Chapter 7
Survey Design and Analysis

Objectives

At the end of this chapter, you should be able to

1. Discuss the elements and problems of network design
2. Carry out simple two-dimensional network design by simulation
3. Perform simple two-dimensional network analysis
4. Design deformation monitoring scheme
5. Discuss the purpose of simulating survey measurements
6. Carry out simple simulation of survey measurements to check if tolerance limits of measurements can be met

7.1 INTRODUCTION

The survey design considered in this chapter is essentially *network design* and the accompanying simulations. Network design is to estimate the confidence of future survey before it is actually carried out. It allows one to experiment with different surveying variables so as to meet or exceed the desired survey accuracy requirements. After an initial design, it may be discovered that the desired accuracy requirements are not met; in this case, there may be a need to iteratively change the surveying variables until the accuracy requirements are satisfied. This is usually done through a process of computer simulation. *Simulation* of survey measurements is an imitation process (before the measurements are made) to see how those measurements would be made under different conditions and also to analyze component measurements of new design. It includes planning and laying out of a project and proper selection of equipment, measurement methods, and procedures. It provides a basis for evaluating the accuracies of the survey measurements and for meeting tolerances that may have been imposed on these measurements. Currently, there is a great demand for more accurate survey measurements, which requires that the surveyor chooses an appropriate survey instrument out of several models of surveying instruments and an appropriate survey technique out of possible survey techniques. For an illustration, consider Figure 7.1, in which the coordinates (X_C, Y_C) of point C are to be determined, given that the coordinates of points A (X_A, Y_A) and B (X_B, Y_B) are known.

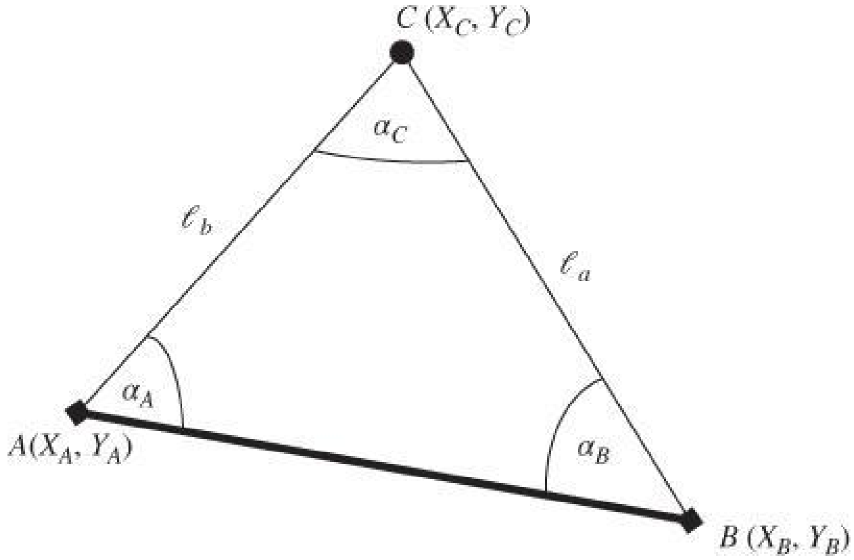

Figure 7.1 A simple surveying problem.

The surveyor can solve the above problem using a number of measuring techniques, such as the following:

- Triangulation – measuring three angles α_A, α_B, and α_C
- Trilateration – measuring two distances ℓ_a and ℓ_b
- Triangulateration – measuring the three angles α_A, α_B, and α_C and the two distances ℓ_a and ℓ_b.

In addition, the surveyor will have to decide on which type of instrument to use out of a number of models of surveying instruments to choose from. The surveyor's choice of survey techniques and instruments must be based on a thorough simulation of the project so that the selected techniques and instruments would satisfy the accuracy requirements of the client in an economical way. The simulation is used in order to predict (or design) what type of instrumentation and what procedure of measurements should be used in order to satisfy the specifications of the client. It should also be mentioned that when a tolerance to be achieved is provided by the client, it is customary for the surveyor to reserve half of the error budget for systematic errors (such as refraction effects that cannot be completely eliminated) and to reserve the other half for random errors, which will be used in determining the type of measurement and instruments to use for the project.

After the design and simulation processes have been completed, a blueprint for the field crew is usually created, such as where the network stations would be located, types of observables to measure at each station, level of accuracy needed for the observations.

7.2 NETWORK DESIGN

The level of accuracy of geodetic positioning has increased in the past few years, requiring that the geodetic surveyors should shift their focus away from just being able to make quick and accurate observations; they should also be able to perform *survey design*, *data processing*, and a*nalysis* in order to produce reliable results. Although the commercial computer software packages for network design and adjustment have made the work of design and analysis easy, a good understanding of the terminology, concepts, and procedures involved are still needed in properly interpreting the results and successfully designing and/or analyzing a survey.

Network design, according to Kuang (1996), will help in identifying and eliminating blunders in network measurements; it will also ensure that the effects of the blunders that are not detected and eliminated are minimal on the network solution. Some of the other benefits of network design can be summarized as follows:

- It helps in reducing the amount of time and effort required in carrying out a field project, which will also result in the reduction of the cost of the project.
- It provides a measure of confidence that the project can or cannot be completed as specified by the client.
- It will afford the surveyor the opportunity to experiment with different design variables such as

  i. network geometry (number and physical location of survey points);

  ii. network accuracy (or precision of measurements);

  iii. reliability (ensuring enough redundant observations in order to be able to assess the accuracy of network); and

  iv. cost of survey (if number of measurement to be made is reduced and if the observation procedure is made simple).

A network must be designed to satisfy the preset precision, reliability, and cost criteria. In order to achieve the network quality set by a client, the network design essentially involves the following:

- Deciding on the best configuration (or geometry) of a geodetic network or deciding on the location of station points
- Choosing the measuring techniques and the types of geodetic observables to be measured
- Making decisions on which instruments to use among hundreds of available models of various geodetic instruments
- Computing the optimal distribution of required observational precisions among heterogeneous observables.

7.2.1 Geodetic Network Design

Much can be done to design a network to ensure that it will achieve its desired aim before any measurements are made. From the least squares adjustment of network survey measurements, the adjusted coordinates of the network points ($\hat{x}$) can be expressed as

$$\hat{x} = x^0 + \delta \qquad (7.1)$$

where x^0 is the vector of approximate coordinates of the network points (taken from a large-scale map or an aerial photograph); and δ is the vector of unknown corrections to the approximate coordinates of the network points. The least squares solution for δ can be given as

notacion nuestra A=B P=W w=f

$$\delta = -(A^{\mathrm{T}}PA)^{-1}A^{\mathrm{T}}Pw \tag{7.2}$$

where A is the first design matrix also known as the configuration matrix, P is the weight matrix (inverse of the covariance matrix of the observations), and w is the vector of the observations computed by using approximate coordinates minus the original observations. From the variance–covariance propagation laws, it can be shown from Equation (7.1) that the covariance matrix ($C_{\hat{x}}$) of the estimated parameters (unknown coordinates of the network points) is as follows:

Notacion nuestra INV(Bt*W*B)

$$C_{\hat{x}} = (A^{\mathrm{T}}PA)^{-1} \tag{7.3}$$

The basis on which a design can be carried out is seen from the covariance matrix of the estimated parameters in Equation (7.3). It can be seen from the equation that the covariance of the estimated parameters ($C_{\hat{x}}$) can be determined before making the actual field measurements if the approximate coordinates of the network stations are known. This situation represents the most usual design problem, which is to decide where to position observation stations and which measurements to make in order to satisfy the defined (precision) criteria. The problem of network design has been divided into the following (Grafarend, 1974; Grafarend et al., 1979): zero-order design (ZOD), first-order design (FOD), second-order design (SOD), third-order design (THOD), and combined-order design (COMD). The characteristics of the different classes of design problems are given in Table 7.1.

