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OPTIMAL DESIGN IN GEODETIC GNSS-BASED NETWORKS

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Abstract

An optimal design of a geodetic network helps the surveying engineers maximise the efficiency of the network. A number of pre-defined quality requirements, i.e. precision, reliability, and cost, of the network are fulfilled by performing an optimisation procedure. Today, this is almost always accomplished by implementing analytical solutions, where the human intervention in the process cycle is limited to defining the requirements. Nevertheless, a trial and error method can be beneficial to some applications. In order to analytically solve an optimisation problem, it can be classified to different orders, where an optimal datum, configuration, and optimal observation weights can be sought such that the precision, reliability and cost criteria are satisfied.

In this thesis, which is a compilation of six peer-reviewed papers, we optimised and redesigned a number of GNSS-based monitoring networks in Sweden by developing new methodologies. In addition, optimal design and efficiency of total station establishment with RTK-GNSS is investigated in this research.

Sensitivity of a network in detecting displacements is of importance for monitoring purposes. In the first paper, a precision criterion was defined to enable a GNSS-based monitoring network to detect 5 mm displacements at each network point. Developing an optimisation model by considering this precision criterion, reliability and cost yielded a decrease of 17% in the number of observed single baselines implying a reliable and precise network at lower cost. The second paper concerned a case, where the precision of observations could be improved in forthcoming measurements. Thus a new precision criterion was developed to consider this assumption. A significant change was seen in the optimised design of the network for subsequent measurements. As yet, the weight of single baselines was subject to optimisation, while in the third paper, the effect of mathematical correlations between GNSS baselines was considered in the optimisation. Hence, the sessions of observations, including more than two receivers, were optimised. Four out of ten sessions with three simultaneous operating receivers were eliminated in a monitoring network with designed displacement detection of 5 mm. The sixth paper was the last one dealing with optimisation of GNSS networks. The area of interest was divided into a number of three-dimensional elements and the precision of deformation parameters was used in developing a precision criterion. This criterion enabled the network to detect displacements of 3 mm at each point.

A total station can be set up in the field by different methods, e.g. free station or setup over a known point. A real-time updated free station method uses RTK-GNSS to determine the coordinates and orientation of a total station. The efficiency of this method in height determination was investigated in the fourth paper. The research produced promising results suggesting using the method as an alternative to traditional levelling under some conditions. Moreover, an optimal location for the total station in free station establishment was studied in the fifth paper. It was numerically shown that the height component has no significant effect on the optimal localisation.

Sammanfattning

En optimal utformning (*design*) av geodetiska nät bidrar till mätningsingenjörernas möjligheter att maximera nätens användbarhet. Detta åstadkoms genom att ett antal fördefinierade kvalitetskrav – t.ex. nätets precision, tillförlitlighet och kostnad – uppfylls genom lösning av ett optimeringsproblem. I dag utförs optimeringen nästan alltid genom implementering av analytiska lösningar, där möjligheterna till mänsklig påverkan på processen är begränsad till att definiera kraven. Ändå kan "trial and error"-metoder vara till nytta i vissa tillämpningar. För att analytiskt lösa ett optimeringsproblem delas det lämpligen upp i flera steg, där ett optimalt geodetiskt datum, en optimal nätkonfiguration och optimala observationsvikter bestäms på ett sådant sätt att kriterierna för precision, tillförlitlighet och kostnad är uppfyllda.

I denna avhandling, som är en sammanställning av sex expertgranskade artiklar, har ett antal svenska GNSS-baserade övervakningsnät optimerats och omformats genom utveckling av nya metoder. Dessutom har optimal design och effektivitet vid totalstationsetablering med RTK-GNSS studerats.

Ett deformationsnäts känslighet för lägesförändringar (*sensitivity*) är av stor betydelse för övervakningsändamål. I den första artikeln definierades ett precisionskriterium så att ett GNSS-baserat övervakningsnät ska kunna detektera 5 mm rörelse i varje nätpunkt. Utveckling av en optimeringsmodell med detta precisionskriterium – samt tillförlitlighet och kostnad – resulterade i en minskning med 17 % av det antal individuella baslinjer som behövde bestämmas; dvs. ett tillförlitligt och noggrant övervakningsnät kunde tas fram till en lägre kostnad. I den andra artikeln antogs att precisionen i observationerna kunde förbättras i kommande mätningar. Därför utvecklades ett nytt precisionskriterium för att testa detta antagande, och en signifikant förändring kunde ses i det optimerade nätet vid efterföljande mätningar. Inledningsvis var de individuella GNSS-baslinjernas vikt i fokus vid optimeringen, men i den tredje artikeln beaktades även effekten av matematiska korrelationer mellan baslinjerna. Därför optimerades observationssessioner som inkluderade mer än två mottagare. Fyra av tio sessioner, med tre samtidiga mottagare, kunde därigenom elimineras i ett övervakningsnät som var utformat för att kunna detektera rörelser på 5 mm. Den sjätte artikeln är den sista som behandlar optimering av GNSS-nät. Det aktuella området delades här upp i ett antal tredimensionella element och precisionen på deformationsparametrarna användes för att utveckla ett nytt precisionskriterium. Genom denna utveckling kunde övervakningsnätet detektera rörelser ned till 3 mm i varje nätpunkt.

Totalstationsetablering i fält kan ske med olika metoder, t.ex. *fri station* eller uppställning över en känd stompunkt. Metoden *realtidsuppdaterad fri station* (RUFRIS) använder RTK-GNSS för att bestämma totalstationens koordinater. Effektiviteten vid höjdbestämning med denna metod undersöktes i den fjärde artikeln. Undersökningen gav lovande resultat som tyder på att man, under vissa förhållanden, kan använda metoden som ett alternativ till traditionell avvägning. Avslutningsvis, i den femte artikeln, studerades den optimala placeringen av en totalstation. I studien visades numeriskt att höjdkomponenten inte har någon signifikant inverkan på stationens optimala placering.

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Alizadeh-Khameneh, M. A., Eshagh, M. & Sjöberg, L. E., 2015. Optimisation of Lilla Edet Landslide GPS Monitoring Network. *Journal of Geodetic Science,* 5(1), pp. 57-66.

Paper II:

Alizadeh-Khameneh, M. A., Eshagh, M. & Sjöberg, L. E., 2016. The Effect of Instrumental Precision on Optimisation of Displacement Monitoring Networks. *Acta Geodaetica et Geophysica,* 51(4), pp. 761–772.

Paper III:

Alizadeh-Khameneh, M. A., Sjöberg, L. E. & Jensen, A. B. O., 2017. Optimisation of GNSS Networks – Considering Baseline Correlations. *Survey Review,* published online. DOI: http://dx.doi.org/10.1080/00396265.2017.1342896

Paper IV:

Alizadeh-Khameneh, M. A., Jensen, A. B. O., Horemuž, M. & Andersson, J. V., 2017. *Investigation of the RUFRIS Method with GNSS and Total Station for Leveling.* Nottingham, UK, IEEE, in Press.

Paper V:

Alizadeh-Khameneh, M. A., Horemuž, M., Jensen, A. B. O. & Andersson, J. V. Optimal Vertical Placement of Total Station. *Journal of Surveying Engineering,* in Review.

Paper VI:

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Acronyms

BOOM	Bi-Objective Optimisation Method
CP	Control Point
FOD	First-Order design
GPS	Global Positioning System
GNSS	Global Navigations Satellite Systems
ITRF	International Terrestrial Reference Frame
MOOM	Multi-Objective Optimisation Method
OF	Objective Function
RTK	Real-Time Kinematic
RUFRIS	Real-time Updated Free Station
SOD	Second-Order Design
SOOM	Single-Objective Optimisation Method
SWEPOS	Swedish national network of permanent GNSS reference stations
SWEREF	Swedish Reference Frame
THOD	Third-Order Design
TS	Total Station
VC	Variance-Covariance
ZOD	Zero Order Design

1 Introduction

1.1 Motivation

A geodetic network consists of a number of ground points that are tied together by performing some geodetic observations. In traditional terrestrial surveying, the observations are angle and distance measurements. However in recent decades, satellite and laser technologies contributed new observations to geodetic networks. Moreover, a network is established for different purposes in surveying engineering such as deformation monitoring, coordinate estimation of unknown points, construction control and many other applications. Therefore, it is of importance to choose the most proper surveying method between the terrestrial and satellite-based, or the triangulation and trilateration methods for establishing a network. It is also imperative to decide proper locations for the network points. Survey engineers usually undertake these tasks by intuitive methods based on their own experiences. For instance, a geodetic control network for an infrastructure project should cover the object, control points should be placed on stable locations and in case of using terrestrial measurements, there must be direct view between the control points for measuring angles and distances. However, the surveyors have yet to answer questions such as: if all observations, regardless of their type, are necessary to be performed for the network's purpose; if the location of control points can influence the quality of the network, and a number of similar questions. The cost of establishing a network is also important and draws attention to feasibility of its execution. Therefore, the observation plan should not be designed to fulfil neither more nor less than the requirements of the network.

1.2 Research Objectives

Basically, all geodetic networks should be constructed on the basis of a designed observation plan. Today, designing geodetic networks are scrutinised from quality control and economical points of view to utilise the designed plan in performing flawless surveying. Although previously the networks were designed by the surveyors own experience and intuition, nowadays, engineers do to larger degree use developed analytical methods to design an optimal observation plan, where all network quality requirements are fulfilled.

This thesis concentrates on optimal design and optimisation of geodetic networks. Some new methods in optimal design of geodetic networks were developed in the course of this research based on the concept of analytical solutions. A major problem in the optimisation of a geodetic network is how to define a proper design criterion. Therefore, new solutions for the design criterion were introduced in this research. For instance, we considered sensitivity, correlated weight matrix of observations, as well as deformation parameters (strains and rotations) in the definition of the precision criterion. Different optimisation models and constraints were also investigated and the best models were suggested to the readers. The developed methods were implemented in geodetic networks with measurements from the Global Navigation Satellite Systems (GNSS) to build an optimal observation plan for them.

When using the GNSS measurements in establishment of a Total Station (TS), it is of interest to use an optimal number of control points with optimal height distributions. We performed both trial and error, and analytical approaches to investigate the optimal conditions for the TS establishment and came up with recommendations for survey engineers. It is also interesting for the surveyors to see whether it is possible to replace the conventional levelling approach with combined GNSS/TS measurements for height determination. To answer this question, a method was developed for GNSS/TS combination and implemented in a real application in Sweden to numerically investigate the efficiency of this method in practical surveying projects.

1.3 Thesis Structure

The thesis is written as a compilation of four published, and two submitted journal papers. It consists of two major parts, where the first part includes a brief description of all methodologies that are developed in this work plus numerical results that are achieved by implementing these methods. In the second part, one may find attached all the original papers.

Recalling the first part, it contains six chapters, starting with an introduction in the first chapter. The principles of geodetic network design and optimisation is described by the second chapter. To be more realistic in presenting the developed methodologies, almost all of them were applied to real applications. Totally, three different study areas in Sweden are used for this purpose. A brief verbal description and graphical presentation of each area is provided in the third chapter. The different optimisation models that were developed for GNSS monitoring purposes are explained in chapter four. This chapter starts by presenting the background of previous works in optimisation of geodetic networks, and contains a short description of methodologies that have been used in four articles. The chapter ends by presenting some main results and discussions of the articles. An optimal design for TS establishment using GNSS measurements is the subject of the fifth chapter. Very similar to chapter four, this chapter starts with reviewing previous studies on TS establishment. Thereafter, it presents the developed methodologies and eventually, the results. All the discussions and final results are concluded in the sixth and last chapter followed by a few suggestions for future works.

1.4 Author's Contributions

The first paper concerns our study on the monitoring network of a village in the Southwest of Sweden – Lilla Edet. The monitoring procedure was conducted by using Global Positioning System (GPS) measurements. The municipality of Lilla Edet provided us with some initial information in the form of a number of annual reports from a consultant, who had epoch-wisely performed the monitoring task. Different optimisation models were developed by considering all network quality criteria (i.e. precision, reliability and cost) plus a sensitivity of 5 mm in detecting possible displacements at each network point. Based on this research we could suggest new observation plans to the municipality for their monitoring network if they intend to perform observations by using two GNSS receivers. Our redesigned network is capable to fulfil the demanded requirements, while it can be economically beneficial as it removes unnecessary observations from the plan.

A new methodology was developed in the second paper to investigate the case in optimisation of monitoring networks, where more precise instruments can be available in subsequent epochs. We applied the methodology again to the Lilla Edet GPS monitoring network. This investigation was more theoretical as we do not know whether the municipality can use more precise instruments in carrying out the next epochs. However, the numerical results show that it is possible to design a network with fewer measurements for future observation plans if better instruments could be used. More specifically about "precise instruments" in GNSS networks, one can mention longer observation time and the use of data from multiple GNSS.

Usually, correlations between GNSS baselines are ignored to simplify the methodology and numerical results. In contrary to the previous two papers, where uncorrelated observations were assumed, the correlations were considered in the third paper. We developed an innovative methodology to consider correlations between static GNSS baselines that result from using more than two receivers simultaneously. In the approach, variance factors of sessions of GNSS observations were optimised instead of single baselines. The methodology was implemented on a GPS monitoring network in Skåne in the southernmost part of Sweden. As this monitoring network is not active anymore, and due to the fact that the area is subject to deformation because of an active fault zone, this study was applied to this area to hopefully bring back some interests to revive the monitoring work.

An investigation was conducted in the fourth paper to find an alternative method for the traditional levelling method in surveying projects. A height can be determined from a combined method of GNSS/TS. To verify the efficiency of this novel method in height determination, levelling and GNSS/TS data from a high-speed railway construction project – the East Link Project – in Sweden were analysed. All data were provided by the Swedish Transport Administration. Furthermore, a program package was developed to manipulate the data and proceed with adjustment and estimation of heights. A report was prepared at WSP Group and submitted to the Transport Administration presenting our numerical results and suggested using the GNSS/TS method in projects with limited access to height control points.

The fifth paper consists of two parts, where the role of the height in optimum establishment of a TS was investigated analytically in the first part and numerically with a trial and error method in the second part. The former part explains the developed analytical solution, where the Symbolic Math Toolbox of MATLAB scripting language was used partially to simplify the equations. Thereafter, a computational program was developed to estimate the three-dimensional coordinates of the TS when using the free-station method. The results of this study could answer the question from surveyors on how to optimally distribute their control points in three-dimensions for a TS establishment purpose.

The outcome of the last paper is a new solution to an optimisation problem of deformation monitoring networks, where the precision of deformation parameters are of interest. By using the finite element strain analysis and developing further methods, we involved the deformation parameters in optimisation and design of a monitoring network. Similar to the third paper, the methodology was implemented in the Skåne GPS monitoring network.

2 Geodetic Network Design

To establish a geodetic network, it is imperative to design the network beforehand. To start with the design process, various a priori information is needed. Depending on the purpose of the geodetic network, this information can be, for instance: required precision of the network, strength of the network in detecting blunders, a priori information on geological status of the area if the network is for monitoring purposes, and so on. A design process is usually yielding an observation plan for the network with some recommendations on the performance of measurements. The proper datum, the number of needed control points, the type of observations (i.e. angle, distance, satellite positioning observables, etc.), as well as the number of observations can be involved in the recommendation list for the client. Amongst many advantages of the network design, the usability of an observation plan in reviving a destroyed or inactive network can be mentioned. It is also possible to redesign and optimise a current network if the observation plan is available.

