mathrm{i}}}, \ldots\right]$ is the least intensity of the points displacement and ${\mathbf{d}^{\mathrm{c}}_{\min}}^{\mathrm{T}} = [5\ \mathrm{mm}, \ldots, 5\ \mathrm{mm}]$ is the criterion vector of the least intensity of movement of the points in the network. The least intensity of the points displacement $\mathrm{d}_{\min_{\mathrm{i}}}$ is determined according to Eq. (8) for the accepted significance level α and the power of the test 1-β. Defined limits Eq. (9) are integrated into the objective function using the method of the penalty function. For any exceedance of the defined limits, the appropriate penalty g_{ij} is formed as:

$$\mathrm{g}_{\mathrm{ij}} = \begin{cases} |\mathrm{p}_{\mathrm{i}} - \mathrm{p}_{\max}|, & \mathrm{p}_{\mathrm{i}} > \mathrm{p}_{\max} \\ 0, & \mathrm{p}_{\min} \leq \mathrm{p}_{\mathrm{i}} \leq \mathrm{p}_{\max} \\ |\mathrm{p}_{\min} - \mathrm{p}_{\mathrm{i}}|, & \mathrm{p}_{\mathrm{i}} < \mathrm{p}_{\min} \end{cases} \tag{11}$$

where p_{i} is the weight of the apposite observation and $\mathrm{p}_{\min}$ and $\mathrm{p}_{\max}$ are the minimal and the maximal permissible weight of the observation. Finally, the objective function has the following shape:

$$\mathrm{f} = \|\mathbf{d}_{\min} - \mathbf{d}^{\mathrm{c}}_{\min}\|_1 + \sum_{\mathrm{i}}\sum_{\mathrm{j}} \beta \cdot \mathrm{g}_{\mathrm{ij}} \tag{12}$$

where β is the weight coefficient of the penalty, which is determined empirically and j is the constraint index (the limit for the weight of the direction j = 1, the limit for a weight of the length j = 2). The minimum of the defined objective function can be determined using the GA or PSO algorithm.

6. CONCLUSION

Projecting geodetic networks is a very demanding task from the aspect of achieving high accuracy, reliability and sensitivity of the geodetic network. When it comes to the control geodetic networks for monitoring ground and objects, the sensitivity is a key parameter of quality. Today, global optimization methods are increasingly being used in the process of designing geodetic networks. This paper presents the optimization method based on GA and PSO algorithms for second order design of the control geodetic network of the Liverovići dam. For the weight of the planned observations, the restrictions Eq. (9) are defined by the method of penalty functions and then included in the objective function Eq. (12). It is important to emphasize that the objective function is defined based on the smallest intensity of displacement of network points. Optimal weight values are determined by finding the minimum of the objective function defined in this way. Observations with very low weight values should be excluded from the observation plan. On the basis of the obtained results, the final plan of observation and selection of the appropriate equipment and accessories for the realization of measurements in the geodetic network can be adopted.

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ПРОЈЕКТОВАЊЕ ГЕОДЕТСКИХ МРЕЖА ПРИМЕНОМ ГЛОБАЛНИХ ОПТИМИЗАЦИОНИХ МЕТОДА

Rезиме: *Традиционално, при решавању проблема оптимизације геодетских мрежа обично се користе аналитичке методе попут линеарног и нелинеарног програмирања или симулационе методе. Међутим, данас се при решавању проблема оптимизације геодетских мрежа све више користе глобалне оптимизационе технике попут генетског алгоритма и алгоритма оптимизације ројем честица. У овом раду дат је предлог примене наведених алгоритама у процесу оптимизације дизајна другог реда геодетске мреже бране Ливеровићи. С обзиром да је предмет анализе геодетска мрежа за мониторинг бране, критеријум оптималности дефинисан је на основу параметара осетљивости геодетске мреже.*

Кључне речи: *оптимизација геодетских мрежа, генетски алгоритам, алгоритам оптимизације ројем честица*