Table 7.1 Problem of Network Design

Problem Order	Unknown (To Be Determined)	Known (or Provided A Priori)
Zero-order design (ZOD) – also known as *datum problem*	Reference coordinate system is unknown so that the optimal values of unknown parameters (x) and their covariance matrix ($C_{\hat{x}}$) are unknown	Matrix indicating the configuration or geometry of network (A) and weight matrix of observations (P)
First-order design (FOD) – also known as *configuration problem*	Optimal locations (or configuration) of network stations (A) and observation technique or plan (how network points are to be connected)	Weight matrix of observations (P) and covariance matrix of parameters ($C_{\hat{x}}$) are known
Second-order design (SOD) – also known as *weight problem*	What type of observations to make and their precisions or weight matrix (P)	Network geometry (A matrix) and covariance matrix of parameters ($C_{\hat{x}}$)
Third-order design (THOD) – also known as *improvement problem*	Improvement of existing network configuration (A) and weight or precision of observations to be made (P). Combining modified FOD and modified SOD	Covariance matrix ($C_{\hat{x}}$) of parameters are known
Combined design (COMD)	Solves combined FOD and SOD simultaneously (A and P are not known)	Covariance matrix ($C_{\hat{x}}$) of parameters are known

The design problem is not only limited to solving the problem of meeting precision criteria, but it also includes providing the minimum-cost solution. When a design satisfies both the precision and the minimum-cost criteria, it is often referred to as the *optimum design*. The cost element can be very difficult to quantify so that designs are usually assessed subjectively by considering the previous experience of the surveyor.

7.2.2 Design of GNSS Survey

One of the significant advantages of the Global Navigation Satellite System (GNSS) survey technique over conventional techniques is that survey stations may be placed where they are required, irrespective of whether intervisibility between stations is preserved, provided there are no obstructions between the stations and the satellites to be tracked. It should be remembered that the GNSS technology is continually evolving and the following are continually changing:

- Requirements for classification of geodetic control surveys by GNSS techniques
- Definitions for GNSS accuracy standards

- Experience in performing GNSS surveys
- GNSS surveying equipment improvement
- GNSS field procedures
- Refinements to processing software.

The design of GNSS surveys is a component of the GNSS specifications. The specifications are for control surveys performed by relative positioning techniques where two or more receivers are collecting carrier phase measurement data simultaneously; and they include network design and geometry, instrumentation, calibration procedures, field procedures, and office reduction (processing) procedures. Some of the guidelines for GNSS network design, geometry, and connections are given for the highest orders of surveys by FGCC (1989) as shown in Table 7.2.

Table 7.2 Guidelines for GNSS Network Design, Geometry and Connections

Geometric Accuracy Standards	
1. Minimum number of stations of the horizontal network control of a reference system to be connected to	4 to 3
2. Minimum number of stations of the vertical network control of a reference system to be connected to	5
3. For continuously tracking stations (master or fiducials): minimum number of stations to be connected to	4 to 2
4. Station spacing (km) between old network control and center of project should not be more than (and 50% not less than $\sqrt{5d}$): (*where d is the maximum distance (km) between the center of the project area and any station of the project*)	100d to 7d
5. Station spacing (km) between old network control located outside of the project's outer boundary and edge of the boundary, not more than	3000 to 100
6. Location of network control (relative to center of project): number of “quadrants,” not less than	4 to 3
7. Direct connections should be performed, if practical, between any adjacent stations (new or old, GNSS or non-GNSS) located near or within the project area, when spacing is less than (km)	30 to 5

7.2.3 Design of Deformation Monitoring Scheme

Deformation monitoring schemes of objects are designed to help in accurately determining the expected deformation model and deformation parameters (which must be known a priori) of the object. In practice (Chen, 1983; Secord, 1985), the initial design is usually based on single-point displacement models with x and y displacements set to zero for each reference network point and each object network point is given constant *x* and *y* displacements whose variances and covariances are later solved for. With some modifications based on the different purpose of the geodetic network (which is to provide absolute positions of network points) compared to that of the monitoring network (which is to determine displacements of network points between epochs), the design of deformation monitoring networks can be classified based on the design orders for geodetic networks given in section 7.2.1, as follows (Kuang, 1996):

- **1.** The Zero-order design (ZOD) problem is about confirming the stability (in position and orientation) of reference network points between monitoring epochs; the reference network that remains stable over several epochs is considered optimal.
- **2.** The First-order design (FOD) problem is about locating monitoring points where maximum deformations are expected and ensuring that the reference network points are located in stable regions. The locations of points with expected maximum deformations can be determined by modeling the deformations using finite element method.
- **3.** The Second-order design (SOD) problem is to determine the types of observables and their accuracies that will provide accurate deformation parameters. The Third-order design (THOD) is about improving the accuracies of the deformation parameters.

The important requirements to be satisfied in the solution for the design parameters of the monitoring scheme are accuracy, reliability, separability (or discriminability), and cost-effectiveness.

7.2.3.1 Accuracy Requirement

The sources of errors causing uncertainty in engineering survey measurements are numerous and diverse. The main concerns include factors such as physical instability of observation stations; atmospheric refractions along the line of observation, thermal effects on the mechanical, electronic, and optical components of the instrument used; instrument malfunction and human ability to fail. Since on many occasions, particularly precision observations are required, special attention must also be paid to matters such as centering, targeting, and leveling of instruments.

Generally, the main problem of deformation monitoring scheme is not to define an optimum datum (or reference system) for the initial epoch of measurements but to confirm the stability of the datum. For example, if a set of reference points (serving as reference datum) used to constrain the network adjustment are erroneously assumed stable while they are not, a biased displacement pattern that can be misinterpreted as monitoring results could be obtained. Unstable reference points must be identified prior to data acquisition stage based on the knowledge of boundaries of deformation zone or during data processing by using appropriate algorithm.

There are several ways of attempting to reduce the errors due to the effects of systematic errors on surveying measurements. It may be possible to calibrate the instrument concerned, quantify the error, and apply corrections to subsequent measurements. The effects of random errors on measurements, which are represented by the precision (or internal accuracy) of the measurements, cannot be completely eliminated.

As a general rule, the accuracy of monitoring ground displacements at 95% confidence level should be at least three times smaller than the expected (or predicted) average displacements over the observation time span. The frequency of observations will then depend on the expected rates and magnitudes of the practically detectable deformations. In this case, the standard deviation of monitoring displacements can be taken as the predicted displacement reduced by a factor of 3 × 2.48 for horizontal displacements and reduced by a factor of 3 × 1.96 for vertical displacements. This requires that the predicted maximum ground displacement over the total period of the deformation activities be available in order to be able to determine the accuracy of monitoring surveys. For example, from the predicted maximum deformation over a period in which gas will be withdrawn from an underground reservoir, one can determine the annual rate of subsidence and the accuracy of monitoring surveys. Since the predicted values, however, may be different from the actual values, the accuracy requirement for the surveys will have to be revisited time to time depending on the depth and geometry of the mine and also on the mining method being adopted.

The ability of a monitoring scheme (configuration of stations and object points and observables) to reveal relative movement is related to the relative positioning error (relative 95% confidence error ellipse) between any pair of stations (Chrzanowski and Secord, 1985). In this case, relative movement must be greater than $a_{95} \times \sqrt{2}$ (where a_{95} is the semi-major axis value of the relative 95% confidence error ellipse) in the case of two-dimensional networks in order to be able to detect the movement.

Relative confidence regions provide the accuracy of coordinate differences among monitoring network stations and are a measure of the internal accuracy of the network. The relative error ellipses between the object points and selected reference stations become indicators of the ability of the scheme, configuration, and observables to monitor the behavior of the structure represented by the object points with respect to the reference created by the network stations. For example, assuming a network is to be designed such that it is capable of detecting the minimum horizontal displacement of $d_{min} = \pm 3.0$ mm and assuming no correlation between a pair of measurement epochs, it can be expressed that

$$d_{min} \geq \sqrt{(a_{I}^{2} + a_{II}^{2})} \quad \textbf{7.4}$$

where a_I and a_{II} are the semi-major axis values of the standard confidence ellipse for epochs I and II, respectively. Assuming the same value (a_{std}) for the semi-major axis values for the two epochs, then $d_{min} \geq a_{std}\sqrt{2}$. Thus, for the detection of ±3.0 mm horizontal displacement at the 95% confidence level, the tolerance for the semi-major axis of the standard confidence ellipses becomes

$$a_{\text{std}} = \pm \frac{3.0}{\sqrt{\chi^2_{0.95,\text{df}=2}}\sqrt{2}} \qquad \textbf{7.5}$$

for the positional accuracy in a single campaign, where $\sqrt{\chi^2_{0.95,\text{df}=2}} = 2.4484$ is the Chi-distribution value at 95% confidence level for the degrees of freedom of df = 2 (for two-dimensional cases). Similarly, for the detection of ±3.0 mm vertical displacement at the 95% confidence level, the tolerance for the standard confidence level becomes

$$a_{\text{std}} = \pm \frac{3.0}{Z_{0.975}\sqrt{2}} \qquad \textbf{7.6}$$

for the positional accuracy in a single campaign, where $Z_{0.975} = 1.9600$ is the standard normal distribution value at 95% confidence level (for one-dimensional cases). The amount of displacement to be detected (e.g., ±3.0 mm) can be predicted based on the annual rate of displacement determined initially from the geotechnical measurements.