There are two approaches for designing a network: a trial and error method, where the experience and intuitive feelings of the designer are involved, and an analytical method, where the mathematical solutions form the basis of the method. A distinctive difference between these two methods is minor human intervention in an analytical design procedure. However, to perform an analytical method, it is of course also needed to obtain some a priori information from the field reconnaissance, thus the human intervention is inevitable, but it is limited. Regardless of the pros and cons of these two methods, the efficiency, optimality, and swiftness of the method are of importance in different applications.

The ultimate goal of designing a network is to obtain an optimal network in the sense of precision, reliability, and economy. These are the predefined criteria in each geodetic network, and the designed network should be able to fulfil these requirements. Other criteria can be added to the requirements of the network based on the purpose of that network, e.g. sensitivity criterion in deformation monitoring networks. By an optimal design of a network, we can decide how the network datum, configuration, and observations can influence the quality of the network. For instance, an observation can be eliminated from an observation plan of a network, if the observation has an insignificant role in fulfilling the required quality of the network.

In this chapter, the main quality requirements of a geodetic network and different stages of the design problem will be described in detail.

2.1 Network Quality Criteria

The essence of designing and optimising a geodetic network is to enable the network to fulfil predefined quality requirements. The major requirements consist of precision, reliability, and economy. Precision is the measure of how the network propagates random errors, reliability is the capability of the redundant observations in detecting model errors, and economy expresses the cost of a network, i.e. cost of building control points, performing measurements, transportation, etc. (Teunissen, 1985a). An optimally designed network should be precise and reliable enough to fulfil the pre-set requirements of the network. At the same time, this network should have an economic feasibility to be constructed in practice. However, considering all these criteria simultaneously in a design procedure may lead to some inconsistencies between them. A possible solution for this problem will be discussed in following chapters. As we do not need observations yet in the design stage, the mentioned quality criteria should be independent from any observations, and instead dependent on the weights of the observations and on the configuration of the network. The principles of these criteria will be described in more detail in this section.

Precision Criterion

In order to design an optimal geodetic network, the precision criterion is the most demanded factor in this process. Rationally, it is of great importance for surveyors to design or perform precise enough observations in a network. The precision of network points is affected by the observational precision and the network geometry. Generally, the Variance-Covariance (VC) matrix of the network coordinates $\mathbf{C_x}$, is the best form of representing the network precision, where its diagonal elements are the variances of the coordinate components, and the off-diagonal elements show the covariance amongst them. As the VC matrix is datum dependent, so by assuming the minimum constraint datum for a network, it can be written as (Kuang, 1996, p. 221):

$$\mathbf{C}_{\mathbf{x}} = \sigma_0^2 \left[\left(\mathbf{A}^{\mathrm{T}} \mathbf{P} \mathbf{A} + \mathbf{D} \mathbf{D}^{\mathrm{T}} \right)^{-1} - \mathbf{H} \left(\mathbf{H}^{\mathrm{T}} \mathbf{D} \mathbf{D}^{\mathrm{T}} \mathbf{H} \right)^{-1} \mathbf{H}^{\mathrm{T}} \right] \tag{1}$$

where σ_0^2 is the a priori variance factor, which is usually set to 1 at the design stage. The design and weight matrices of the observations are represented by $\mathbf{A}$ and $\mathbf{P}$, respectively. The $\mathbf{P}$ matrix is a fully populated matrix, if the correlations between all observables are considered; otherwise, it is a diagonal matrix. $\mathbf{D}$ and $\mathbf{H}$ are the matrices with the minimum and inner constraint datum information for the network, respectively. If we consider the inner constraint datum for a network, the $\mathbf{D}$ matrix in Eq. (1) would be replaced by the $\mathbf{H}$ matrix (Koch, 2010, p. 62).

In the design stage, we usually assume that the configuration matrix in Eq. (1) includes all possible observations in the network. Hence, the obtained VC matrix represents the precision of the network with all observations. However, this is not an ideal precision that we are seeking. The ideal case will be introduced to the optimization procedure by defining a criterion matrix. This matrix has the same dimension as the VC matrix in Eq. (1), and can be either defined by theoretical methods such as a Taylor-Karman structure (Grafarend, 1974), or derived from empirical solutions (Cross, 1985). The elements of the matrix can be computed based on the requirements of the client, for instance a desired value for the precision of estimated parameters can be considered as a criterion (Schmitt, 1985). In case of working with large networks, where it is not practically feasible to go through all precision elements in the VC matrix separately, a scalar precision function can be considered as a reasonable alternative. Some widespread examples are the norm, trace, determinant, etc. of the VC matrix of the network points, which can be applied according to the user requirements (Kuang, 1991).

In this research, we developed different methods to define a precision criterion matrix. According to different purposes, the criterion was defined by considering for instance sensitivity to displacement detection, precision of deformation parameters, and a fully populated weight matrix.

Reliability Criterion

In addition to the precision criterion, an optimal network is supposed to be reliable enough in order to detect gross errors and minimise the effects of undetected ones on the coordinates of network points. Baarda (1968) introduced the concept of reliability to perform quality control for geodetic networks. He used a statistical hypothesis to test if the outliers are detectable or not. The reliability matrix, $\mathbf{R}$, has the following structure (Kuang, 1996, p. 122):

$$\mathbf{R} = \mathbf{I}_n - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A} + \mathbf{D}\mathbf{D}^{\mathrm{T}}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P} \tag{2}$$

where $\mathbf{I}_n$ is an $n \times n$ identity matrix with n observations, and the other parameters are the same as introduced in Eq. (1).

The *i*-th diagonal element of the reliability matrix is called the redundancy number of the *i*-th observation. The redundancy number r_i will be in the range between 0 and 1. The redundancy number equals to zero (minimum case), when the number of observations is the same as the number of unknowns. Hence, $\mathbf{A}$ in Eq. (2) is an $n \times n$ square and invertible matrix, and therefore $\mathbf{R}$ is zero. In this extreme case, the network has no capability in detecting gross errors. On the other hand, the redundancy number equals to 1 (maximum case), when representing the ability of the network in detecting all gross errors, regardless of their size. This occurs when there are only observations and no unknowns in the system of equations, thus $\mathbf{A}$ becomes a zero matrix and consequently, $\mathbf{R} = \mathbf{I}_n$. A unit reliability matrix means that the residual of an observable is not affected by the error of other observables, but only with its own error (Amiri-Simkooei, et al., 2012). From the least squares adjustment point of view, if we assume that the redundancy number equals to zero, then all errors in the observations will be directed to estimated parameters and nothing will be absorbed by residuals, i.e. it is impossible to detect errors (Teunissen, 1985a). On the contrary, the observation has the highest degree of reliability and controllability, if $r_i = 1$, and this happens only when the true value of the observation is known. However, as already mentioned, the design stage does not deal with observations and estimations of unknown parameters. It should be mentioned that the least squares method is not only sensitive to gross errors, but it is also very efficient in hiding large errors and in distributing their effects over measurements, thus making them undetectable. Erroneous measurements are not necessarily those, which have the largest residuals after adjustment (Krarup, et al., 1980). In other words, the least squares method will give the best unbiased estimates of the unknown parameters only if the true errors of data are randomly distributed. Furthermore, the presence of a large gross error may damage the linearisation process of the least squares method and cause the iteration procedure to diverge, which may lead to no or a wrong solution.

Therefore, performing a data screening procedure such as a pre-adjustment or post-adjustment step is necessary to obtain reliable results. Moreover, the systematic errors are assumed to be treated separately from the design stage.

Generally, the reliability of a network can be divided into internal and external ones. The former describes the ability of a network in detecting its gross errors, and the capability of observations in controlling each other, while the latter investigates the influence of undetected gross errors on estimated parameters. Recalling Eq. (2), the dependency of reliability on configuration of a network and the weight of observations is obvious. Therefore, the reliability of a network should be considered as one of the criteria in optimal design of a network.

Cost Criterion

The third criterion that ensures the optimality of a network is the cost of the project. Contrary to the previous two criteria, the desired cost of a project is its minimum value. The relation of precision and reliability criteria with design of a network is somehow clear in Eqs. (1) and (2), while there is no specific equation to describe the cost criterion. Based on the type of project, different factors are subjected to cost. For instance, in levelling networks, the length of the levelling run can affect the economy. More instrument set-ups and labour force is needed to transfer and distribute the known height of a benchmark to other points in a large levelling network. All these efforts would raise the cost of the project. It is of interest to mention that in this thesis (section 5), we investigate an alternative method for levelling in large-scale networks, which could probably reduce the project cost.

Another example is the cost of a GNSS monitoring network. The cost criterion in this case can be related to transportation, the observation length, labour and equipment costs, etc. In case of using static GNSS measurements, the time that should be spent on performing the measurements can also be included in definition of the cost criterion.

Practically, it is difficult to come up with a solution that provides the highest precision and reliability as well as the lowest cost for a project. The more precise and reliable the network is, the more expensive is the project. Theoretically, fewer observations, fewer observation iterations, or observations with lower precision result in lower project expenses. However, a network with a reasonable precision and reliability can be the economically most optimum one (Teunissen, 1985b).

Sensitivity Criterion

Dealing with deformation or displacement monitoring networks, the sensitivity criterion can be introduced in addition to the abovementioned criteria. The sensitivity of a network describes the ability of the network in detecting the possible displacements or deformations. Therefore, we need to implement the sensitivity criterion within the optimisation procedure to enable the network to detect a specified magnitude of deformation. It is of importance to investigate whether an assumed displacement is detectable in the network or not. For this purpose, we follow the statistical model presented by Kuang (1996, p. 302) as:

$$\lambda_j = \hat{\mathbf{d}}_j^{\mathrm{T}} \mathbf{C}_{\hat{\mathbf{d}}_j}^{-1} \hat{\mathbf{d}}_j \sim \chi_{1-\alpha}^2\left(df\right) \tag{3}$$

where λ_i denotes a test variable, $\hat{\mathbf{d}}_j$ is the adjusted displacement vector at network point j, $\mathbf{C}_{\hat{\mathbf{d}}_j}$ represents the corresponding displacement VC matrix, and $\chi_{1-\alpha}^2\left(df\right)$ is the chi-square distribution with the significance level of $1-\alpha$, where typically $\alpha = 0.05$ and df is the degrees of freedom.

The statistical hypotheses for this experiment can be defined as:

$$\begin{cases} \mathrm{H}_0 : \mathrm{E}\left\{\hat{\mathbf{d}}_j\right\} = 0 \\ \mathrm{H}_1 : \mathrm{E}\left\{\hat{\mathbf{d}}_j\right\} \neq 0 \end{cases} \tag{4}$$

where $\mathrm{E}\{\bullet\}$ is the statistical expectation operator. The null hypothesis will be rejected if $\lambda_j \geq \chi_{0.95}^2\left(df\right)$, which means that a detectable displacement occurred at point j. Hence, $\hat{\mathbf{d}}_j$ cannot be considered as a random error.

In designing a deformation monitoring network, the sensitivity can be either used as a condition to define a precision criterion for deformation parameters, or involved in the optimisation procedure as a separate criterion. For instance, in the former case, a postulated displacement of certain magnitude can be used in the displacement vector $\mathbf{d}$ in Eq. (3), and a precision criterion for displacements $\mathbf{C}_{\mathbf{d}}$ can be defined based on a chi-square test. Alternatively, the sensitivity can be considered as a single design criterion where the goal is to maximise it. Now, the network configuration and observation weights should be optimised in such a way that the network will have maximum sensitivity in detecting a specific magnitude of deformations or displacements.

2.2 Network Design Orders

To design an optimal geodetic network, Grafarend (1974) proposed a four-step procedure to solve the design problem. These steps cover the methodology of the design procedure. However, to establish an optimally designed network, some other practical efforts should be added to these theoretical steps. For instance, a comprehensive reconnaissance of the network area is essential at the beginning. Establishing the network and performing the measurements according to the observation plan (as obtained from design procedure) is the final step to execute the design network in practice.

The four-step design procedure starts by performing a Zero-Order Design (ZOD). In this step, an optimum datum is sought for the network. The datum of a geodetic network is defined as the minimum required number of parameters to connect the configuration and observations of the network to a known coordinate system. It is worth mentioning that the datum parameters are defined based on the type of network, e.g. one translation in the z-direction is needed for a levelling network, while three translations, in x-, y- and z-directions, are required in a three-dimensional GNSS network (Kuang, 1996, p. 100). The datum of a geodetic network can be defined by, for instance, minimum or inner constraints. In the former case, a number of fixed constraints, such as positions, directions and distances of actual stations are used in the datum definition, while in the latter, some constraints of an artificially made station in the centre of the network is fixed. When performing the ZOD step in a network with minimum constraints datum, the best station positions, directions, and distances are determined to be fixed in the network. By fixing these constraints the highest precision can be sought in adjusting the network.

An optimal configuration of a network is another problem that can be solved in the First-Order Design (FOD). The approximate locations of network points are usually decided in the initial reconnaissance, mainly according to the topography of the area. The visibility between the points in case of designing for a traditional terrestrial network and the open sky visibility in case of GNSS networks are of importance. However, these approximate point locations can be altered to increase the quality of the network. Based on the pre-defined set of criteria in the beginning of the design and optimisation procedure, some changes – within a pre-set boundary – are applied to the approximate coordinates of the network points in the FOD step.

To answer the question raised in the Motivation section (1.1) about the type and the number of observations that are needed in an optimal network, Second-Order Design (SOD) should be conducted. The SOD step deals directly with the weight of observations ($\mathbf{P}$ matrix in Eqs. (1) and (2)) and determines whether an observation should be performed, and if so, with which precision. Observations that receive low weights after running the optimisation procedure can be removed from the network, implying that their absence cannot diminish the required network quality. Furthermore, the type of required observations can also be decided based on the obtained weight matrix. In a three-dimensional network with distance, horizontal and vertical angle measurements, the ones with higher weights would be retained in the observation plan. Seemingly, the SOD contributes more than the other steps to fulfil the cost criterion of the optimal design by eliminating unnecessary observations.

We performed mainly the SOD step in this thesis to optimise the various geodetic networks (Chapter 4). However, the FOD was also used in our research, when we investigated the free station method for total station establishment in Chapter 5.

Finally, an already designed geodetic network can be improved with respect to precision, reliability, and sensitivity by adding or removing some observations. This step is the last order of the design process and known as Third-Order Design (THOD). If the quality requirement of a network is subject to change before the next measurement campaign, then the THOD can provide a new suggestion for the observation plan. Adding some new network points (network densification), adding some new observations, and/or changing the observation types can be implemented in the network to fulfil any new criteria.