7.2.3.2 Reliability Requirement

Reliability of a monitoring network is a measure of the ability of a measuring scheme to detect and eliminate blunders from observations. It is a function of both the network geometry and the precision of observations. An observation that is reliable is unlikely to contain an undetected blunder, and, conversely, a blunder is unlikely to be detected in an unreliable observation. The reliability of the monitoring network will generally improve if the design is capable of producing redundant observations and if the sources of errors are well understood and well taken care of. An unreliable and poorly designed monitoring system will lead to false conclusions and misinterpretation of deformation of the monitored object. The design of monitoring network should also make sure that at least a monitoring point is located at the point of expected maximum movement; otherwise, the deformation analysis based on this design may become inconclusive.

7.2.3.3 Separability or Discriminability Requirement

In a specific case of a deformation monitoring network, the design may not only be required to meet precision (e.g., variances of point positions or derived quantities) and reliability criteria, but also to be *sensitive* to the deformation pattern that is expected to take place. If such a pattern of a deformation can be formulated in a mathematical model, then network designs can be quantitatively assessed as to their capability to identify the true deformation. Such ability is sometimes referred to as *separability* (*or discriminability*) of the network. When considering design for deformation analysis, it is important to consider the separability of the resulting network to the particular deformation expected. This is because the purpose of such a network is usually not only to detect possible movements but also to try and establish the general mechanism of the motion taking place.

Separability is to indicate if the network is sensitive enough to detect and discriminate possible causative factors and the mechanisms postulated as responsible for the deformation of the object. This concept was extended into what is known as *discriminability* of the monitoring network (Ogundare, 1995). Discriminability incorporates all possible causative factors and the postulated deformation mechanisms in its formulation while separability considers only one possible causative factor and a postulated mechanism at a time.

7.2.3.4 Cost-Effectiveness Requirement

The network design must satisfy the required accuracy, reliability, and three-dimensional monitoring criteria in the most economical way. The choice of monitoring technology or monitoring system, however, must be chosen according to the accuracy requirement, reliability of the network design, and cost requirements.

7.3 SOLUTION APPROACHES TO DESIGN PROBLEMS

Once the design problem has been formulated, there are two basic approaches to its solution:

- Computer simulation or trial and error methods
- Analytical methods, which attempt to mathematically formulate the design problem in terms of equations or inequalities and then explicitly solve for the optimal solution.

The *trial and error* method uses personal judgment at every step of the design. It requires repeated postulation of solution until a satisfactory (unlikely to be optimal) network is found. With the development of modern computers, the *trial and error method* is now referred to as the *computer simulation method*.

7.3.1 Simulation Steps for Network Design

In simulation, the problem of propagation of errors is reversed in order to determine the accuracy of measurements that will satisfy a specified tolerance limit for the unknown quantities to be determined. A simulation may tell us that the requirements for the accuracies of measurements are within or beyond our capabilities. If the requirements are beyond our capabilities, the client must be told that the tolerance limits specified are beyond what can be satisfied. The common steps for carrying out a network design by computer simulation method are as follows (cf. Cross, 1985):

1. Specify precision and reliability desired for the new network.

2. Choose a measurement scheme, such as station locations, types of observations, and weights (from precisions) of observations.

3. Select preliminary locations of control stations on an existing map or on an existing photographs based on the specific needs of the survey project control required by the client.

4. Perform a preliminary field reconnaissance, and based on the available instrumentation, determine the possible interconnection of stations by geodetic observables.

5. The proposed station locations and geodetic observables in steps 1–4 constitute an initial design of the network. According to this initial design, hypothetical precisions or weights of observables are then used to simulate the quality of the network. This is done by computing the covariance matrices of the desired least squares estimates and deriving the values of the quantities specified as precision and reliability criteria (such as standard deviation, standard error ellipse, and relative error ellipse or redundancy number).

6. If the values derived in step 5 are close enough to those specified in step 1, go to step 7; otherwise, alter the observation scheme in step 2 (by removing observations or decreasing weights if the selected network is too good, or by adding more observations or increasing weights if it is not good enough) and return to step 5.

7. Compute the cost of the network and consider the possibility of returning to steps 2 and 3 and restarting the process with a completely different type of network (e.g., using traverse instead of using triangulation, etc.). Stop when it is believed that the optimum (minimum cost) network has been achieved.

8. Perform a field reconnaissance to examine the physical possibilities of the simulated network. Control stations are temporarily marked on the ground. If conventional terrestrial geodetic observables are proposed, intervisibility of control stations must be ensured. If the GNSS technique is to be used, the station site should be wide open with no obstructions to block the GNSS satellite signal between the stations and the satellites (within 10–15° above the horizon).

9. If step 8 is successfully done, the network will be monumented and surveyed.

This method has been used for decades, and a number of commercial software packages, such as *Microsearch GeoLab* and *Star*Net Pro*, are available for the network simulations. The main advantage of this method over analytical methods is that there is no need of evaluating any complex mathematical formulation, unlike in the analytical methods. The main disadvantage of the method is that an optimum (minimum-cost) solution may never be achieved.

Some of the properties of a well-designed control network should include the following:

a. Stations must be as evenly spaced as possible; ratio of the longest length to the shortest should never be greater than 5.

b. Adjacent pairs of stations should be connected by direct measurements.

c. There should be reasonable number of redundant measurements in the network.

d. Good a priori estimates of the accuracies of various instruments used with various techniques must be available in order to design a network that will achieve required accuracies. Accuracy of a horizontal control can be assessed properly from the results of a rigorous least squares adjustment of the measurements.

Typical examples of simulation problems are given as follows. Consider a case where you are to design a survey scheme (i.e., deciding on the best choice of equipment and procedures) for horizontal positioning by the process of trial and error or simulation assuming the following:

- Two types of observables (angles and distances) are to be measured.
- Standard deviation of each angle is σ_{β_i}, and of each distance is σ_{s_i}.
- Potential geometry is expressed as approximate coordinates, x^0.
- Ninety-five percent relative positioning tolerance (relative ellipses at 95% confidence) is to be achieved in the survey, that is, the semi-major axis of the relative error ellipses should be a_{95}.

The steps for carrying out the network design by the computer simulation method can be given as follows:

1. Specify precision and reliability desired of the new network:

- Semi-major axis of the relative error ellipses expected is a_{95}.

2. Choose a measurement scheme, such as station locations, types of observations, and weights (from precisions) of observations.

- Standard deviations of observations are provided ($\sigma_{\beta_i}, \sigma_{s_i}$).
- Use them to form the cofactor matrix Q (matrix of variances of observations, assuming observations are uncorrelated).
- Form the weight matrix (P) from Q, assuming $\sigma_0 = 1$, giving $P = Q^{-1}$.

3. Select preliminary locations of control stations on an existing map or on an existing photographs based on the specific needs of the survey project control required by the client.

- Use approximate coordinates (x^0) of network points to create the first design matrix (A) based on the observation equations of distances and angles as functions of unknown coordinates of network points.

4. Compute from the available data in steps 2–3, the achievable semi-major axis of the relative error ellipses a'_{95} as follows:

i. Create the covariance matrix ($C_{\hat{x}}$) of the adjusted coordinates of the network points from Equation (7.3).

ii. Determine the relative standard deviations and covariances of pairs of points (x_1, y_1) and (x_2, y_2) involved in the network from the $C_{\hat{x}}$. Refer to Equations (2.32)–(2.40) for a typical covariance matrix $C_{\hat{x}}$ and for the relative variances and covariances of a pair of points.

iii. Obtain the standard relative error ellipses from Equations (2.41)–(2.44) as $a_s = \sqrt{\lambda_1}$, $b_s = \sqrt{\lambda_2}$ and θ, where a_s is the semi-major axis of the standard relative error ellipse, b_s is the semi-minor axis of the standard relative error ellipse, θ is the orientation of the semi-major axis of the standard relative error ellipse with λ_1 and λ_2 as the maximum and the minimum eigenvalues of the relative covariance matrix.

iv. Obtain the 95% relative error ellipses from Equations (2.26)–(2.28), which can also be expressed as follows:

$$a'_{95} = k_{95} a_s \quad (7.7)$$

$$b'_{95} = k_{95} b_s \quad (7.8)$$

$$\beta = \frac{1}{2} \arctan \left[\frac{2 s_{\Delta x \Delta y}}{s^2_{\Delta y} - s^2_{\Delta x}} \right] \quad (7.9)$$

$$k_{95} = \sqrt{\chi^2_{\alpha=0.05, df=2} (\text{upper-tail area})} \quad (7.10)$$

where a'_{95} is the computed semi-major axis of the 95% relative error ellipse, b'_{95} is the semi-minor axis of the 95% relative error ellipse, β is the orientation of the semi-major axis of the 95% relative error ellipse, and k_{95} is obtained from the Chi-square statistical table. Note that Equations (7.7)–(7.10) give identical results as those given in Equations (2.26)–(2.28); the above equations are just to show the variations in formulas commonly used.

v. The semi-major axis of the relative error ellipses computed is a'_{95} in Equation (7.7).

5. Compare the obtained a'_{95} in step 4 with the limit on the relative ellipses (a_{95}) from step 1; if a'_{95} is less than the tolerance a_{95}, then the precision of potential observables and potential geometry are considered acceptable. If a'_{95} in step 4, however, is greater than the tolerance a_{95}, modify the precision and the geometry of observables or one of them and repeat the simulation until a'_{95} is less than the tolerance (a_{95}).

Referring to the above illustration, one can ensure that the intended standard deviations σ_{β_i} and σ_{s_i} are realized during the observations by taking the following steps:

1. Confirm that the instruments are well calibrated and their standard deviations quoted are correct.

2. Avoid sources of systematic errors such as

- refractions (avoid temperature variations or make measurement at different atmospheric conditions and average the results);
- consider leveling and tilting axis error (use dual-axis compensators and ensure that instrument is in good adjustment);
- design allowable discrepancy for testing the acceptability of the set of measurements and implement this during the data acquisition stage.

3. Use face left and face right positions of instrument for direction measurements.

4. Targets must be well designed and appropriate for the project.

5. Targets must be well illuminated and visible from the instrument stations.

6. Use appropriate centering devices; well-adjusted optical/laser plummet or use forced centering procedure.

7. Minimize pointing error by using experienced instrument persons.

8. Use instruments with slightly better precision than those designed.

Simulation of 3D Traverse: Modern precision total stations can be used in three-dimensional traversing to result in Easting (E), Northing (N), and orthometric height (H) simultaneously, provided that points with known E, N, and H are available. The 3D design is accompanied using appropriate computer software simulation. The design (especially the network configuration) may be modified based on the outcome of the reconnaissance survey. The following steps may be taken:

1. In the simulation process, the approximate coordinates (N, E, H) of network points are to be used to determine the standard deviations of the horizontal and zenith angles to be measured.

2. For simulation purpose, the typical height of instrument (HI), height of reflector or target (HR or HT) can be assumed to be 1.600 m; and heights above pillar plates used as reference can be assumed to be 0.300 m.

3. In simulation and later in the least squares adjustment of measurements, two measurements between two stations taken at both ends will not necessarily be of the same observable since the HIs, HRs, and, possibly, the meteorological conditions at the time of measurements may be different. If this is the case, the average measurements corrected for meteorological conditions and reduced to mark to mark can be used or the two measurements involved can be treated as separate observables.

4. Use the input from the design in the simulation software and generate station (or relative) error ellipses and confidence intervals at 95%. Take note of the following with regard to relative error ellipses:

- They are unaffected by the choice of origin of a network in the minimally constrained adjustment.
- They are the precisions of relative positions of two points; they represent the relative precision of each station pair.
- They can be smaller than the absolute (station) error ellipse on each end, that is, the coordinates for each station could be completely wrong (e.g., based on incorrectly used fixed coordinates), but the relative errors between stations give the best estimate of the precision of the survey regardless of the coordinates. For example, in terms of GPS measurements, the station coordinates determined using GPS may be off by meters, but

the vector (the difference between these coordinates) can be accurate to centimeter level or better. The error in this vector is the best indicator as to the quality of the measurement.

5. Impose minimal constraints on the traverse by fixing (E, N, H) one of the control points available and an azimuth to another control point (assuming distances have been measured).

6. Often the quality of a traverse depends on its not exceeding a maximum allowable linear misclosure or the ratio of misclosure. It should be mentioned that the ratio of misclosure only implies general quality of relative precision of a closed traverse; it does not evaluate scale, rotational errors, blunders, and positional errors of the traverse. The product of a simulation suggesting that the expected quality would meet that ratio of misclosure criterion is the 95% confidence relative error ellipse between a pair of points or the relative distance accuracy estimates between the points in the network. The relative distance accuracy estimate is determined by error propagation using the positional standard errors at each end of the given line. If only approximate adjustments are being performed, the relative distance accuracies may be taken as a function of position misclosure.

7. The generated relative error ellipses are to suggest how well the 3D connection between every pair of points should be determined by the scheme that has been designed.

7.4 NETWORK ADJUSTMENT AND ANALYSIS

According to Kuang (1996), network analysis in surveying is about processing and analyzing survey data and reporting the outcome with its quality to the client. The steps involved in network analysis are given as follows (cf. Kuang, 1996):

1. Accuracy analysis of observations

2. Observation data preprocessing

3. Preadjustment data screening

4. Least squares network adjustment

5. Postadjustment data screening

6. Quality analysis of the results

7. Reporting network results and their quality to the user.

In general terms, items 1–3 can be considered as *preanalysis of measurements*; and items 5 and 6 as *postanalysis of measurements* and results.

Basic problem in surveying is to determine coordinates of a network of points using various types of measurements that establish a known geometrical relationship between them. Points with unknown spatial coordinates are connected to the network by the measurements. Network adjustment permits all of the available survey measurements to be processed together to determine a weighted mean value for the coordinates. Coordinate accuracy is determined by the application of error propagation to the observation equations. A predetermined uncertainty (standard deviation) is assigned to each measurement, which then propagates to the coordinates during the adjustment. The probable error in the coordinates (or positioning accuracy) is reported by the absolute confidence ellipse for each point or by the relative confidence ellipse between two points. It is essential to determine the positioning accuracy; without an adequate knowledge of the positioning accuracy, the survey (and the network adjustment) should be considered incomplete.

7.5 ANGULAR MEASUREMENT DESIGN EXAMPLE

In monitoring a dyke system along a certain coast, direction, distance, and height difference measurements were made to a network of stations on the dyke system. Directions were observed using a Kern DKM3 optical-mechanical theodolite (with adapter to fit onto a Wild tribrach) and Wild traversing targets, both onto Wild trivets; the distances were measured using a Tellurometer MA-100 infrared distance meter; and the height differences were measured using a Wild N3 tilting level and invar staves. The approximate coordinates, heights of instrument (DKM3, tilting axis above pillar plate), and pillar plate elevations are given in Table 7.3 for three of the stations involved in the network. TJ-8B required tripod setup (using Wild tribrach with optical plummet) since the pillar top was too close to the ground to be used directly.

Table 7.3 Approximate Coordinates, Heights of Instrument and Pillar Plate Elevations

Station	*x*/*E* (m)	*y*/*N* (m)	*H* (m above msl)	HI (m)
TJ-8	2037.384	1197.560	1.464	0.300
TJ-8A	2050.536	1138.241	1.249	0.300
TJ-8B	2051.170	1075.133	0.646	1.080

If the directions at station TJ-8A are to be observed to each of the two other stations, what would you expect to be the standard deviation of each of the directions, measured in one set using DKM3?

Solution

Some of the specifications of Kern DKM3 are as follows: $M = 45\times$; micrometer = 0.5″; plate vial sensitivity = 10″/2 mm. Figure 7.2 and Equations (7.11)–(7.13) can be used in computing the corresponding horizontal distance (HD), change in elevation (ΔH), and slope distance (s):

$$\text{HD} = \sqrt{(\Delta E)^2 + (\Delta N)^2} \tag{7.11}$$

$$\Delta H = H_B + \text{HT} - H_A - \text{HI} \tag{7.12}$$

$$s = \sqrt{(\text{HD})^2 + (\Delta H)^2} \tag{7.13}$$

where HI is the height of instrument, HT is the height of target, and *s* is the slope distance.