3 Study Areas

The real application is essentially complementary to that of the theory. A theory has much high impact in the real life if it can be tested and verified in a real application. This impact can be even more highlighted in engineering sciences, where methodologies are developed to improve the human technology. As a survey engineer, we deal more or less with real applications in our daily life. Therefore, we tried to experiment with all our developed methodologies in optimal design of geodetic networks on real geodetic networks. However, Paper V in this thesis was written based on simulated data. Totally, we used three different study areas in Sweden, and in this chapter, we describe each of them in detail.

3.1 Lilla Edet GPS Monitoring Network

Since the year 2000, a GPS monitoring network has been established in the Lilla Edet region. The village of Lilla Edet is located in Västra Götaland County, the Southwest of Sweden. This region is well-known for its landslides and subductions (SGI, 2012). An illustration of the region and the GPS network points is provided in Fig. 1. As can be seen in the figure, the village is divided into two parts by the Göta River, one of the main rivers in Sweden. According to previous studies, there are numerous areas with high risk of landslides along this river, mostly between Lilla Edet and Trollhättan, and the risks will increase with the effects of climate change. Annually, several landslides of different sizes occur along the river, and the area belongs to those with the most frequent landslides in the country. The risky areas along the river are depicted in the figure by a darker colour based on the data from the Swedish Geodata portal[1]. Meanwhile, the figure shows that many residential areas are located within this risky zone.

The municipality of Lilla Edet, therefore, hired a consultant to monitor and report the possible landslides of the region, regularly with specific time

[1] https://www.geodata.se/

intervals. Due to size and vastness of the area, the consultant chose GPS measurements to establish a geodetic monitoring network. The network surrounded the village and consisted of 35 stations, where six of them were fixed stations (shown by red triangles in the figure) on top of the hills around the area, and the rest (shown by red circles in the figure) were distributed inside the village. Moreover, 245 independent GPS baselines were observed in each epoch of observation. This monitoring network was active between 2000 and 2013, and totally 8 epochs of observations were carried out based on a fixed structure of the observation plan during these years. It should be mentioned that the coordinate system used in the computations was SWEREF 99 12 00 and the height reference system was the Swedish national height system, RH 2000.

Due to the importance of continuing this monitoring network, it was decided to redesign its observation plan according to the concept of analytical optimisation solution. Our developed optimisation models (Papers I and II) were implemented in this network by considering predefined quality criteria provided by the consultant (Nordqvist, 2012).

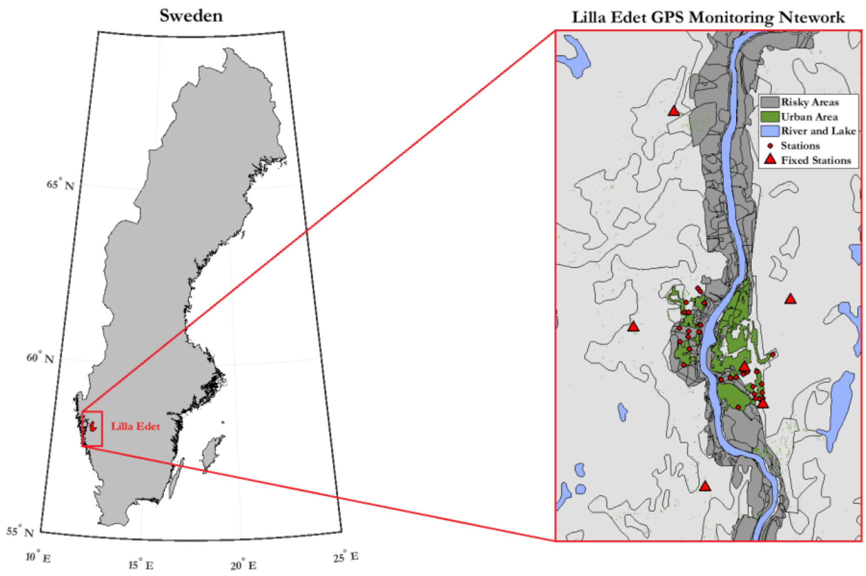

***Figure 1**. Lilla Edet study area and the established GPS stations for monitoring purpose. It is also overtly clear in the figure that some residential areas are located within the risky zone.*

3.2 Skåne GPS Monitoring Network

Amongst several active fault zones in Sweden, the Tornquist zone in Skåne, the southernmost landscape of Sweden, is of interest to be investigated due to its activity during the recent decades. A part of this zone extends diagonally from the Northwest of the Skåne region to its Southeast (see Fig. 2). To estimate crustal deformations of the area, a GPS monitoring network was established by KTH-geodesy covering the fault zone. The network consists of seven GPS stations with a maximum distance of 80 kilometres. Five of these stations were distributed around the zone, and two stations were located inside (shown in Fig. 2 by red triangles). Totally, three epochs of observations were performed in 1992, 1996 and 1998 (Pan, et al., 2001). The estimated coordinates of the points are available in ITRF96 as geocentric Cartesian coordinates for these three epochs. To build a local coordinate system, we transformed the coordinates to the Transverse Mercator map projection. Also, ellipsoidal heights of the network points were used in our study.

The Skåne network has fewer points than the Lilla Edet network, and therefore it is proper to implement some other optimisation ideas (Papers III and VI) that could be easily introduced and tested with this network.

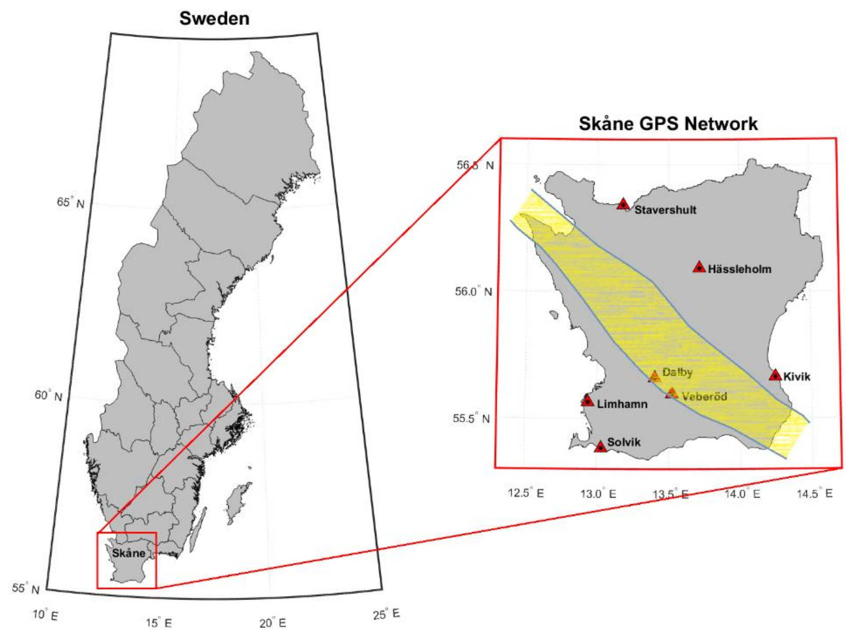

***Figure 2**. The GPS monitoring network of Skåne in the South of Sweden. The right figure shows the location of established GPS stations and the fault zone in the area.*

3.3 Geodetic Networks in the East Link Project

The East Link project is the first step towards a first generation of high-speed railways in Sweden. The construction of this railway, for about 150 kilometres, provides a fast and sustainable transportation between the cities of Järna and Linköping (Fig. 3) and the inauguration is planned for 2028. Totally 97 geodetic ground stations are established along the project corridor by using the combined GNSS/TS method. These stations are depicted in Fig. 3 by small filled circles. The colour difference of plotted symbols in the figure is to illustrate the two different map projection zones of the project, i.e. SWEREF 99 16 30 and SWEREF 99 18 00.

Furthermore, in a separate survey campaign, the heights of these points are determined by using the traditional double-run levelling. In Paper IV, we used the obtained data from this project to carry out our study on verification of the efficiency of the combined GNSS/TS method in height determination.

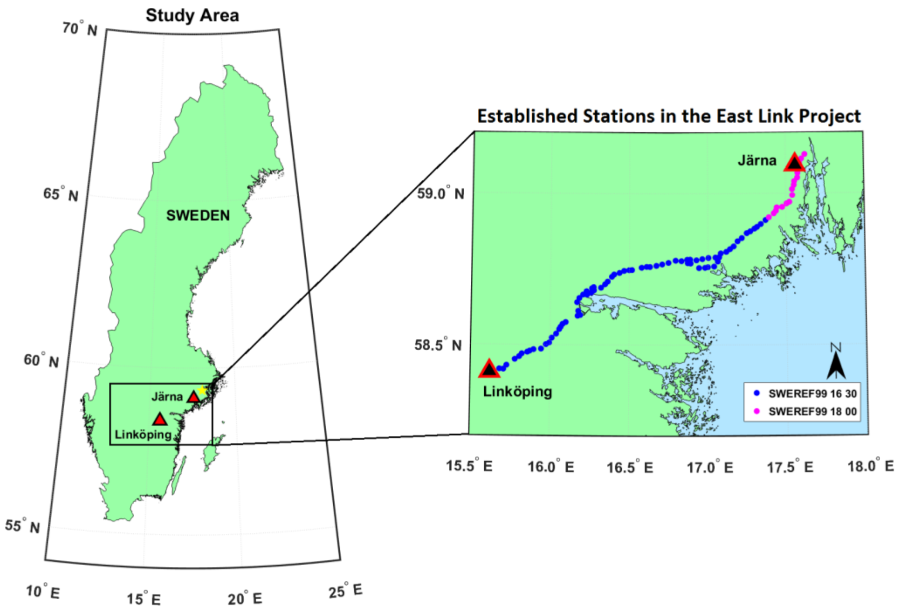

***Figure 3**. Illustration of the study area and established stations along the East Link high-speed railway project in Sweden.*

4 Optimisation of Geodetic Networks

4.1 Background

An optimal geodetic network should satisfy the pre-defined quality requirements of the network, which can be assigned by the network users. In other words, an optimal network should provide high precision for the network points, high reliability for detecting possible gross errors and low cost for execution of the network. A number of salient studies conducted on optimisation and design problem are addressed in this section.

Optimality Criteria

An ideal precision of a geodetic network can be presented by a VC matrix of the point coordinates. Now, the precision criterion can be derived as a scalar function of the elements of this matrix or introduced as a criterion matrix. A number of common precision scalar functions can be defined for optimisation of geodetic networks (Kuang, 1996, p. 206), for instance: the A-optimality (minimising the trace of the VC matrix), D-optimality (VC matrix determinant is being minimised), and E-optimality (minimising the maximum eigenvalue of the VC matrix).

Furthermore, an optimal network should have the capability to detect gross errors in the observations and minimise the effect of the undetected ones on the adjustment results (Fan, 2010). Baarda (1968) proposed a global test for outlier detection and data snooping for the localisation of gross errors and introduced the concept of reliability. The reliability criterion is widely used as a requirement in designing optimum networks. As an example, Amiri-Simkooei (2004) optimally designed a geodetic network at the SOD stage to meet maximum reliability. In his work, the weights of the observations were improved so that the redundancy numbers for all observations became the same. In another study, the effect of less reliable observations on a deformation monitoring network of a dam was inspected by Amiri-Simkooei (2001b). To confront the probable distortions in the network, due to weak observations from a reliability point of view, he came up with a solution to decrease their weights in the SOD stage to reach a reasonable range of reliability.

In addition to the reliability criterion, robustness analysis of a geodetic network is an efficient technique to handle the effect of errors on the network (Amiri-Simkooei, 2001a). This method is defined based on the concept of strain and reflects the geometrical strength or weakness of the network. A comparison between the reliability and geometrical strength criteria reveals a correlation between these two, where the largest robustness parameters are due to observations with minimum redundancy number (Amiri-Simkooei, 2001a). This implies that a network has high geometrical strength if it is designed considering high reliability.

The cost of a project is another criterion to be considered in optimising a geodetic network. Using the GNSS measurements in establishing a monitoring network, one can reduce the cost for instance by considering transportation distances between network points and the length of the observation times. Generally, the monitoring projects are carried out either using continuous GNSS stations (e.g. Naito, et al., 1998) or by establishing the GNSS receivers temporarily and repeatedly at pre-set network points. Dare and Saleh (2000) performed an epoch-wise survey consisting of many observation sessions in which the GPS receivers were moved between the network points. In their work, the instrument shifts were addressed as more costly than the observation time of each session. The difficulty of finding an optimal solution for large networks induced Dare and Saleh (2000) to use a simulated annealing solution based on a heuristic approach to define the session schedule.

Optimisation Models

Based on the priority of each of these criteria, any of them can be considered as an Objective Function (OF) in developing an optimisation model. Considering only one OF will yield a Single-Objective Optimisation Model (SOOM), while involving two or more OFs in constructing the optimisation model leads to Bi-Objective Optimisation Model (BOOM) and Multi-Objective Optimisation Model (MOOM), respectively (Xu, 1989). Moreover, these models could or should be bound to some other constraints. For instance, SOOM of precision constrained to reliability has the precision criterion as the OF and should control the reliability of the network keeping it within the defined range. The previous studies on the efficiency of these models show that the risk of encountering inconsistencies between constraints is high when using the SOOMs. A comparison between different SOOMs was performed by Bagherbandi, et al. (2009), who realised inconsistencies in a SOOM of cost.

This model could hardly meet the demanded requirements because of precision and reliability constraints. However, they found that a SOOM of reliability, which is constrained to precision, is the best model as it could maximise the reliability of the network, while it satisfies the precision. Furthermore, the SOOM of reliability has more dependency on the configuration of a network and applies more changes to it to fulfil the network requirements. The possible inconsistencies are inconspicuous, when more than one OF is included in the optimisation model, i.e. BOOM or MOOM (Kuang, 1996, p. 250). The capability of the BOOM versus SOOM was presented in Eshagh (2005), in which the possible inconsistency between constraints in the SOOM was eliminated by using a bi-objective model. The effect of constraints on a BOOM was investigated by Eshagh & Alizadeh-Khameneh (2015). It was shown numerically that a BOOM of unconstrained precision and reliability is more efficient than the constrained models. As this model has both of the precision and reliability criteria as an OF, it could satisfy the network demands. Furthermore, the unconstrained BOOM of precision and reliability removes more observations from the observation plan and is therefore economically beneficial in practice.

Design Orders

A general review of the network design orders was provided in a book, where Teunissen (1985b), Koch (1985) and Schmitt (1985) explained respectively the ZOD, FOD and SOD concepts. However, the idea of categorising the design problem to a number of steps was introduced by Grafarend (1974) and developed in consecutive years, e.g. Grafarend (1975), Milbert (1979) and Schmitt (1980).

In recent years, more researches have concentrated on optimisation and design problems. For instance, a closed-form analytical solution was sought by Blewitt (2000) to perform the FOD step and seek for the best configuration of a geodetic network by optimising the precision of geophysical parameters. The ZOD and FOD problems were investigated in simple trilateration and triangulation networks by Amiri-Simkooei, et al. (2012). They realised that the optimal configuration of the trilateral network may become different than the triangulation network when designing a network to meet pre-set precision and reliability criteria. In short, the type and number of observations play a significant role in optimal design of geodetic networks.