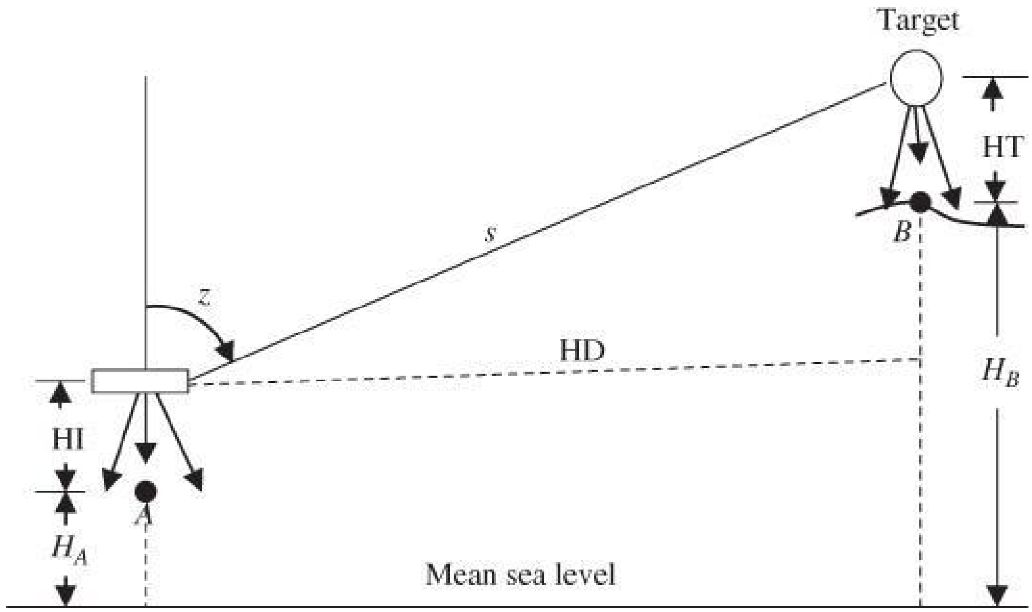

Figure 7.2 A typical direction measurement to a target.

Using Equations (7.11)–(7.13), the following are calculated:

$$\text{Direction TJ-8A} \rightarrow \text{TJ-8}: \text{HD} = 60.760\,\text{m};\ \Delta H = 0.215\,\text{m};\ s = 60.760\,\text{m}$$

$$\text{Direction TJ-8A} \rightarrow \text{TJ-8B}: \text{HD} = 63.111\,\text{m};\ \Delta H = 0.177\,\text{m};\ s = 63.111\,\text{m}$$

Compute the total random error in a single direction due to centering, pointing, reading, leveling as follows:

Stations TJ-8 and TJ-8A: Use forced-centering device, $\sigma_c = \pm 0.0001\,\text{m}$;

Stations TJ-8B: Use optical plummet, $\sigma_c = \pm 0.0005 \times \text{HI}\ \ (\text{m})$.

Direction TJ-8A to TJ-8B:

For direction TJ-8A to TJ-8B, the chosen centering error for each station are as follows:

$$\sigma_{c\text{TJ-8A}} = \pm 0.0001\,\text{m};\ \sigma_{c\text{TJ-8B}} = 0.0005 \times 1.080\,\text{m}\ \ (\text{or}\ \ 0.00054\,\text{m});$$
$$\text{Distance} = 63.111\,\text{m}$$

Using Equation (4.48), the centering for a direction measurement can be given as

$$\sigma_c = \frac{206,265}{63.111}\sqrt{(0.0001)^2 + (0.00054)^2} = 1.795'' \quad \textbf{7.14}$$

From Equation (4.21) and using $C = 45''$, the pointing error for direction measurement for $n = 1$ set can be calculated as

$$\sigma_P = \frac{45}{45\sqrt{2}} = 0.707'' \quad \textbf{7.15}$$

From Equation (4.32), the reading error for direction measurement for $n = 1$ set can be given as

$$\sigma_R = \frac{2.5(0.5)}{\sqrt{2}} = 0.884'' \quad \textbf{7.16}$$

From Equations (4.34) and (4.36), the leveling error for direction measurement in one set can be given as

$$\sigma_l = 0.2(10) \times \text{cotan}(Z) \quad \textbf{7.17}$$

where $\text{cotan}(Z) = (\Delta H/HD)$ and the level bubble sensitivity per division is 10"/div. This gives the leveling error of direction as $\sigma_l = 0.0056''$. The total error due to centering, pointing, reading, and leveling is calculated to be 2.12″.

Direction TJ-8A to TJ-8: For direction TJ-8A to TJ-8, the chosen centering error for each station is ±0.0001 m; Distance = 60.760 m.

Using Equation (4.48), the centering for a direction measurement can be given as

$$\sigma_c = \frac{206,265}{60.760}\sqrt{(0.0001)^2 + (0.0001)^2} = 0.48'' \quad \textbf{7.18}$$

Using the same approach as in the case of direction TJ-8A to TJ-8B, the calculated pointing error is 0.707″; the reading error is 0.884″; the leveling error is 0.0071″; and the total error is 1.23″.

7.6 DISTANCE MEASUREMENT DESIGN EXAMPLE

For visible and near infrared radiation and neglecting the effects of water vapor pressure, the refractive correction, ΔN, can be determined by

$$\Delta N_i = N_D - N_i = 281.8 - \frac{0.29065p}{1 + 0.00366086t} \quad \textbf{7.19}$$

The meteorological correction is in the sense that $s = s' + C_{met}$, with $C_{met} = \Delta N_i s'$.

Temperature and pressure are to be measured at each end of an 1800 m distance, the refractivity correction at each end will be calculated, and the average value of ΔN_i will be used to determine the meteorological correction, C_{met}. The instrument being used has a design $n_D = 1.0002818$ (so that $N_D = 281.8$) and the average temperature and pressure during the measurements are expected to be +35 °C and 1000 mb. What would be the largest values of σ_t and σ_p that, together with equal contribution to $\sigma_{\Delta N}$, would result in a meteorological correction that would contribute uncertainty of no more than 2 ppm to the corrected distance? (Reproduced by permission of CBEPS.)

Solution

From Equation (5.40), the first velocity correction (or meteorological correction) can be given as $C_{met} = \Delta\overline{N}s'$ (where $\Delta\overline{N}$ is in ppm). Uncertainty in meteorological correction (m) by error propagation of C_{met} can be given as $\sigma_{C_{met}} = s'\sigma_{\Delta\overline{N}}$. The error propagation of the average value of refractive correction ($\Delta\overline{N} = (\Delta N_1 + \Delta N_2)/2$ with ΔN_1 and ΔN_2 as the refractive corrections at the two ends of the measured line) can be given as

$$\sigma^2_{\Delta\overline{N}} = \frac{1}{4}(\sigma^2_{\Delta N_1} + \sigma^2_{\Delta N_2}) \quad (7.20)$$

Assuming equal contribution of error with $\sigma^2_{\Delta N} = \sigma^2_{\Delta N_1} = \sigma^2_{\Delta N_2}$, then

$$\sigma^2_{\Delta\overline{N}} = \frac{1}{2}(\sigma^2_{\Delta N}) \rightarrow \sigma_{\Delta N} = \sigma_{\Delta\overline{N}}\sqrt{2} \quad (7.21)$$

Applying the laws of variance–covariance propagation on Equation (7.19) with pressure (p) and temperature (t) as variables gives the following:

$$\sigma^2_{\Delta N_i} = \left(\frac{0.29065}{1 + 0.00366086t}\right)^2 \sigma^2_p + \left(\frac{0.29065 \times 0.00366086}{(1 + 0.00366086t)^2}P\right)^2 \sigma^2_t \quad (7.22)$$

In order to solve for the unknown quantities σ^2_p and σ^2_t in Equation (7.22), it will be assumed that each term in Equation (7.22) contributes equally to $\sigma^2_{\Delta N_i}$, resulting in the following relationships:

$$\left(\frac{0.29065}{1 + 0.00366086t}\right)^2 \sigma^2_p = \left(\frac{0.29065 \times 0.00366086}{(1 + 0.00366086t)^2}P\right)^2 \sigma^2_t = \frac{\sigma^2_{\Delta N_i}}{2} \quad (7.23)$$

Using the given uncertainty in meteorological correction ($\sigma_{\Delta\overline{N}} = 2\,\text{ppm}$) in Equation (7.21) and substituting the value (ppm) for $\sigma^2_{\Delta N_i}$ in Equation (7.23) give the following:

$$\left(\frac{0.29065}{1 + 0.00366086t}\right)^2 \sigma^2_p = 4 \text{ or } \left(\frac{0.29065}{1 + 0.00366086 \times 35}\right)\sigma_p = 2 \quad (7.24)$$

$$0.257639\sigma_p = 2 \text{ or } \sigma_p = 7.76\,\text{mbar} \quad (7.25)$$

Similarly for the other term in Equation (7.23):

$$\left(\frac{0.29065 \times 0.00366086}{(1 + 0.00366086t)^2}P\right)\sigma_t = 2 \quad (7.26)$$

$$0.83605543\sigma_t = 2 \rightarrow \sigma_t = 2.39^\circ\text{C} \quad (7.27)$$

The values $\sigma_t = 2.39°\text{C}$ and $\sigma_p = 7.76$ mbar are the largest errors expected.

7.7 TRAVERSE MEASUREMENT DESIGN EXAMPLES

Example 7.1

A closed-loop traverse of 5 points is to be run in a fairly flat and homogeneous terrain. Assume that the traverse legs will be approximately equal to 300 m and the specified allowable maximum misclosure of the five angles is to be 15″. Design the measurement scheme and the type of theodolite to be used for this traverse.

Solution

Let $3\sigma = 15''$; the permissible standard deviation of closure of the traverse will be 5″. Assuming the same precision (σ_θ) of angular measurements at each station (with five stations), from error propagation rule:

$$5\sigma_\theta^2 = (5'')^2 = 25 \qquad (7.28)$$

The permissible standard deviation of the angle measurements at each station will be

$$\sigma_\theta = \sqrt{\frac{25}{5}} = \sqrt{5} \qquad (7.29)$$

The permissible standard deviation of the angle measurements at each station will be due to reading error ($\sigma_{\theta r}$), pointing error ($\sigma_{\theta p}$), and centering error ($\sigma_{\theta c}$) (assuming the instruments are in good adjustment and the targets will be well designed). The leveling error will be ignored since the terrain is fairly flat. The permissible standard deviation for each angular measurement becomes

$$\sigma_\theta = \sqrt{\sigma_{\theta r}^2 + \sigma_{\theta p}^2 + \sigma_{\theta c}^2} \qquad (7.30)$$

Let us assume (for the sake of preanalysis) that the error components will have equal contribution, so that $\sigma_{\theta r} = \sigma_{\theta p} = \sigma_{\theta c} = \sigma$. In this case, each error will be equal to $\sigma_\theta/\sqrt{3}$ (or $\sqrt{5/3}$):

$$\sigma_{\theta r} = \sigma_{\theta p} = \sigma_{\theta c} = \sqrt{\frac{5}{3}} \qquad (7.31)$$

From Chrzanowski (1977) and Section 4.5.2.2, the reading error for an angle measured (based on directional method) in n sets for theodolites with optical micrometers and with the smallest division of 1″ or 0.5″ is given as

$$\sigma_{\theta r} = \frac{2.5d''}{\sqrt{n}} \tag{7.32}$$

where d is the nominal value of the smallest division of the instrument (in arc seconds).

In the current problem, the least count of the instrument to be used can be estimated as follows:

$$\sigma_{\theta r} = \sqrt{\frac{5}{3}} = \frac{2.5d''}{\sqrt{n}} \tag{7.33}$$

or

$$d'' = \frac{\sqrt{5n}}{2.5\sqrt{3}} \tag{7.34}$$

From Equation (7.34), it can be deduced that for n = 1 *set*, $d'' = \sqrt{5}/(2.5\sqrt{3})$ or 0.5″, meaning that 0.5″ theodolite should be used; for n = 4 *sets*, $d\varepsilon = \sqrt{20}/(2.5\sqrt{3})$ or 1″, meaning that 1″ theodolite should be used.

Considering a case of an average atmospheric condition (average visibility and thermal turbulence over short traverse legs) and the use of well-designed targets, it is understood from Chrzanowski (1977) and that provided in Section 4.5.1 that the pointing error for angular measurement can be expressed by

$$\sigma_{\theta p} = \frac{45''}{M\sqrt{n}} \tag{7.35}$$

Given in Equation (7.31) that $\sigma_{\theta r} = \sigma_{\theta p} = \sigma_{\theta c} = \sqrt{5/3}$, the magnification of instrument telescope can be computed as follows:

$$\sigma_{\theta p} = \sqrt{\frac{5}{3}} = \frac{45''}{M\sqrt{n}} \tag{7.36}$$

or

$$M = \frac{45''\sqrt{3}}{\sqrt{5n}} \tag{7.37}$$

From Equation (7.37), it can be deduced that for n = 1 set, $M = 45''\sqrt{3/5}$ or 35, meaning that a theodolite with a magnification of 35× should be used; for n = 4 sets, $M = 45''\sqrt{3/20}$ or 18, meaning that a theodolite with a magnification of 18× should be used.

The influence of centering errors (σ_c) on an angle measurement is given by

Chrzanowski (1977) and can be deduced from Equation (4.46) from Section 4.5.4 by assuming the angle measurement $\theta = 180°$; centering errors of target and instrument are the same; and the distances (D) are the same. From the appropriate substitution into Equation (4.46), the instrument centering error on angle measurement is deduced as

$$\sigma_{\theta c_i} = \frac{2\sigma_c}{D}(206,265) \tag{7.38}$$

and similarly, the target centering error on angle measurement can be deduced from Equation (4.44) as

$$\sigma_{\theta c_t} = \frac{\sigma_c}{D}(206,265)\sqrt{2} \tag{7.39}$$

Given in Equation (7.31) that $\sigma_{\theta r} = \sigma_{\theta p} = \sigma_{\theta c} = \sqrt{5/3}$ and assuming the centering errors of target and instrument are the same, the error due to each component will be $\sigma_{\theta c}/\sqrt{2}$ or $\sqrt{5/6}$. The type of instrument centering device to be used can be determined from the estimated centering error (σ_c) as follows:

$$\sqrt{\frac{5}{6}} = \frac{2\sigma_c}{D}(206,265) \tag{7.40}$$

so that by rearranging Equation (7.40):

$$\sigma_c = \frac{D}{2(206,265)}\sqrt{\frac{5}{6}} \tag{7.41}$$

Substituting D = 300,000 mm into Equation (7.41) gives σ_c = 0.7 mm or 0.4 mm/m if the height of instrument is taken as 1.6 m. The centering device (such as forced centering with tripod) that will give a centering error of 0.7 mm at the height of the instrument (about 1.6 m) above the ground will be suitable. Similarly, solving Equation (7.39) gives target centering error σ_c = 0.9 mm or better than 0.6 mm/m for the height of target of 1.6 m; a plumb line will be suitable as a centering device if it is not windy. The summary of the design is given in Table 7.4.

Table 7.4 Summary of Traverse Design.

Option	Magnification (M)	Least Count (d)	Number of Sets (n)	Type of Suitable Theodolite
1.	35	0.5″	1	Kern DKM3 (M = 45, d = 0.5″)
2.	18	1″	4	Wild T2 (M = 28, d = 1″)

In each option, optical plummet, laser plummet, or plumbing rods (0.5 mm/m) can be used as a centering device for the target and a forced-centering device with tripod for the instrument. If there will be recentering of the instrument between sets, then the optical plummet, laser plummet, or plumbing rods can be used as the centering device for the instrument.

Example 7.2

The maximum allowable angular misclosure (m_θ) in a traverse of 10 angles is 50″ at 99% confidence level, what is the expected standard deviation of measuring each of the angles of the traverse, assuming equal error for each angle?