Alzubaidy, et al. (2012) discussed the problems of the FOD and SOD in a micro-geodetic network. Kuang (1992) proposed an approach to solving the SOD problem, where an optimal solution for observation weights was obtained by the best approximation of a defined criterion matrix, and not its inverse matrix. Kuang (1993) presented another approach to the SOD leading to maximum reliability using linear programming. To fulfil the postulated precision of a GPS network, Mehrabi & Voosoghi, (2014) developed an analytical method to find a solution for the SOD problem in their network. They concluded that by using optimisation techniques it is possible to cut down the number of observed baselines by 36% and save on the measurement cost, but at the same time achieve the defined precision.

In order to find an analytical solution for an optimisation problem according to the defined model, either linear or quadratic programming is used. These mathematical techniques are explained in detail in, for instance, Lemke (1962), Milbert (1979) and Cross (1985).

GNSS Networks

The efficiency and many advantages of GNSS measurements in comparison to conventional surveying techniques made it very popular over the last few years. Therefore, the GNSS measurements and their effective design play a crucial role in many different applications of surveying engineering. For instance, Gerasimenko, et al. (2000) designed two-dimensional geodynamic GPS networks for monitoring crustal movements. They constructed a VC matrix by ignoring the correlations between GPS baselines and used it in the definition of an optimality criterion.

Amongst different types of observables in GNSS, the carrier phase double-difference observations yield better performance than undifferenced observations for short to medium length baselines. The VC matrix of GNSS observations contains not only information about the precision of observations, but also on the correlations amongst them. Two types of correlations can affect the double-difference phase observations, i.e. mathematical and physical ones. The former is resulted from differencing the phase observations and the latter is created due to environmental effects on the observations which make them spatially and/or temporarily correlated (Hofmann-Wellenhof, et al., 2008). A comprehensive introduction was provided in Santos, et al. (1997) on the evolution of researches – from 1985 to

1994 – considering the mathematical and physical correlations in GNSS-based applications. It is mentioned in this text that the effect of considering within-baseline mathematical correlation is insignificant in positioning, while considering between-baseline correlation may lead to a little improvement. However, their own investigation on the effect of mathematical correlation on GPS networks by considering baselines of hundreds of kilometres yielded the conclusion that taking the mathematical correlations into account would provide better estimation in the reliability of baseline components, as well as more realistic uncertainty estimates. The same conclusion was derived from other similar works, e.g. Ding, et al. (2004) or Fotiou, et al. (2009), emphasising the importance of considering the mathematical correlations on performing geodetic measurements, particularly for control purposes. The insignificant effect of physical correlation on the estimation of coordinates and ambiguities was elucidated by El-Rabbany & Kleusberg (2003). However, they mentioned the efficiency of long observations in decreasing the effect of physical correlations on coordinate estimations.

Deformation Monitoring Networks

For monitoring deformations with the help of geodetic methods, usually a network is established to cover the deformable body. The networks can be built either as a relative or a reference network according to the purpose of monitoring. In the former type, all the network points are located on the deformable body, while in the latter, some points are established outside of the deformable body as reference points to determine the absolute displacement of the object points (Chrzanowski & Secord, 1983). In absolute measurements, the number of reference points is directly related to the reliability of the network. For instance, in GNSS monitoring networks, as the number of reference points increases, the less error is propagated into the coordinates of the points (Kutoglu & Berber, 2015).

Deformation surveys provide a priori information from different measuring techniques to build a proper deformation model for an area for analyses of the corresponding deformation parameters. A number of typical deformation models in two-dimensional space are introduced and discussed in Chrzanowski & Secord (1983) and Setan & Singh (2001). It is quite common in some of the previous studies on deformation analysis to consider the whole deformable body as one object and compute the deformation parameters of that (see for instance: Doma, 2014, and Kuang, 1996, pp. 292-300). Thus, the obtained strain components describe the deforming condition of the whole body. However, a simple solution to find a more precise deformation model for a deformable object is to split the body into smaller pieces (Dermanis & Grafarend, 1992). For instance, Welsch (1983) divided the area of interest to finite elements and determined the horizontal strain patterns in each element by geodetic observations. Moreover, Kiamehr & Sjöberg (2005) investigated the three-dimensional finite element method to analyse surface deformation patterns in the Skåne area in Southern Sweden.

A precision criterion matrix for designing a deformation network can be defined by using several methods. For instance, the eigenvalues and eigenvectors of the VC matrix of a network were used in Crosilla (1983) to define the criterion matrix in solving the SOD problem. The criterion, in this study, was constructed in a way that the eigenvectors, i.e. greatest semi-axis of the error ellipse, should be as orthogonal as possible to the direction of predicted deformations.

Other Optimisation Methods

Besides using the traditional analytical approaches for solving the optimisation problems, some metaheuristic algorithms have recently been developed. Well-known examples are genetic algorithms (Haupt & Haupt, 2004), ant colonies and particle swarm optimisation. For instance, the social behaviour of some creatures living in a group is the source of inspiration for developing the particle swarm optimisation method. Doma (2013) used the particle swarm optimisation method to optimise a similar simulated GPS network as Kuang (1996, p. 338) concluding that his method is more efficient in optimising the network due to the elimination of more baselines by fulfilling the precision criterion. Also, this method was used by Singh, et al. (2016) to solve the FOD problem in densification of GPS networks. The high convergence rate of the swarm optimisation method compared with the traditional optimisation solutions was highlighted in their research.

A kind of nature-inspired method called the shuffled frog leaping algorithm was used by Yetkin & Inal (2015) to design an optimal deformation monitoring network. They used this method to find the optimal reference point locations such that the reliability of the network became a maximum. They found this algorithm easier to perform than traditional methods as it does not need either linearisation or differentiation of the OF. The FOD problem was solved in another study by applying a simulated annealing method in Berne & Baselga (2004). The OF was defined based on the D-optimality criterion, and the method could successfully fulfil it.

4.2 Optimisation Models and Constraints

Basically, an OF in optimisation models should be minimised or maximised in order to satisfy the quality requirements of the network. The OFs are usually constructed by considering precision, reliability and cost criteria – one, any pair of these criteria, or all of them. Generally, to design an optimal network, the precision and reliability criteria should be maximised, while the cost should be a minimum. Hence a general OF can be presented as (Schaffrin, 1985):

$$f_{precision} + f_{reliability} + f_{cost}^{-1} = \max \tag{5}$$

where f denotes a function of precision, reliability or cost. Equation (5) expresses a MOOM, but it can be degraded to a BOOM or SOOM by removing one or two of the functions. Each OF in this equation will be explained in detail below.

If we consider an ideal VC matrix as a precision criterion matrix ($\mathbf{C}_c$), then the OF for precision can be defined in such a way that the best fit of the existing VC matrix ($\mathbf{C}_\mathbf{x}$) of a network to the criterion is obtained. In other words, the criterion matrix is introduced to the optimisation procedure to push the current precision of the network towards the required one, i.e.

$$\left\| \mathbf{C}_\mathbf{x} - \mathbf{C}_c \right\|_2 = \min . \tag{6}$$

The VC matrix ($\mathbf{C}_\mathbf{x}$) in Eq. (6) is the one in Eq. (1), and the criterion matrix ($\mathbf{C}_c$) can be defined according to the purpose of the network. The use of an S-transformation is indispensable to transform the criterion matrix to the datum of the network (Baarda, 1973). The L_2 norm is shown by the symbol $\|\bullet\|_2$ in the equation. However, Eq. (6) should be constrained to a precision control to avoid achieving better VC matrix than the criterion after running the optimisation procedure. Hence, $\text{vecdiag}\left(\mathbf{C}_\mathbf{x}\right) \le \text{vecdiag}\left(\mathbf{C}_c\right)$, where the operator of "vecdiag" produces a vector containing the diagonal elements of a matrix.

The reliability function in Eq. (5) should also be a maximum to assure the capability of a network in detecting and resisting gross errors, i.e.

$$\|\mathbf{r}\|_{\infty} = \max \tag{7}$$

where $\|\bullet\|_{\infty}$ represents the L_{∞} norm and $\mathbf{r}$ is a vector containing the diagonal elements of the $\mathbf{R}$ matrix in Eq. (2).

The last term in Eq. (5) belongs to the cost of a network. It is obvious that this function should be a minimum to increase the feasibility of executing the designed network. As already mentioned, there are many factors that are involved in the determination of the project costs. However, to simplify and facilitate the problem, we minimise the weight matrix of the observations ($\mathbf{P}$). The low weights imply less precise observations, which perhaps are obtained from less precise instruments, shorter observation times and/or less repeatability in the measurements. Hence,

$$\|\mathbf{P}\|_{1} = \min \tag{8}$$

with $\|\bullet\|_{1}$ denoting the L_{1} norm. Different norm choices are selected according to Kuang (1996) because of the advantage of each norm operator in fulfilling the optimisation purpose.

Similar to the precision control, we should define reliability and cost controls as well. The reliability could be constrained to a pre-set minimum redundancy number so that it does not fall below this value, but exceed it. Furthermore, the cost control should guarantee that the weights are determined below the assigned value after the optimisation procedure.

In addition to precision, reliability and cost inequality constraints, physical constraints should also be considered in an optimal design. When performing the FOD step, usually some improvements are added to the initial positions of the network points to satisfy the optimality criteria. Therefore, it is required to constrain the optimisation model to the datum of the network (translation, rotation, and scaling), so the FOD step cannot affect the datum. Moreover, there are usually restrictions in moving the network points for instance because of invisibility conditions where topography or man-made obstacles near the network points prevents line of sight. Therefore, another constraint should be introduced to the optimisation model, where the maximum possible movement of each network point (boundary value) is specified. To complete the set of physical constraints, a boundary value should also be defined for the observation weights. This constraint is valid for all the orders of the design problem. The observation weight cannot be a negative value, and it can definitely not receive a larger value than the one corresponding to the maximum precision of the instrument used.

4.3 Optimisation Procedure

An analytical optimisation procedure in a geodetic network commences with defining an OF. Based on the purpose of the network, the OF may contain any single or combined functions of Eqs. (6) to (8). However, the VC, reliability and weight matrices in the presented equations consist of non-linear functions of both network point coordinates and observation weights. By expanding these matrices using Taylor series to first orders, one can obtain the linearised form of them as (Kuang, 1996, pp. 221-232):

$$\begin{cases} \mathbf{C_x} = \mathbf{C_x^0} + \sum_{j=1}^{m} \frac{\partial \mathbf{C_x}}{\partial \mathrm{x}_j} \Delta \mathrm{x}_j + \sum_{j=1}^{m} \frac{\partial \mathbf{C_x}}{\partial \mathrm{y}_j} \Delta \mathrm{y}_j + \sum_{j=1}^{m} \frac{\partial \mathbf{C_x}}{\partial \mathrm{z}_j} \Delta \mathrm{z}_j + \sum_{i=1}^{n} \frac{\partial \mathbf{C_x}}{\partial \mathrm{p}_i} \Delta \mathrm{p}_i \\ \mathbf{R} = \mathbf{R}^0 + \sum_{j=1}^{m} \frac{\partial \mathbf{R}}{\partial \mathrm{x}_j} \Delta \mathrm{x}_j + \sum_{j=1}^{m} \frac{\partial \mathbf{R}}{\partial \mathrm{y}_j} \Delta \mathrm{y}_j + \sum_{j=1}^{m} \frac{\partial \mathbf{R}}{\partial \mathrm{z}_j} \Delta \mathrm{z}_j + \sum_{i=1}^{n} \frac{\partial \mathbf{R}}{\partial \mathrm{p}_i} \Delta \mathrm{p}_i \\ \mathbf{P} = \mathbf{P}^0 + \sum_{i=1}^{n} \frac{\partial \mathbf{P}}{\partial \mathrm{p}_i} \Delta \mathrm{p}_i \end{cases} \tag{9}$$

where $\mathbf{C_x^0}$ and $\mathbf{R}^0$ are the approximate VC and reliability matrices that can be computed by using Eqs. (1) and (2) with respect to initial values of the coordinates (x^0, y^0 and z^0) and weight (p^0), respectively. The approximate weight matrix is also denoted as $\mathbf{P}^0$. Equation (9) contains the coordinate improvements of each network point j, i.e. $\Delta\mathrm{x}_j$, $\Delta\mathrm{y}_j$ and $\Delta\mathrm{z}_j$, and weight improvement for each observation i, i.e. $\Delta\mathrm{p}_i$. Moreover, m and n are respectively the number of network points and observations in the network.

Now, by inserting the linearised matrices from Eq. (9), into Eqs. (6) to (8), and reformulating the results in a compact matrix form, the objective functions of the network can be written as:

- Precision function:

$$\left\| \mathbf{C}_1 + \mathbf{C}_2 \mathbf{w}_c \right\|_2 = \min \tag{10}$$

where $\mathbf{C}_1 = \text{vec}\left(\mathbf{C}_\mathbf{x}^0\right) - \text{vec}\left(\mathbf{C}_c\right)$, by using the vec operator to stack the columns of a matrix below each other to construct one vector. $\mathbf{C}_2$ is a matrix consisting of the derivatives of $\mathbf{C}_\mathbf{x}$ with respect to point coordinates and weights. The vector $\mathbf{w}_c$ contains the unknown parameters, which are coordinate and weight improvements.

- Reliability function:

$$\left\|\mathbf{R}_1 + \mathbf{R}_2\mathbf{w}_r\right\|_\infty = \max \tag{11}$$

  where the diagonal elements of the approximate reliability matrix ($\mathbf{R}^0$) are stacked in the $\mathbf{R}_1$ vector, and the derivatives of $\mathbf{R}$ with respect to point coordinates and observation weights are collected in the $\mathbf{R}_2$ matrix. The unknown vector $\mathbf{w}_r$ is the same as $\mathbf{w}_p$.

- Cost (Economy) function:

$$\left\|\mathbf{E}_1 + \mathbf{E}_2\mathbf{w}_e\right\|_1 = \min \tag{12}$$

  where $\mathbf{E}_1$ is a vector containing the diagonal elements of the approximate weight matrix ($\mathbf{P}^0$), and $\mathbf{E}_2$ is a matrix including the derivatives of the weight matrix $\mathbf{P}$ with respect to weights of each observation. $\mathbf{w}_e$ is an unknown vector with improvements of the observation weights.

Any of these OFs presented in Eqs. (10) to (12) can be considered as a SOOM and solved by quadratic or linear programming subject to a number of control and physical constraints. It is the choice of a norm that specifies the programming solution. For instance, Eq. (6) or (10) is developed based on the concept of best fitting to the user's pre-set precision in a least squares sense, and therefore, the L_2 norm is applied because of its computational efficiency. The best mathematical approach for this problem can be sought by quadratic programming (Milbert, 1979). However, linear programming methods minimise or maximise linear functions subject to a number of linear constraints. Thus, linear programming can be used in Eqs. (11) and (12) to ensure that the reliability and cost of the network will meet the user's desired values (Cross, 1985).

One can overcome possible inconsistencies between constraints by including more than one optimality criterion in the OF, namely forming a MOOM and subjecting it to a number of inequality constraints. Quadratic programming can be used for solving this optimisation problem.

Basically, designing an observation plan for a geodetic control or monitoring network does not depend on the values of observations, but their precision. Hence, an optimisation procedure may start with considering all possible observations (angles, distances, baseline components …) in the observation plan. Evidently, the approximate positions of the network points must be decided at the beginning of the procedure. Although the ZOD and FOD steps are considered as the prior steps to the SOD, they are not necessarily performed beforehand. In other words, depending on the purpose and condition of a geodetic network, the optimal observation plan can be sought only by performing the SOD step. In fact, this step should merit more attention in the optimization procedure due to mainly two reasons. The first reason is because of a number of practical difficulties (see. Section 2.2) that decreases the feasibility and efficiency of the FOD step. Secondly, the cost of a project, which is important for decision makers, can be determined by solving the SOD problem. However, the result of the FOD in changing the location of network points can be of interest in establishing large-scale networks with the possibility of moving points within hundreds of metres or even kilometres.

In essence, the coordinate and weight improvements are the unknowns of an optimisation procedure. If an optimal solution is sought to estimate the coordinate improvements, then a FOD step is performed. Hence, the optimal positions of the network points can be determined by applying these small improvements in each iteration until the solution converges and meets the pre-set defined criterion. On the other hand, the improvements of the observation weights should be estimated, if the SOD step is executed. Recalling Eq. (9), one can see that by using an analytical approach, it is possible to perform simultaneously the FOD and SOD, i.e. determine the coordinate and weight improvements in one step. In other words, there is no sharp separation between the design stages – at least in the analytical solution – and the result of one stage may affect the other one.

The optimisation procedure is an iterative solution and after running each loop, the improvements are applied to the coordinates and weights. Should the quality of the network be met, the procedure stops, otherwise, it continues in another loop. Eventually, optimal positions for the network points as well as an optimal observation weight matrix are obtained. Observations with low weights can be excluded from the final observation plan.

4.4 Optimisation of GNSS Networks

Amongst different design orders, usually the FOD step is not performed in the design process of a network with GNSS measurements (Kuang, 1996, p. 260). The configuration of the GNSS network is defined by the geometry formed by the ground stations and the satellites. Since we cannot change the configuration of the satellite constellation, the concept of FOD in this topic is impractical. This is specifically the case in establishing a small- or medium-scale geodetic network, where moving the ground points (GNSS receivers) within some metres does not affect significantly the quality of the observations. Nevertheless, the optimisation procedure can proceed with seeking for an optimal solution for the SOD problem.

An optimal observation plan can be designed for a GNSS network, where the number and precision of baseline observations as well as optimal time-duration for operating receivers are determined. Due to skipping the FOD step in the optimisation of GNSS networks, the optimal positions of the network points would not be involved in the observation plan. Therefore, recalling Eq. (9), the terms expressing the derivatives of the VC or reliability matrix with respect to the coordinates are eliminated from the optimisation procedure.

To define a proper OF for an optimisation problem, the initial design matrix of observables and their approximate weights should first be determined. Let us define a baseline vector in a GNSS network by $\mathbf{\Delta}_i$ as a vector containing the coordinate differences between the two unknown network points j and k:

$$\mathbf{\Delta}_i = \mathbf{x}_j - \mathbf{x}_k, \quad i = 1,2,\ldots,n \tag{13}$$

where $\mathbf{x}_j$ and $\mathbf{x}_k$ represent the three-dimensional coordinate vectors of points j and k, respectively. Assuming that this network has m points, the total number of baselines will be $n = m(m-1)/2$, where only $m-1$ of the baselines are independent. Furthermore, from the least squares point of view, an observation equation for a GNSS network can be written as follows by considering Eq.(13):

$$\mathbf{\Delta} - \boldsymbol{\varepsilon} = \mathbf{A}\mathbf{x} \tag{14}$$

where $\mathbf{\Delta} = \begin{bmatrix} \mathbf{\Delta}_1 & \mathbf{\Delta}_2 & \cdots & \mathbf{\Delta}_n \end{bmatrix}^{\mathrm{T}}$, $\mathbf{A}$ is the design matrix that relates the observables to the unknown coordinates $(\mathbf{x})$ with +1, -1 and 0 coefficients, and $\boldsymbol{\varepsilon}$ is the vector of residuals.

To estimate the unknowns in Eq. (14), either of single- or multiple-baseline adjustment method can be used. The former considers a baseline by baseline computation and ignores the possible correlations between the baselines, while the latter takes the correlations into account by considering all of the network points at once (Hofmann-Wellenhof, et al., 2008, p. 258). The correlation assumption influences the adjustment procedure by the weight matrix. Therefore, in introducing a weight matrix to the optimisation of uncorrelated GNSS baselines, the off-diagonal elements of the matrix are considered as zero. On the contrary, a fully populated weight matrix must be defined with the correlation assumption.

Here we introduce an initial weight matrix to the optimisation procedure by ignoring possible correlations. This weight matrix is being updated during the process according to the defined criteria, and hopefully, it turns into an optimum weight matrix and provides us with the decisively needed baselines. In GNSS measured baselines, we have three components for each baseline, thus, three weights should be defined for each baseline. Mathematically, we form the total weight matrix as:

$$\mathbf{P} = \mathrm{diag}\left(\mathbf{p}_i\right), \quad i = 1,2,\ldots,n \tag{15}$$

where

$$\mathbf{p}_i = \sigma_0^2 \begin{bmatrix} \sigma_{\Delta x_i}^2 & 0 & 0 \\ 0 & \sigma_{\Delta y_i}^2 & 0 \\ 0 & 0 & \sigma_{\Delta z_i}^2 \end{bmatrix}^{-1} \tag{16}$$

and the precision of the baseline components are shown by $\sigma_{\Delta x_i}$, $\sigma_{\Delta y_i}$ and $\sigma_{\Delta z_i}$, and σ_0^2 is the a priori variance of unit weight, which is often assumed to be 1 in the design stage. Less precision in the up-direction of the GPS measurements in each baseline *i* can be tuned in Eq. (16) by considering

$$\sigma_{\Delta x_i}^2 = \sigma_{\Delta y_i}^2 < \sigma_{\Delta z_i}^2 . \tag{17}$$

Now, recalling the introduced OF in Eqs. (9) to (12), and based on the purpose of the network, a proper optimisation model can be defined to design an optimal GNSS network.

The described concept of GNSS network optimisation was used in implementing the ideas of the first, second, third and sixth paper of this thesis in real GNSS monitoring networks. The assumption of observing uncorrelated baselines was made in Papers I, II and VI, while the effect of mathematical correlations was explicitly investigated in Paper III.

4.5 GNSS-Baseline Correlations in Optimisation

Nowadays, it is very rare to use only two receivers per session of observations when performing static GNSS observation campaigns. Therefore, working with more than two receivers at a time obviously leads to a correlation between baselines. With correlation, the weight matrix cannot be assumed to be a diagonal matrix anymore, and the effect of the mathematical correlation should be considered.

As yet, the weight of baselines in a GNSS network was subject to optimisation because of the assumption of uncorrelated baselines. The baselines with low weights (zero or close to zero) could be removed from the optimal observation plan, and therefore, the output of the GNSS network optimisation was the superfluous baselines. Now, in this section, the case is investigated, where mathematical correlations are considered between GNSS baselines. Hence, the emphasis should be on optimising the observation sessions, and this is carried out by optimising the variance factor of each session to fulfil the pre-defined network requirements.

When the differential phase observations are used, the correlations should be taken into account in computing the VC matrix. This is a solution to introduce a realistic VC matrix in the optimisation procedure. To better explain the idea of this section, a simple network is assumed, where three receivers simultaneously collect data from four satellites. Totally, six double-difference observations can be composed in one epoch for two non-trivial baselines. The correlated weight matrix $\mathbf{P}$ of the observations can be numerically obtained for any session i as follows Hofmann-Wellenhof, et al. (2008, pp. 258-260):

$$\mathbf{P}_i = \sum\nolimits_{\nabla\Delta}^{-1} = \frac{1}{\mu_i}\begin{bmatrix} 4 & 2 & 2 & 2 & 1 & 1 \\ 2 & 4 & 2 & 1 & 2 & 1 \\ 2 & 2 & 4 & 1 & 1 & 2 \\ 2 & 1 & 1 & 4 & 2 & 2 \\ 1 & 2 & 1 & 2 & 4 & 2 \\ 1 & 1 & 2 & 2 & 2 & 4 \end{bmatrix}^{-1} \tag{18}$$

where $\sum_{\nabla\Delta}$ represents the VC matrix of the double-difference phase observables ($\nabla\Delta$), and μ expresses the variance factor of session i. As can be seen in Eq. (18), $\mathbf{P}_i$ is a fully populated matrix that indicates the existing correlations between the two baselines. Ignoring this phenomenon yields a diagonal matrix with zero off-diagonal elements.

To implement the correlated weight matrix in the optimisation procedure, it is required to derive the VC and reliability matrices with respect to the variance factor of each session, rather than the observation weights as in this case only μ is unknown. Rewriting Eq. (9) for a correlated GNSS network yields the following equations for precision and reliability:

$$\begin{cases} \mathbf{C}_{\mathrm{x}} = \mathbf{C}_{\mathrm{x}_0} + \sum_{i=1}^{n} \frac{\partial \mathbf{C}_{\mathrm{x}}}{\partial \mu_i} \Delta\mu_i \\ \mathbf{R} = \mathbf{R}^0 + \sum_{i=1}^{n} \frac{\partial \mathbf{R}}{\partial \mu_i} \Delta\mu_i \end{cases} \tag{19}$$

where all the parameters are as in Eq. (9) except μ_i (the variance factor of session *i*) and its correction $\Delta\mu_i$. In Paper III, these two criteria, i.e. precision and reliability, were used in defining an OF to perform a BOOM. Moreover, a full description of the mentioned methodology is available in Paper III of this thesis.

4.6 Deformation Parameters in Optimisation

One of the important applications for establishing geodetic networks is the monitoring of displacements or deformation networks. Displacement networks provide discrete information about the area at each network point, while the deformation networks contain continuous information about a deformable object. Deformation can occur in an object due to different external forces and cause changes in the shape, dimension, and position of the object. The role of time is imperative to take into account in deformation monitoring. Most of the changes in the appearance of an object are found as time elapsed.

Strain analyses can be used as a solution to obtain some information about the possible deformation of a deformable object. The strain information can be acquired by geodetic (e.g. establishing deformation monitoring networks) or non-geodetic (e.g. measurements from strainmeters) approaches. However, geodetic measurements are adequate for deformation monitoring, and non-geodetic approaches can be supportive in providing, for instance, additional geological information about the deformable object. The concept of strain in a geodetic network can be mathematically defined as the rate of changes in the displacement field $\mathbf{u}(x,y,z)=(u,v,w)^T$ of an object with respect to position $\mathbf{x}=(x,y,z)^T$, where u, v and w represent displacement components in the directions of x, y and z, respectively. A proper deformation model should be fitted to the displacement field determined at the discrete network point i as follows (Vaníček, et al., 1991):

$$\begin{cases} u_i = \varepsilon_x x_i + \varepsilon_{xy} y_i + \varepsilon_{xz} z_i + \omega_z y_i + \omega_y z_i \\ v_i = \varepsilon_{xy} x_i + \varepsilon_y y_i + \varepsilon_{yz} z_i - \omega_z x_i + \omega_x z_i \\ w_i = \varepsilon_{xz} x_i + \varepsilon_{yz} y_i + \varepsilon_z z_i - \omega_y x_i - \omega_x y_i \end{cases} \tag{20}$$

where ε_x, ε_y, ε_z, ε_{xy}, ε_{xz} and ε_{yz} are strain, and ω_x, ω_y and ω_z are differential rotation parameters. Equation (20) can be expressed in a compact matrix form as:

$$\mathbf{d}_{3i\times 1} = \mathbf{B}_{3i\times 9}\, \mathbf{e}_{9\times 1} \tag{21}$$

where $\mathbf{d}$ is the vector of displacements, $\mathbf{B}$ is a matrix consisting of coefficients (x, y or z) for deformation unknown parameters, and finally, $\mathbf{e}$ is a vector containing the deformation parameters (strain and differential rotations).

An optimal deformation monitoring network can be designed to fulfil the precision of the deformation parameters. For this purpose, a criterion matrix should be defined according to the required precision of the deformation parameters. Similar to Eq. (6), the defined criterion matrix should be approximated to the VC matrix of the deformation parameters ($\mathbf{C}_\mathbf{e}$). However, this is easily possible if the whole deformable object follows a similar deformation pattern. In other words, then it is assumed that all those nine deformation parameters are constant all over the object. Although this assumption is valid for small-scale objects, it may not be valid for large-scale objects. For instance, if a large deformable object is divided into a number of elements, not all of them necessarily have the same behaviour. Hence, fragmenting a large-scale object to smaller elements and analysing the deformation parameters individually in each element would be a good solution. Grafarend (1985) showed that the tetrahedron is the fundamental geodetic figure in order to determine the characteristic coefficients of the deformation equations, i.e. strain and differential rotation components in a three-dimensional space. Therefore, a deformable body can be fragmented to tetrahedral elements by implementing, for instance, the three-dimensional Delaunay triangulation technique. The Delaunay triangulation method provides non-overlapping and equilateral triangles/tetrahedrons with vertices at network points.

As described in Paper VI, we proposed a methodology to define a displacement criterion matrix for an optimisation procedure based on the precision of deformation parameters in each three-dimensional element. This leads to designing a monitoring network, where displacements at each network point is a function of the deformation behaviour of its corresponding elements. The precision OF can be defined as:

$$\left\| \mathbf{C}_{\mathbf{d}}^{c} - \mathbf{C}_{\mathbf{d}} \right\| = \min \tag{22}$$

with

$$\mathbf{C}_{\mathbf{d}} = 2\sigma_0^2 \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A} + \mathbf{H}\mathbf{H}^{\mathrm{T}} \right)^{-1} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{H} \right)^{-1} \mathbf{H}^{\mathrm{T}} \tag{23}$$

and

$$\mathbf{C}_{\mathbf{d}}^{c} = \mathbf{B}^{c}\,\mathbf{C}_{\mathbf{e}}\,\mathbf{B}^{c\mathrm{T}} \tag{24}$$

where the components of Eq. (23) are as Eq. (1). Equation (24) is obtained by applying the error propagation law to Eq. (21), where $\mathbf{C}_{\mathbf{e}}$ shows the VC matrix of the deformation parameters ($\mathbf{e}$) and $\mathbf{B}^{c}$ is the coefficient matrix containing a number of $\mathbf{B}$ matrices as given in Eq. (21) for each tetrahedral element (see Paper VI for more detailed information).

4.7 Results and Discussion

In this section, a summary of the research results are presented. Amongst the six papers that were collected in this thesis, four of them, namely Paper I, II, III and VI, are directly related to this chapter. Hence, the results of these four papers are discussed here and the remaining two papers will be reviewed in the next chapter.

Performing the SOD step to optimise and design a GNSS monitoring network is the common goal in these four papers. Neither the zero nor first order designs are required to be performed on GNSS monitoring networks. There is no need for a ZOD, as the problem in monitoring networks is not to define an optimum datum for the reference network, but stabilising it for measurements in subsequent epochs. Then the network will have the same position and orientation in all observation epochs. Moreover, a FOD determines the optimal position for network points, which is somehow impractical in static GNSS networks due to the insignificant effects of moving receivers for small distances in the optimal design of the network. However, the SOD step is carried out based on different criteria, developed in the papers to find the best observation plan for networks.