Solution

Equation (2.15) or (2.16) can be used, but Equation (2.15) will be used as an example as follows:

$$|m_\theta| \le \sigma_{m_\theta} z_{\alpha/2} \tag{7.42}$$

where the misclosure $m_\theta = \bar{x} - \mu$ or 50″, α = 0.01, $z_{\alpha/2} = 2.576$ and σ_{m_θ} is the standard deviation (which should be considered as the SE) of the misclosure; note also from Equation (2.51) that $\sqrt{\chi^2_{\mathrm{df}=1,0.01}} = \sqrt{6.635} \quad (\text{or } 2.576)$. Determine the unknown σ_{m_θ} from the equation as follows:

$$\sigma_{m_\theta} = \frac{|m_\theta|}{z_{1-\alpha/2}} \rightarrow \sigma_{m_\theta} = \frac{50''}{2.576} \quad (\text{or } 19.4'')$$

By error propagation, the standard deviation of the misclosure can be expressed in terms of the standard deviation of individual measured angle as follows:

$$\sigma_{m_\theta} = \sqrt{\sigma^2_{\theta_1} + \sigma^2_{\theta_2} + \sigma^2_{\theta_3} + \cdots + \sigma^2_{\theta_{10}}} \tag{7.43}$$

For this problem, $\sigma^2_{\theta_1} = \sigma^2_{\theta_2} = \sigma^2_{\theta_3} = \cdots = \sigma^2_{\theta_{10}} = \sigma^2_\theta$; the propagated error of misclosure can be given as $\sigma_{m_\theta} = \sqrt{10}\sigma_\theta$ so that

$$\sigma_{m_\theta} = \sqrt{10}\sigma_\theta = 19.4'' \rightarrow \quad \sigma_\theta = 6.1''$$

Each of the traverse angles should be measured with a precision of not more than 6.1″.

Example 7.3

A traverse is to be measured around a rectangular city block, which is 100 m by 210 m as shown in Figure 7.3. For subsequent use, there has to be an intermediate point along each 210 m side so that there would be six angles (one, ~90°, at each corner and one, ~180°, in the middle of each long side) with approximate horizontal “lengths of sight” of ~100 or ~105 m. The two 100 m sides are relatively flat while the other two have slopes of +18% and of −18%. The equipment (theodolite or targets) would be set up on tripods with HI or HT of 1.755 m. Since the survey may extend over one session, forced centering cannot be assumed, but this would be inappropriate anyway since only ground mark points (monumented in the concrete of the sidewalk by either brass plates or finely cut crosses) will be occupied. At least two sets would be observed. Using theodolites with lower precision may require more sets to make the mean values compliant with the misclosure limit or compatible with the result from two sets using the highest precision instrument. Offer one choice of equipment and the associated procedures for observing the angles associated with the given situation, with consideration for the effects of centering, leveling, pointing, and reading. Determine the maximum misclosure in the loop of six angles.

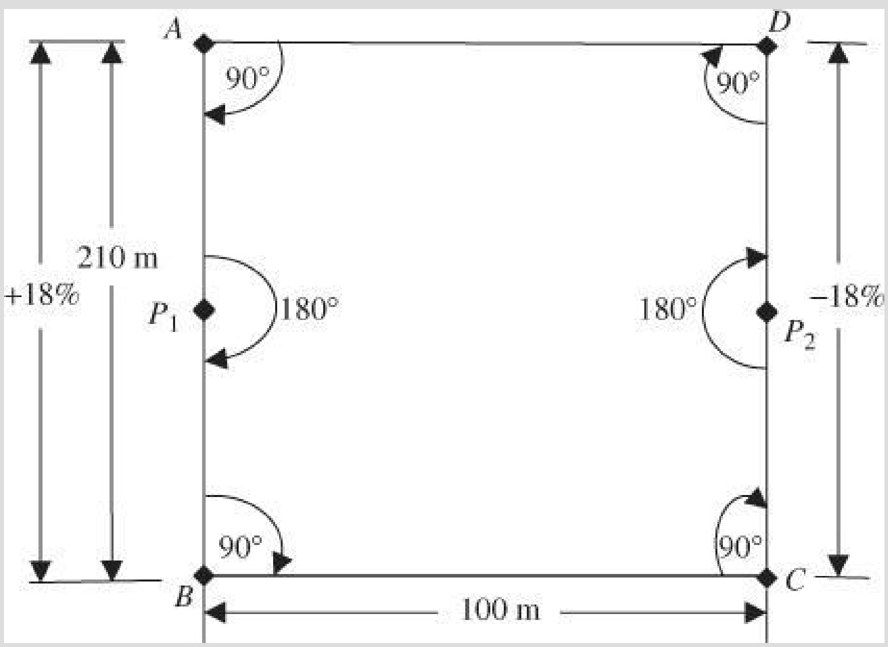

Figure 7.3 A sketch of a traverse around a rectangular city block.

Solution

Information supplied: HI or HT = 1.755 m; at least two sets observed; and forced centering is not assumed since the survey may extend beyond one session.

Horizontal distance of 210 m side is given, at a slope of +18% and −18%. The calculated slope distances AB is 213.375 m; AP_1, P_1B, CP_2, and P_2D are all equal to 106.687 m. Assume the choice of Leica TC2003 with a standard deviation of angle measurement of 0.5" (ISO 17123-3) and electronic dual-axis compensator with a setting accuracy of 0.3".

Centering error of instrument and targets on horizontal angles can be determined from Equation (4.47):

$$\sigma_{\theta c} = (206{,}265'')\sqrt{\frac{\sigma_{\text{cns}}^2}{S_{\text{bs}}^2} + \frac{\sigma_{\text{cfs}}^2}{S_{\text{fs}}^2} + \frac{\sigma_{\text{cst}}^2}{S_{\text{bs}}^2 S_{\text{fs}}^2}(S_{\text{bs}}^2 + S_{\text{fs}}^2 - 2S_{\text{bs}}S_{\text{fs}}\cos\theta)} \quad \mathbf{7.44}$$

Assuming all backsight (bs), foresight (fs), and setup (st) points are all centered and leveled using the same methods ($\sigma_{C\text{bs}} = \sigma_{C\text{fs}} = \sigma_{C\text{st}} = \sigma_c$). If optical plummet will be used, then

$$\sigma_c = (0.0005) \times (\text{HI}) \ \text{ or } \ \sigma_c = (0.0005) \times (1.755)\,\text{m} = 0.0008775\,\text{m}$$

Centering error at stations A, B, C, and D: Substituting S_{bs} = 106.687 m, S_{fs} = 100.000 m, θ = 90°, and σ_c = 0.0008775 m in Equation (7.44) gives the centering error as $\sigma_{\theta c} = 3.51''$. For recentering between two sets, the centering error will be 2.48".

Centering error at stations P_1 and P_2: Substituting S_{bs} = 106.687 m, S_{fs} = 106.687 m, θ = 180°, and σ_c = 0.0008775 m in Equation (7.44) gives the centering error as $\sigma_{\theta c} = 4.16''$. For recentering between two sets, the centering error will be 2.94".

Leveling error on angle measurement is determined from Equation (4.38) assuming electronic dual-axis compensator with a setting accuracy (σ_v) of ±0.3" (for Leica TC2003) will be used:

$$\sigma_{\theta L} = 0.3''\sqrt{[\cot(Z_{\text{bs}})]^2 + [\cot(Z_{\text{fs}})]^2} \quad \mathbf{7.45}$$

Leveling errors at A, B, C, D: For 18% slope, the backsight zenith angle at each station will be 79.796° and the foresight zenith angle will be 90°; substituting these values into Equation (7.45) gives $\sigma_{\theta L} = 0.05''$. For releveling between two sets, the leveling error will be 0.04".

Leveling errors at P_1 and P_2: For 18% slope, the backsight and foresight zenith angles at each station will be 79.796°; substituting these values into Equation (7.45) gives $\sigma_{\theta L} = 0.08''$. For releveling between two sets, the leveling error will be 0.06".

Pointing and reading errors at each station using the chosen Leica TC2003 is 0.5" for angle measurement in two sets:

Total error (for two sets) at each of stations A, B, C, D: $\sigma_\beta = 2.53''$.

Total error (for two sets) at each of stations P_1, P_2: $\sigma_\beta = 2.98''$.

The total error for the six stations is $\sigma_m = 6.59''$; the expected maximum error (at 99% confidence level) for the loop traverse will be determined from Equation (2.15) as $\sigma_m \times 2.57$ or 16.9".

Example 7.4

As part of a special traverse of "*n*" angles around a city block, a total station is to be set up along one side of the block, at one station with sight distances of 50 and 200 m with the angle being very close to 180°. The 50 m sight is nearly horizontal, but the 200 m sight is at a slope of 15%. These are the extreme values for this situation. Accounting for the effects of centering, leveling, pointing, and reading, recommend an instrument that would be capable of meeting the requirement that the block angular misclosure is not to exceed $10''\sqrt{n}$. "Not to exceed" is to be regarded as being at 99%. The values taken in the calculation of the misclosure would be averages from at least two sets (a set being the average of face left and face right sightings)

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Solution

Standard deviation of misclosure of "*n*" angles: $\sigma_m = \sigma_\theta\sqrt{n}$.

Using Equation (2.52), the maximum misclosure (at 99%) can be given as

$$|m| = \sigma_m\sqrt{\chi^2_{\alpha=0.01,\mathrm{df}=1}} \text{ or } |m| = \sigma_\theta\sqrt{n}\sqrt{6.63} \qquad \textbf{7.46}$$

Equate the maximum misclosure to the angular misclosure and solve for σ_θ:

$$\sigma_\theta\sqrt{n}\sqrt{6.63} = 10''\sqrt{n} \rightarrow \sigma_\theta = \frac{10''}{\sqrt{6.63}} \rightarrow \sigma_\theta = \frac{10''}{2.575}(\text{or } 3.88'') \qquad \textbf{7.47}$$

Error propagation for each angle θ due to centering, leveling, pointing, and reading errors:

$$\sigma_\theta^2 = \sigma_c^2 + \sigma_L^2 + \sigma_P^2 + \sigma_R^2 \qquad \textbf{7.48}$$

Assuming equal contribution (σ) of all the errors: $\sigma_\theta = \sigma\sqrt{4}$ so that each error will contribute $\sigma = \sigma_\theta/2$ (or 1.94″), and $\sigma_{\theta p} = \sigma_{\theta r} = \sigma_{\theta c} = \sigma_{\theta L} = 1.94''$.

- From Equation (4.22), the pointing error (two sets):

$$\sigma_{\theta p} = \frac{\sigma_p}{\sqrt{2}} \quad \sigma_p = \frac{60''}{M} \tag{7.49}$$

Solve for M in Equation (7.49):

$$M = \frac{60''}{\sigma_{\theta p}\sqrt{2}} \quad (\text{or } M = 21.9\times) \tag{7.50}$$

- From Equation (4.31), the reading error (two sets):

$$\sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}} \quad \sigma_r = 2.5\text{div} \tag{7.51}$$

Solve for div (given $\sigma_{\theta r} = 1.94''$):

$$\text{div} = \frac{\sigma_{\theta r}\sqrt{2}}{2.5} (\text{or div} = 1.1'') \tag{7.52}$$

- Centering error (due to target and instrument): Since the question is specific about the distances, we cannot make any other assumptions about them but use Equation (4.47) and assume that the worst angle θ will be 180° and the centering error (σ_c) for target and instrument are equal. The centering error σ_c is then solved for as follows:

$$\sigma_{\theta c} = \sqrt{\frac{\sigma^2}{(200)^2} + \frac{\sigma^2}{(50)^2} + \frac{\sigma^2}{(200)^2(50)^2}(200^2 + 50^2 - 2(200)(50)\cos 180)} \tag{7.53}$$

or

$$\sigma_{\theta c}^2 = 0.000025\sigma^2 + 0.0004\sigma^2 + 0.000625\sigma^2 \quad \text{or} \quad \sigma_{\theta c}^2 = 0.00105\sigma^2 \tag{7.54}$$

For recentering between two sets, the centering error on one set of angle will be $1.94''\sqrt{2}$ or 2.74". The error in centering the instrument and the target can be calculated from Equation (7.54) as

$$0.00105\sigma^2 = \left(\frac{2.74}{206,265}\right)^2 \quad \text{or} \quad \sigma^2 = 1.680586011\text{E}{-7} \tag{7.55}$$

with σ = 4.1E−4 m (or 0.41 mm or 0.00041 m) as the expected centering error of the instrument and the target, which requires forced centering device of ±0.0001 m.

- From Equation (4.40), the leveling error (releveling between two sets) can be given as

$$\sigma_{\theta L} = \frac{\sigma_v}{\sqrt{2}}\sqrt{\cot^2(Z_b) + \cot^2(Z_f)} \quad \text{cotan } Z_b = 0 \tag{7.56}$$

Since the slope angle (tangent of the vertical angle (90° − Z_f)) is 15%, cotan(Z_f) = 0.15, so that the following is obtained:

$$\sigma_{\theta L} = \frac{\sigma_v}{\sqrt{2}}(0.15) = 1.94'' \quad \text{or} \quad \sigma_v = 18.3'' \qquad 7.57$$

From Equation (7.57), $\sigma_v = 0.2\text{div}$ or 18.3" so that the sensitivity of plate bubble div is equal to 92"/2 mm. On the basis of this design, the recommended instrument will have the following features: M = 22×; least count = 1″; forced centering with target and instrument interchange on tripods (with equivalent centering error of 0.0001 × height of instrument); and bubble sensitivity better than 92"/2 mm.

7.8 ELEVATION DIFFERENCE MEASUREMENT DESIGN EXAMPLE

Determine, by the propagation of variance, whether a Wild N3 could be used, with double-scale invar staves, for Canadian special-order leveling. If not, suggest the order for which it would be suitable and why.

Solution

Some of the specifications for Wild N3 level are as follows: standard deviation for 1 km double run leveling is 0.2 mm; setting accuracy (split bubble) is 0.25″; parallel-plate micrometer (with a range of 10 mm, interval of graduation of 0.1 mm with estimation to 0.01 mm possible); magnification of telescope, M = 42×; and tubular level sensitivity per 2 mm is 10″.

The sources of error are pointing, reading, and leveling of the instrument; the magnitude of each error is estimated as follows:

Pointing error can be calculated from Equation (6.1) as

$$\sigma_P = \frac{45}{206,265 \times M} S \qquad \textbf{7.58}$$

For the given sight distance S = 50 m and magnification M = 42, the calculated pointing is $\sigma_P = 0.2597$ mm. The reading/plumbing error is estimated using Equation (6.2):

$$\sigma_r = \frac{\ell}{2}\left(\frac{v_r}{206,265}\right)^2 \qquad \textbf{7.59}$$

For the given length of rod $\ell = 3000$ mm and sensitivity of leveling rod v_r = 600″, the calculated reading error is $\sigma_r = 0.0127$ mm. The instrument leveling error is calculated from

Equation (6.3):

$$\sigma_{\mathrm{L}} = \left(\frac{\sigma_{\mathrm{v}}}{206,265}\right) S \quad \mathbf{7.60}$$

Given the error in leveling the instrument as $\sigma_{\mathrm{v}} = 0.25''$ and the sight distance as S = 50 m, the leveling error is calculated as $\sigma_{\mathrm{L}} = 0.0606 \text{ mm}$. The total in an elevation difference measurement in a setup is calculated as

$$\sigma_{\Delta h} = \sqrt{(0.2597)^2 + (0.0127)^2 + (0.0606)^2} \text{ or } 0.267 \text{ mm} \quad \mathbf{7.61}$$

The value calculated in Equation (7.61) is for the average of two leveling runs in one setup as the double-scale invar rod readings suggest. It can be concluded from the above calculations that Wild N3 level with double-scale invar rods will yield a standard deviation of elevation difference of 0.267 mm per setup with 50 m sight lengths. The following relationship can be established for special-order leveling using Equation (3.6) and the specification in Table 3.1:

$$\sigma_{\mathrm{ran}} \sqrt{2} \times 1.96\sqrt{L} = 3\sqrt{L} \text{ mm} \quad \mathbf{7.62}$$

where σ_{ran} is the standard deviation of elevation difference over 1 km, L is the total length of leveling section, and $3\sqrt{L}$ mm is the allowable discrepancy between independent forward and backward leveling runs between benchmarks (at 95% confidence) for special-order leveling. The value of σ_{ran} from Equation (7.62) is $1.0823 \text{ mm}/\sqrt{\text{km}}$. The expected standard deviation of elevation difference ($\sigma_{\Delta h1}$) at every instrument setup (with a total backsight and foresight distance of 100 m per setup or 10 setups in 1 km leveling section) can be calculated as $\sigma_{\Delta h1} = 1.0823/\sqrt{10}$ or 0.342 mm. This is considered as the value for leveling done twice in a setup; leveling with double-scale invar staves involves leveling twice per setup with the average of the two elevation differences determined and used as the elevation difference for that setup. On this basis, 0.342 mm is considered as the standard deviation of the average of two elevation differences at a setup. Since the standard deviation (±0.267 mm per setup) achievable with N3 with double-scale rods is less than ±0.342 mm per setup required by special order, then Wild N3 can be used for the Canadian special-order leveling.