Paper I:

The core idea of this paper was to apply sensitivity in developing the precision criterion for optimisation of a GNSS monitoring network. All optimisation models, i.e. SOOM, BOOM, and MOOM, were developed considering this criterion, and were applied to a real application. The current GPS monitoring network of the Lilla Edet region was selected as a study area. Detailed information about the area and the network was already described in Section 3.1. As the area is subject to landslides, an optimal network should be designed to be sensitive to possible displacements. By defining the sensitivity of a network, we can guarantee the capability of the network in detecting and revealing displacements or deformations.

A single point statistical test was performed according to Eqs. (3) and (4) to find the maximum sensitivity of the Lilla Edet network. The value obtained was 5 mm. The displacement VC matrix in Eq. (3) was replaced by

$$\mathbf{C_d} = \mathbf{C_{x_1}} + \mathbf{C_{x_2}} \tag{25}$$

where no correlations between GNSS baselines ($\mathbf{x}_i = [\mathrm{x}_i \quad \mathrm{y}_i \quad \mathrm{z}_i]^{\mathrm{T}}$) are assumed. That is because only two receivers are considered to be simultaneously used in the network. Furthermore, we assumed a similar quality for observations in both epochs of measurements, thus $\mathbf{C}_{\mathbf{x}_1} = \mathbf{C}_{\mathbf{x}_2}$, and $\mathbf{C}_{\mathbf{d}} = 2\mathbf{C}_{\mathbf{x}}$. Then a precision criterion was developed to enable the network to recognise a displacement of 5 mm at each network point. A threshold of 0.7 was also considered for the reliability criterion.

To start with the optimisation procedure, it is required to form the current configuration of the network, matrix $\mathbf{A}$, and an initial weight matrix, $\mathbf{P}$. However, in the definition of the weight matrix for independent single baselines in GNSS, three elements are assigned for each baseline. Namely, considering n baseline observations, the dimension of the weight matrix is $3n \times 3n$. Since in optimisation of a GNSS network we cannot consider each component of the baseline independent from the other components, we have to define a weight matrix for each baseline rather than for each component. In this case, the baseline, including all its three elements, will either be removed or kept after optimisation.

Two different SOOMs were developed: a SOOM of Precision (denoted as P in Table 1) and a SOOM of reliability (denoted as R in the table), where it was constrained to precision. A BOOM of precision and reliability (PR in the table), and a MOOM of precision, reliability, and cost (PRC in the table) were also defined in this study.

Figure 4 illustrates the network before and after optimisation by the MOOM of precision, reliability and cost. The left panel shows the original GNSS network with 245 independent single baselines. The middle and right panels depict, respectively, the remaining and eliminated baselines in the optimal observation plan.

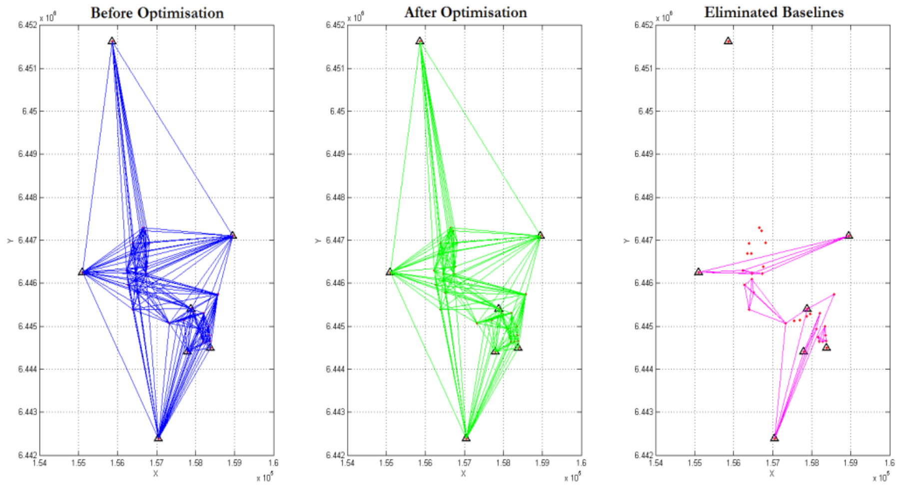

***Figure 4**. Optimised observation plan for the Lilla Edet GPS monitoring network (a MOOM of precision, reliability, and cost was applied).*

As can be seen in Table 1, the most economical optimisation solution for this network is the SOOM of precision, which can eliminate 19 percent of the baselines. And on the other hand, the worst model from an economical point of view is the SOOM of reliability, which keeps almost all the baselines. Nevertheless, referring to Figs. 5 and 6, one can have a different perception.

***Table 1**. Results of different optimisation techniques performed on the GPS network. Total number of baselines before optimisation is 245.*

	After optimisation			
	SOOM (P)	SOOM (R)	BOOM (PR)	MOOM (PRC)
Remaining baselines	199	240	206	203
Eliminated baselines	19%	2%	16%	17%

Figure 5 delineates the precision of network points before and after the optimisation procedure. The results from three different optimisation models, i.e. SOOM of precision, SOOM of reliability and BOOM of precision and reliability are compared in this figure. The uppermost graph shows the initial precision of the network and the red horizontal line introduces the precision criterion. The reliability of baseline observations is depicted in Fig. 6. Similarly, the red horizontal line in the figure expresses the pre-defined threshold for reliability. It should be mentioned that the graph colours are identically chosen for optimisation models in these two figures.

It is obvious from Figs. 5 and 6 that the SOOM of reliability yields the best result for this network. As mentioned, this model tries to keep all the observations in the plan in order to be able to provide the best possible precision and reliability for the network. Referring to Fig. 5, one can see that the model is more precise than our requirement, which was the detection of 5 mm possible displacements. Thus, if we come up with a model that is close to our required precision, it can be the desired one. Here, the SOOM of precision, BOOM, and MOOM models are acceptable for our purpose. The outcome of MOOM is not impressive due to the extra cost constraint of the MOOM. In the definition of this constraint, we considered the distances between receivers to place stress on the weight of that observation. Nevertheless, all the distances in this network are shorter than 10 kilometres, which actually cannot affect the result of the MOOM too much.

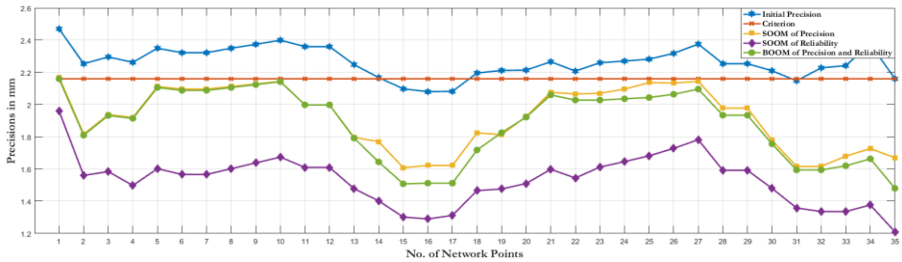

***Figure 5**. Precision of network points in detecting displacements by implementing different optimisation models.*

It can be seen in Fig. 5 that the SOOM of precision provides acceptable precision for the network, but at the same time it is clear in Fig. 6 that this model cannot preserve the reliability requirement of the network. As the model is not constrained to the reliability, it delivers an irregular behaviour that decreases the capability of this model in practical use. Unlike this SOOM model, the BOOM and MOOM are successful models in retaining the reliability of the network within the specified range.

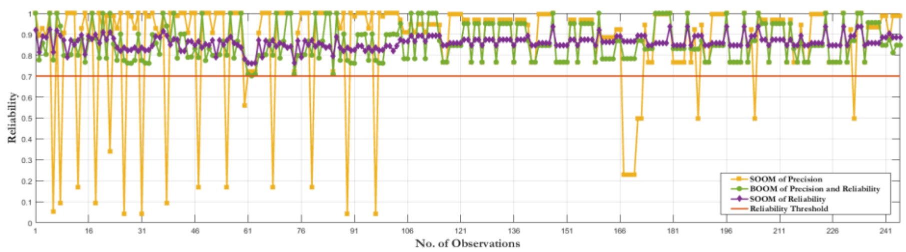

***Figure 6**. Reliability of network by implementing different optimisation models.*

Paper II:

The Lilla Edet monitoring network was used in the second paper as well. In this research, a case was investigated in the optimal design of geodetic networks, where more precise measuring instruments could be used in subsequent epochs. It was concluded in the previous paper that the SOOM of precision yields acceptable results in fulfilling the pre-set precision criterion. To cover its drawback in fulfilling the reliability criterion, a SOOM of precision was defined here, where it was bound to reliability. With this constraint, the developed model ensured the robustness of the network as well as its precision.

Since we wanted to redesign a network that should be able to detect displacements by more precise instruments, the definition of the precision criterion matrix had an important role. A two-epoch observation plan was considered with an assumption of uncorrelated GNSS baselines. However, the assumption of equal VC matrix of both epochs was infringed, $\mathbf{C}_{\mathbf{x}_1} \neq \mathbf{C}_{\mathbf{x}_2}$. Instead, the VC matrix of the second epoch was defined by multiplying the variances of the network points in the first epoch by a coefficient $k < 1$, implying a higher precision of the observations for the second epoch, $\mathbf{C}_{\mathbf{x}_2} = k\mathbf{C}_{\mathbf{x}_1}$.

The results of our test for several values of k in the criterion matrix are summarised in Table 2. The assumption of $k = 1$ delivered the optimised observation plan for both epochs while using $k < 1$, the optimised plan for the second epoch was obtained. It is obvious from the table that the larger the weight matrix in the second epoch becomes, the less number of baseline measurements are required.

Table 2*. The number of required observations in the first and second epochs after optimisation procedure according to precision improvements. The number of baselines before optimisation is 245.*

$k < 1$	$P_2 = \frac{1}{k} P_1$	No. of observations in	
		first epoch	second epoch
1	$P_2 = P_1$	215	215
0.9	$P_2 = 1.1\, P_1$	215	204
0.8	$P_2 = 1.2\, P_1$	215	193
0.7	$P_2 = 1.4\, P_1$	215	175
0.6	$P_2 = 1.7\, P_1$	215	154
0.5	$P_2 = 2\, P_1$	215	143

Figure 7 shows the optimisation results for $k = 0.5$, where we need 215 baselines to be observed in the first epoch and 143 baselines in the second one to be able to build a monitoring network, which is sensitive to detect a 5 mm displacement. It should be clarified here that it is improbable these days to double the weight matrix for the next epoch. It is obvious that with available precise measuring devices in the market, it is very rare to be able to increase the precision of the instruments this much within a time interval of some months. However, the case was investigated theoretically in this work to express the idea and also bring up the thoughts around this issue. Moreover, the effect of even small precision improvements (obtained by for instance using multi GNSS and longer observation time in GNSS networks) on the number of baselines is recognisable enough to consider this method as an efficient one.

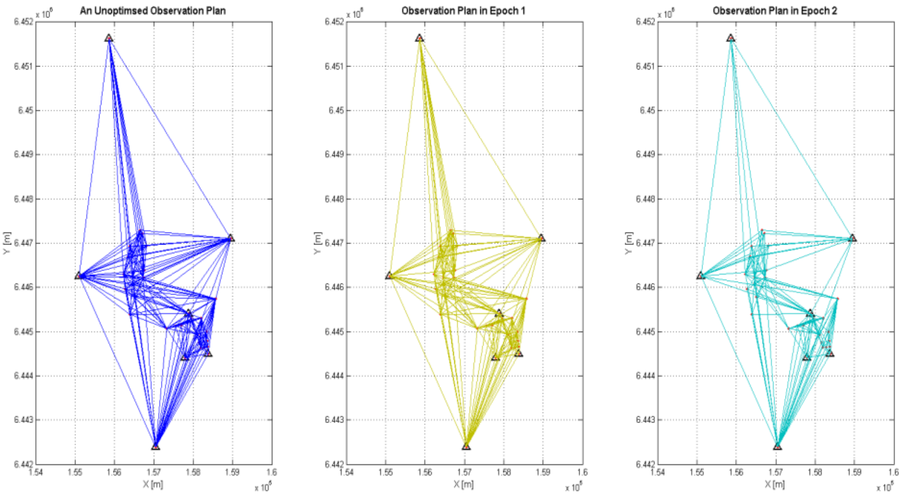

Figure 7*. Observation plan of Lilla Edet GPS monitoring network in two epochs by assuming instrument precision increase in the second epoch.*

Paper III:

As yet in our research, the correlations between GNSS baselines were ignored in the optimisation of geodetic networks. The assumption of ignoring correlations yields a diagonal observation weight matrix. Therefore, what we optimised in our previous two papers was the weight of observations. Based on the optimised values, we determined which single-baselines can be removed from the optimisation plan. In the third paper, a new idea was developed, where the effect of mathematical correlations was taken into account, and the methodology was developed further by considering the correlation between receivers and satellites. This yielded a fully populated weight matrix for the observations.

To test the idea in a real application, the GPS monitoring network of the Skåne region (see Section 3.2 for further information) was selected. This network has 7 stations, where we excluded the two internal stations (as in Fig. 8) from the network only to reduce the number of possible session combinations. According to the permutation formula, in a network with five points, one can separate ten cases of three points. It was assumed that we use three receivers simultaneously within this network.

The methodology of involving correlations in the optimisation of GNSS networks is explained briefly in Section 4.5 and with further details in Paper III.

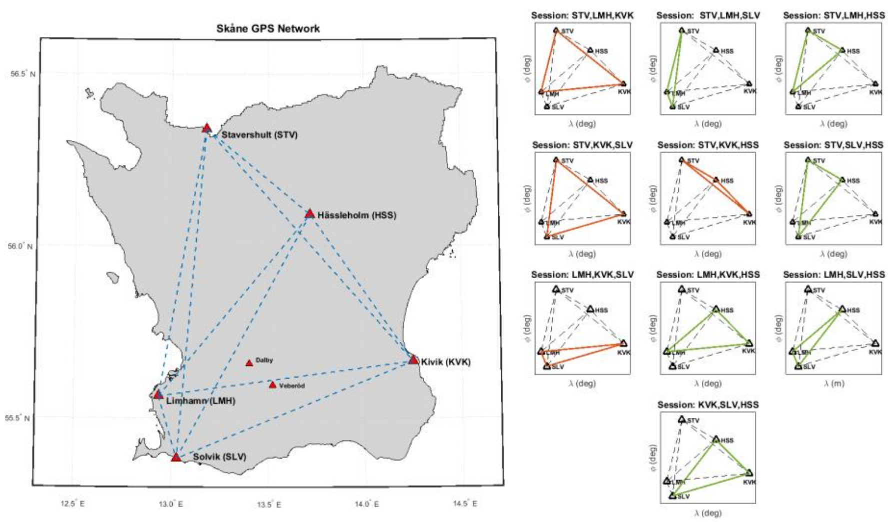

***Figure 8**. Session-wise optimisation of Skåne GPS monitoring network, when correlations between GPS baselines are considered.*

A BOOM of precision and reliability was used to optimise this network. The precision criterion was developed based on the chi-square statistical test to detect a 5 mm displacement of each point at a 95% confidence level, and by considering a fully populated weight matrix in computing the VC matrix. The reliability threshold was set to 0.7 as an empirically sufficient value for obtaining a robust network.

Concisely, we optimised the variance factor of each session in this study. As can be seen in Eq. (18), the variance factor is inversely related to the weight matrix of each session. Hence, the sessions with relatively large values of the variance factor were eliminated from the plan. Table 3 describes the names of the stations in the Skåne GPS network that made up each session and the optimised value of variance factors. The eliminated sessions, i.e. 1, 4, 5 and 7 in Table 3, are illustrated in red colours in Fig. 8. After optimisation, the number of necessary sessions decreased from ten to six, and the number of single baselines, from 20 to 12.

***Table 3**. GNSS sessions and their corresponding optimised variance factors.*

Session No.	**Observed baselines**	**Initial variance factor (μ^0)**	**Optimised value (μ)**	**Weight matrix ($\mathbf{P}$)**
1	STV-LMH, STV-KVK		356	≈ 0
2	STV-LMH, STV-SLV		1.34	$\mathbf{P}_{\text{STV, LMH, KVK}}$
3	STV-LMH, STV-HSS	⋮	1.83	$\mathbf{P}_{\text{STV, LMH, HSS}}$
4	STV-KVK, STV-SLV		338	≈ 0
5	STV-KVK, STV-HSS		355	≈ 0
6	STV-SLV, STV-HSS	1.00	1.39	$\mathbf{P}_{\text{STV, SLV, HSS}}$
7	LMH-KVK, LMH-SLV		306	≈ 0
8	LMH-KVK, LMH-HSS	⋮	1.36	$\mathbf{P}_{\text{LMH, KVK, HSS}}$
9	LMH-SLV, LMH-HSS		1.16	$\mathbf{P}_{\text{LMH, SLV, HSS}}$
10	KVK-SLV, KVK-HSS		1.15	$\mathbf{P}_{\text{KVK, SLV, HSS}}$

One more case was investigated, where the correlations were neglected, and therefore, the weight matrix in Eq. (18) turned into a diagonal matrix. Re-running the optimisation procedure yielded an observation plan with almost the same precision as the correlated case, but with two more retained sessions. It was concluded that as the methodology was developed based on the correlations between double-difference observations, it would be incorrect to ignore the correlation effect when optimising the network.

Paper VI:

The last research on the analytical optimisation of GNSS monitoring networks was performed on the Skåne monitoring network, and by considering deformation parameters. The methodology is explained in Section 4.6, where the precision of deformation parameters in each split element is used in definition of the displacement precision criterion.

As explained in Section 3.2 about the Skåne region, there is an expectation of deformations within the risky Tornquist zone. Therefore, designing a monitoring network by considering the precision and magnitude of the deformation parameters are of importance. Due to lack of geological information in our research, we used the previous measurements and data from this network to estimate the deformations (strains and differential rotations) between 1996 and 1998. This estimation was considered as the basis of our optimisation procedure. The VC matrix of deformation parameters ($\mathbf{C}_e$) in Eq. (24) is obtained as the variance of the estimated deformation parameters from the least squares adjustment. Additionally, a deformation primitive – dilation – can be derived from estimated strain parameters to analyse the average expansion/contraction (+/-) of a deformable object to obtain overall information on the deformation of the area.

It should be noted that the Skåne area was split into ten tetrahedral elements by using the three-dimensional Delaunay triangulation technique. Thereafter, the deformation parameters were estimated for each separated element. For instance, Fig. 9 illustrates one of the three-dimensional elements in the network.

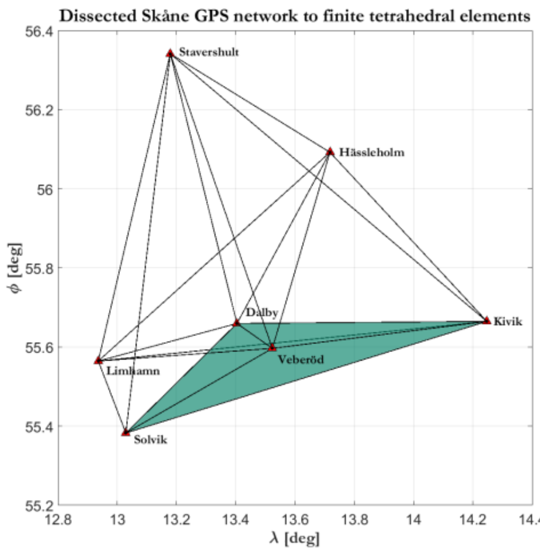

***Figure 9**. The Skåne area, which is discretised to ten subareas with the help of three-dimensional Delaunay triangulation technique.*

By introducing the variances of estimated deformation parameters to Eq. (24), the VC matrix of displacements was computed. This matrix indicated a displacement detection of 3 mm at each network point. Also, the reliability threshold was set to 0.7. Then the optimisation procedure was developed by applying three different optimisation models: SOOM of precision, SOOM of reliability and BOOM of precision and reliability. Almost the same conclusion as in Paper I can be derived from Table 4. The superiority of the SOOM of reliability was confirmed in this research as well due to its capability in retaining more observations. Moreover, the BOOM can be considered as the optimal solution because it can concurrently satisfy all of the pre-defined criteria and remove a reasonable number of unnecessary baselines.

Table 4*. Comparison between different optimization models. Total possible number of baselines is 21. P: Precision, R: Reliability, PR: Precision and Reliability.*

Objective Function	$f_{precision}$	$f_{reliability}$	**Eliminated Baselines**
SOOM (P)	8.34	1.16	9
SOOM (R)	8.53	1.95	1
BOOM (PR)	8.42	1.41	4

The dilation parameters shown in Table 5, show that the minimum deformation occurs in the elements IX and X. These two elements belong to the northern part of the Tornquist zone, and this is in agreement with the previous deformation analysis of this area by Pan, et al. (2001). They concluded that the southern part of the Tornquist zone is more exposed to deformation than the northern part. According to Table 5, the major deformations belong to the elements III and IV which are both located in the southern part.

Table 5*. Estimated dilation parameters for each tetrahedral element.*

Element No.	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**
Dilation	1e-7	1e-7	-9e-7	-6e-7	1e-7	-1e-7	-5e-7	2e-7	4e-8	2e-8

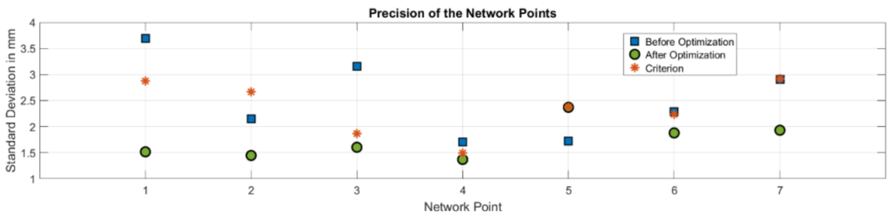

Figure 10*. The precision of network points before and after performing the bi-objective optimization model.*

It can be seen in Fig. 10 that the value of the precision criterion for each network point is different (shown by orange stars). This is because of splitting the area into a number of elements and considering unique deformation parameters for each of them. Therefore, the displacement precision of each network point is under the influence of the precision of deformation parameters of the adjacent tetrahedral elements.

The results in Table 5 are in compliance with Fig. 10. For instance, point number 4 has higher precision in Fig. 10 as it was used in constructing almost all of the tetrahedral elements that are shown in Table 5 as areas with maximum deformations. Thus performing precise measurements from this point is essential for fulfilling the precision of deformation parameters in those risky elements. Furthermore, point number 7 has received relatively low precision because this point was used in the forming of the elements IX and X and as mentioned these two elements have the minimum deformations.

5 Optimal Design for Total Station Establishment

5.1 Background

When measuring with a Total Station (TS), it must first be established by performing distance and angle measurements towards a number of Control Points (CPs) – points with known coordinates. The optimal horizontal and vertical distribution of the CPs is of importance to achieve a low establishment uncertainty. These issues will be discussed in greater length in this chapter. Moreover, an application of the TS establishment for height determination is described. This section starts by explaining a motivation for using the TS for height determination, then a number of studies on TS establishment are reviewed, and eventually, a motivation for combining the TS and GNSS measurements is provided.

Traditional Levelling

The traditional differential levelling method is usually used to determine heights precisely in geodetic networks. With this method, the height difference of two points is computed by measuring their vertical distances from a horizontal line of sight and subtracting the first distance from the second one (Kahmen & Faig, 1988, p. 321). In practice, a levelling instrument is needed to provide such line of sight and two levelling rods for reading the vertical distances. Typically, the instrument is set up at approximately equal distances from the two points to avoid possible systematic errors, e.g. collimation and refraction errors (Torge, 2001, p. 207). By using a benchmark (a point with known height) in a network, the height of other points can be obtained by transferring this known height to others by performing the differential levelling method and applying the measured height differences. To determine the height differences over longer distances, a number of temporary points and the transfer of heights through them may be required.

Precision of the levelling method decreases with the square root of the number of set-ups (Kahmen & Faig, 1988, p. 381). When transferring a known height from a benchmark located far from a geodetic network of interest many set-ups may be needed, thus this method may lead to lower precision. Furthermore, increasing the number of set-ups is laborious surveying work. Therefore, an alternative method to the levelling can be useful and advantageous in networks with low accessibility to benchmarks. Such method, which is developed by combining the Real-Time Kinematic (RTK) GNSS and TS measurements, will be introduced in Section 5.2.

TS Measurements

Today, terrestrial measurements by TS is considered as one of the main sources for collecting geodetic data. Before performing measurements with a TS, it is required to establish this instrument. This can be achieved by different approaches. For instance, a TS can be established by performing observations (i.e. distance, horizontal and vertical observations) from the TS towards minimum two CPs. This method is known as a free station or resection method and is associated with a simple geodetic network. A number of studies have been conducted to investigate the FOD problem (optimal configuration) in resection networks, where the optimal location of the TS is sought. Amiri-Simkooei, et al. (2012) investigated a two-dimensional simulated resection case by considering three horizontal angle observations. They distributed the CPs on a hypothetical circle. Their numerical results indicated that the best location of the unknown point, in sense of precision, is either in the centre of the circle or is a point in the largest distance from the circle in the simulated grid of points. The independency of their reliability criterion on the optimal location of the unknown point was also shown in their results. Another study was recently conducted by Song, et al. (2016), where the effect of the geometric configuration of a resection network on the precision of a TS establishment was analysed. They aimed to seek for an optimal configuration for a distance-only resection network, where all A- D- and E- optimality criteria (see Section 4.1) can be simultaneously achieved. They proposed a regular sector configuration, where the angle between any two adjacent distances is π/n, with n number of CPs. The optimal horizontal location of a TS in the establishment of a free station was investigated by Horemuž & Jansson (2016), who used both analytical and trial and error approaches in their study. The results from both approaches propose an optimal location for the TS in the centre of gravity of all CPs. They concluded that the uncertainty in orientation and reliability of observations has no effect on the optimal location of the TS. However, this location can be influenced by the geometry (number and distribution of the CPs) of a network.

RTK-GNSS/TS

In order to establish a TS, there is a need for a number of physically marked CPs with a direct view (line of sight) towards the TS. These conditions are not always available in surveying projects, because of for instance, obstacles in the field or an inadequate number of known points. Therefore, GNSS-aided solutions can be of interest here. However, amongst many practical advantages of the GNSS technique (e.g. observing long baselines, continuous measurements, its viability in all weather conditions, etc.); there are a few drawbacks that restrict its usage. For instance, the GNSS measurements cannot be carried out in areas with limited sky view or low satellite coverage. Furthermore, it is time-consuming in case of using static GNSS observations, which may not be economically beneficial in some surveying projects.

Using a recently developed method, where the GNSS and TS measurements are combined, it is possible to overcome the addressed limitations of each individual measuring technique. This combined method can also be used in a free station computation to determine position and orientation of a TS. The concept is similar to the traditional resection approach, where a TS is set over an unknown point, and the coordinates of that point are computed by measuring to a minimum of two known points (Kahmen & Faig, 1988, p. 252). However, in the combined method, the position of the TS is estimated by measuring a number of common points that do not have known coordinates on beforehand, but their coordinates are estimated using the GNSS measurements.

Brown, et al. (2007) performed free stationing by using the combined GNSS/TS to monitor deformations in an open-pit mine. Their CPs were distributed over the mine area and were equipped with GNSS receivers and prisms. Both the TS and CPs were located on unstable points. By post-processing the GNSS data, they achieved about 2 mm precision in estimation of the TS position. Therefore, it was concluded in their study that the combined method is efficient for monitoring purposes in the absence of stable locations for placing the TS and CPs.

Using the combination of RTK-GNSS/TS, one can determine the position of the TS immediately in the field by performing a free station. In this approach, the coordinates of the common points are obtained from the RTK measurements. RTK is a differential positioning technology that uses GNSS phase and code corrections from one or more reference stations. The achievable positioning accuracy with using the RTK method can be around one centimetre in the horizontal plane and 2 centimetres in the height (Lantmäteriet, 2015). The required base station for the RTK measurements can be a temporary or permanent reference station. A network RTK can be built up in an area by establishing several permanent reference stations and interconnecting them. Thus, a user needs only GNSS receivers as the rovers. For instance, in Sweden RTK GNSS can be performed based on the Swedish national network of permanent reference stations (SWEPOS), see for instance Jonsson, et al. (2003) or the SWEPOS website[2].

In order to reach the possible highest precision in establishing a TS with the help of RTK-GNSS/TS, the RUFRIS method, which is an abbreviation for the Swedish term *RealtidsUppdaterad FRI Station* (meaning real-time updated free station), was introduced. The essentials of the RUFRIS method were described and internationally published by Horemuž & Andersson (2011). In their research, the minimum number of common points to perform a resection was suggested as 10 to 30 points in order to obtain a reasonable precision and reliability, albeit measuring more common points would yield a higher precision. Moreover, they proposed to distribute the common points in a sector size of 180 degrees around the TS. For instance, an establishment precision of 3 mm can be obtained, if 30 common points are distributed in a 180 degrees sector. Finally, they recommended distributing the common points at different distances from the TS, if the topography of the area does not allow for a radial distribution of 180 degrees.

5.2 Real-time Updated Free Station in Levelling

Due to many advantages of the RUFRIS method in establishing a total station, it is a popular method amongst many surveyors in Sweden to set up their instruments. A number of these benefits are addressed in the previous section,

[2] https://swepos.lantmateriet.se

such as independency from physically marked CPs and immediate coordinate determination. Also, the provided horizontal precision by this method is acceptable for many surveying applications. However, the precision of the RUFRIS method in the vertical component is in question. In this section, we introduce the concept of the three-dimensional free station with RTK measurements, and in Section 5.4 implementation and testing in a real application is described.

To implement the RUFRIS method in a free station computation of the position and orientation of a TS, a number of common points should be measured in the field. Any of these points, which are equipped with a prism and a GNSS receiver, is measured simultaneously by the TS and RTK-GNSS. Therefore, we deal with two sources of observations, i.e. TS observations and RTK-GNSS observations. Total station observations are composed of distances as well as horizontal and vertical angle measurements and are expressed by the following equations (Kuang, 1996, pp. 85-92):

- Slope distances (S) between a TS and n common points (cp):

$$S_i - \varepsilon_{S_i} = \sqrt{\left(x_{cp_i} - x_{TS}\right)^2 + \left(y_{cp_i} - y_{TS}\right)^2 + \left(z_{cp_i} - z_{TS}\right)^2} \tag{26}$$

  where x, y and z represent the three-dimensional coordinates of a TS or a common point according to their sub-indices.

- Horizontal angle (ψ):

$$\psi_i - \varepsilon_{\psi_i} = \arctan\frac{x_{cp_i} - x_{TS}}{y_{cp_i} - y_{TS}} - \omega_i \tag{27}$$

  where the orientation of the TS is denoted by ω. This is an angle between the zero direction of the TS and the North axis of the coordinate system.

- Vertical angle (v):

$$v_i - \varepsilon_{v_i} = \arctan\frac{z_{cp_i} - z_{TS}}{\sqrt{\left(x_{cp_i} - x_{TS}\right)^2 + \left(y_{cp_i} - y_{TS}\right)^2}}\,. \tag{28}$$

  In Eqs. (26) to (28), the sub-index i is used as a counter for the number of observations between the TS and the i-th common point, and $i = 1, 2, \ldots, n$.

It should be mentioned that Eqs. (26) to (28) must be linearised to be used in a least squares adjustment. Further details on the linearised forms are available in Paper IV of this thesis.

The second set of equations belongs to the RTK-GNSS measurements. The determined coordinates of each common point by the RTK measurement (rtk) can be written as:

$$\begin{aligned} x_{rtk} - \varepsilon_x &= x_{cp} \\ y_{rtk} - \varepsilon_y &= y_{cp} \\ z_{rtk} - \varepsilon_z &= z_{cp} \end{aligned} \tag{29}$$

assuming that the GNSS coordinates are transformed into a cartographic coordinate system combined with a height system. In Eqs. (26) to (29), ε represents the residuals, i.e. differences between adjusted and measured values.

The least squares solution for the system of equations in Eqs. (26) to (29) can be written for n common points as:

$$\mathbf{X} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{L} \tag{30}$$

where $\mathbf{X}$ is the vector of unknowns containing the three-dimensional coordinates and orientation of the TS, and the coordinates of all measured common points. The design matrix $\mathbf{A}$ is constructed by coefficients of the linearised equations and Eq. (29). Weights of the observations are collected in the $\mathbf{P}$ matrix by considering the technical specifications of the TS and uncertainty of the RTK measurements. Eventually, the vector $\mathbf{L}$ contains the observations.

5.3 Optimal Vertical Placement of Total Station

An optimal configuration of a free station network is needed to obtain a reasonably high precision in establishment of a TS. An overview of previous work on optimal horizontal distribution of the CPs was provided in Section 5.1. Now, in this section the optimal vertical distribution of the CPs are discussed by considering an analytical solution.

Before writing the resection formulas, let us introduce an xyz coordinate system for TS measurements, and an external ENH coordinate system. In spite of identical z and H axes, these two systems differ from each other by a rotation angle around the z/H axis and a translation vector. A new coordinate system

$E'N'H'$ can be defined by paralleling the axes of the xyz system to the ENH system. In other words, the $E'N'H'$ system is identical to ENH, but with different origo.

We aim for an optimal location of the TS in relation to a number of CPs. A precision optimality criterion can be defined such that the positional and rotational uncertainties of a TS establishment is minimised. Following the general resection formulas, linearising, and eventually rearranging them in a matrix form, the relation between the coordinates of a CP and a TS can be expressed as:

$$\underbrace{\begin{pmatrix} E_{CP_i} - E'_{CP_i} \\ N_{CP_i} - N'_{CP_i} \\ H_{CP_i} - z_{CP_i} \end{pmatrix}}_{\mathbf{L}} - \boldsymbol{\varepsilon} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & x_{CP_i}\cos\omega_0 - y_{CP_i}\sin\omega_0 \\ 0 & 1 & 0 & -x_{CP_i}\sin\omega_0 - y_{CP_i}\cos\omega_0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} E_{TS} \\ N_{TS} \\ H_{TS} \\ \Delta\omega \end{pmatrix}}_{\mathbf{X}} \tag{31}$$

where

- E_{TS}, N_{TS} and H_{TS}: coordinates of the TS in the ENH system.
- E_{CP_i}, N_{CP_i} and H_{CP_i}: coordinates of the i-th CP in the ENH system.
- E'_{CP_i} and N'_{CP_i}: coordinates of the i-th CP in the $E'N'H'$ system.
- x_{CP_i}, y_{CP_i} and z_{CP_i}: coordinates of the i-th CP in the xyz system.
- ω_0 and $\Delta\omega$: approximate rotation angle and its correction.
- $\mathbf{L}$, $\boldsymbol{\varepsilon}$, $\mathbf{A}$ and $\mathbf{X}$: observation vector, residual vector, design matrix, and unknown vectors, respectively.

All steps that leads to Eq. (31) are described in detail in Paper V of this thesis. The covariance matrix of the observations ($\mathbf{Q}_\mathbf{L}$) can be written as:

$$\mathbf{Q}_\mathbf{L} = \mathrm{E}\left\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^\mathrm{T}\right\} = \mathbf{Q}_{ENH} + \mathbf{Q}_{E'N'H'} \tag{32}$$

where $\mathbf{Q}_{ENH}$ is the covariance matrix of the coordinates of CPs in the ENH system, and $\mathbf{Q}_{E'N'H'}$ is the covariance matrix of the coordinates of the CPs determined by TS in the $E'N'H'$ system. The operator of expectation is shown as $\mathrm{E}\{\bullet\}$.

If equal uncertainties are assumed for the coordinates of CPs measured by TS in both the ENH and the $E'N'H'$ systems, and also independent distance and angle measurements are considered, Eq. (32) can be simplified to:

$$\mathbf{Q_L} = \mathbf{P}^{-1} = \sigma_0^2\, \mathbf{I}_{n\times n} \tag{33}$$

where $\mathbf{P}$ is a weight matrix and $\sigma_0^2\, \mathbf{I}_{n\times n}$ implies a diagonal n by n matrix with equal diagonal elements. n is the number of CPs. The above-mentioned assumptions are considered here to simplify the problem for obtaining a closed-form solution. Moreover, the coordinates of CPs are usually determined by precise geodetic measurements, thus the coordinate uncertainties as well as correlations between coordinates are very small and reasonable to be ignored.

The VC matrix of the unknowns in Eq. (31) is now obtained by

$$\mathbf{Q}_{\mathrm{x}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1} \tag{34}$$

where it contains the positional variances on its first three diagonal elements, and the fourth element belongs to the variance of the estimated rotation angle. Following up the optimality criterion, the trace of coordinates and orientation variances should be a minimum, i.e. $\mathrm{tr}\left(\mathbf{Q}_{\mathrm{x}}\right) \to \min$. In order to fulfil this condition, the first derivations of it with respect to E_{TS}, N_{TS} and H_{TS} should equal zero. The solution yields:

$$\begin{cases} E_{TS} = \sum_{i=1}^{n} E_{CP_i} \Big/ n \\ N_{TS} = \sum_{i=1}^{n} N_{CP_i} \Big/ n \\ H_{TS} = c \end{cases} \tag{35}$$

with $c \in \mathbb{R}$.

It can be seen in Eq. (35) that the optimal horizontal location for the TS – considering the minimum trace of the variance – is in the centre of gravity of all CPs, which was already mentioned by Horemuž & Jansson (2016). However, this conclusion is here enhanced by considering also the vertical component in the optimisation problem. Recalling Eq. (35), it is noted that the height of a TS has no effect on the optimal location of the establishment. In other words, moving the TS, or distributing CPs, at different heights does not affect the result.

5.4 Results and Discussion

This section presents the main results achieved in Papers IV and V. The methodology behind the numerical computations was briefly explained in Section 5.2.

Paper IV:

The main goal of this paper was to investigate the efficiency of the RUFRIS method in height determination. In order to verify the estimated height component of each point, we compared them with corresponding heights obtained from the traditional levelling method. The raw data for this research were made available from the East Link project in Sweden (further details about the project is available in Section 3.3).

To estimate the three-dimensional coordinates of each marked ground point by a free station computation, a program package was developed in the MATLAB scripting language. The program was able to read the polar measurement file for each point provided by the surveying instrument, i.e. TS, and estimate the coordinates and orientation of it as well as the uncertainties of the elements. The measurement at each point was performed by considering minimum 15 common points according to the RUFRIS recommendations to ensure achieving a reasonable precision on the coordinate estimation. Moreover, the coordinates of these common points were obtained by using the RTK-GNSS measurements as supported by the SWEPOS network.

We would rely more on our results, if we could compare it to another solution for estimating the coordinates. Therefore, a surveying engineering software, SBG Geo (SBG, 2015), was used. The results from both computations were to a great extent in compliance with each other.

In a parallel surveying measurement, a traditional double-run levelling had been performed along the project corridor to obtain the height of points. The collected data were adjusted in the SBG Geo software to estimate the heights of the points. Statistics of the differences between heights determined by the RUFRIS and levelling methods are depicted in Table 6 and Fig. 11.

Two geoid models were used in this research. We started the height determination procedure by using SWEN08_RH2000, the national Swedish geoid model. For more information on Swedish reference systems, see for instance Ågren (2009). Then, a tailored geoid model, which is developed by the National Surveying and Mapping Authority of Sweden, was used. The new model, SWEN08_OSTL, is similar to the national model, but with improvements along the East Link project. Therefore, the expected uncertainty of the new model is reduced from about 10-15 mm in the national model to about 3-5 mm (Ågren & Ohlsson, 2016). The effect of this improvement can be clearly seen in Fig. 11 in which the schematic illustration of residuals by considering these two models is provided (left panel: results from SWEN08_RH2000, and right panel: results from SWEN08_OSTL).

It can be seen in the left panel of the figure that the residuals are following a systematic trend from the northern point towards the south, i.e. small positive residuals in the north, increasing values in the middle, and finally turning to negative values in the southern part of the area. Numerically inspecting the mean values of residuals in Table 6, an offset of 6 mm can be seen in case of using the national geoid model implying the existence of a systematic error. Seemingly, this problem is vanished by applying the newly developed geoid model. Furthermore, the standard deviation of the residuals as well as the standard deviation of the mean value is provided in this table. Using the national geoid model in computations yielded a standard deviation of 11 mm according to Table 6. However, this value can be improved if the new geoid model is taken into account. A standard deviation of 7 mm is then obtained. A smaller value of the standard deviation of the mean when using the new geoid model indicates a more precise model for our computations.

The reason for the remaining residuals in Fig. 11 can be due to both uncertainties of the developed geoid model, the RUFRIS method, and the levelling. Assuming an uncertainty of 5 mm for the geoid model and maximum 1 mm for the levelling, and applying the error propagation law, the uncertainty of the RUFRIS method itself can be estimated to be about 5 mm in determination of the heights.

Table 6*. Statistical analysis of differences between RUFRIS and levelling heights. Units are in millimetres.*

	SWEN08 RH2000	SWEN08 OSTL
Mean Value	6	0

Standard Deviation	11	7
Standard Deviation of the Mean	1.14	0.73

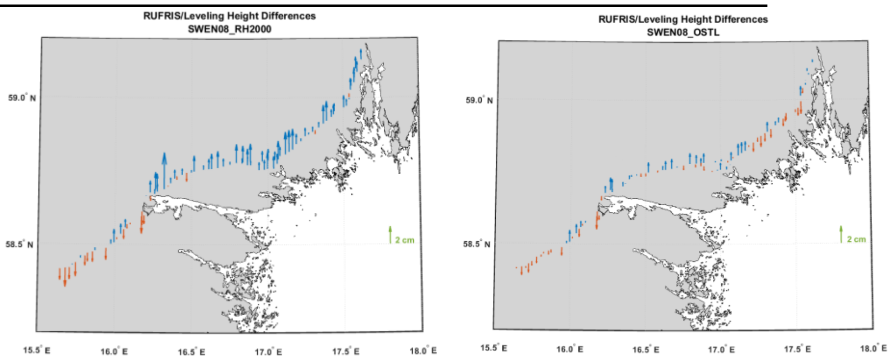

Figure 11*. Differences in height estimation of stations by using RUFRIS and levelling. The used geoid models are SWEN08_RH2000 in the left plot and SWEN08_OSTL in the right plot.*

Paper V:

Despite the fact that many terrestrial surveying projects are conducted in the horizontal plane, there are some applications (e.g. mining or dam monitoring projects or surveying in areas with large variations in topography) that induce us to delve into the vertical component in TS measurements.

In this paper, we specifically investigated the effect of height in optimal free station establishment of a TS. Previously, in Section 5.3, it was analytically proved that distribution of CPs in different height layers has no effect on the optimal location of the TS. However, in that solution, an assumption of equal uncertainties of TS observations was used for simplicity purposes. Therefore, this optimisation problem was also investigated by a trial and error method on completion, where all observations (distance, horizontal and vertical angles) had reasonable uncertainties. In fact, Paper V solved the problem by performing two different approaches.

The trial and error method was implemented using simulated data. A TS was assumed, where it could move in a grid of 100×100 metres with an interval of 10 metres between set-up points. On each location, the TS measured four CPs distributed symmetrically in one of four height layers. The height layers were zero, 10, 30 and 60 metres above the TS. Furthermore, this test was repeated with fifteen random CPs to comply with RUFRIS recommendations.

The numerical results from the trial and error method are in agreement with the ones from the analytical solution. The height component has in practice an insignificant role in the optimal location of a TS in free station establishment. This can be concluded by referring to the results illustrated in Fig. 12. In fact, the presented results here show the microscopic behaviour of TS establishment uncertainties, when the CPs are distributed at different heights. The horizontal uncertainties shown by error ellipses in the first column of Fig. 12 confirm the optimal horizontal location of the TS to be in the middle of CPs. On the contrary, a very small change of the vertical uncertainties is seen in the second column of the figure. However, at the zero layer, when the vertical angle is zero, only the distance and precision of the vertical angle affect the uncertainty of the height. That is why the middle points have smaller uncertainty than the corner points as they are closer to all four CPs than the other set-up points. As we increase the height of the CPs, the vertical angles and the uncertainty of the distance measurements start to influence the height uncertainty. Therefore, the set-up points around the CPs provide less precise locations for the TS in the higher layers.

The figure shows also that the least height uncertainty belongs to the zero-height layer, where the TS and CPs are placed at the same height – suggesting an optimum location for the TS. However, moving the CPs with respect to TS from the zero-height layer to the topmost (60-meter) layer has an influence of only a few micrometres which is negligible in practice compared with the uncertainty of the TS measurements. Hence, it can be concluded that the free station establishment of a TS can be optimally performed with respect to CPs located in any reasonable heights.

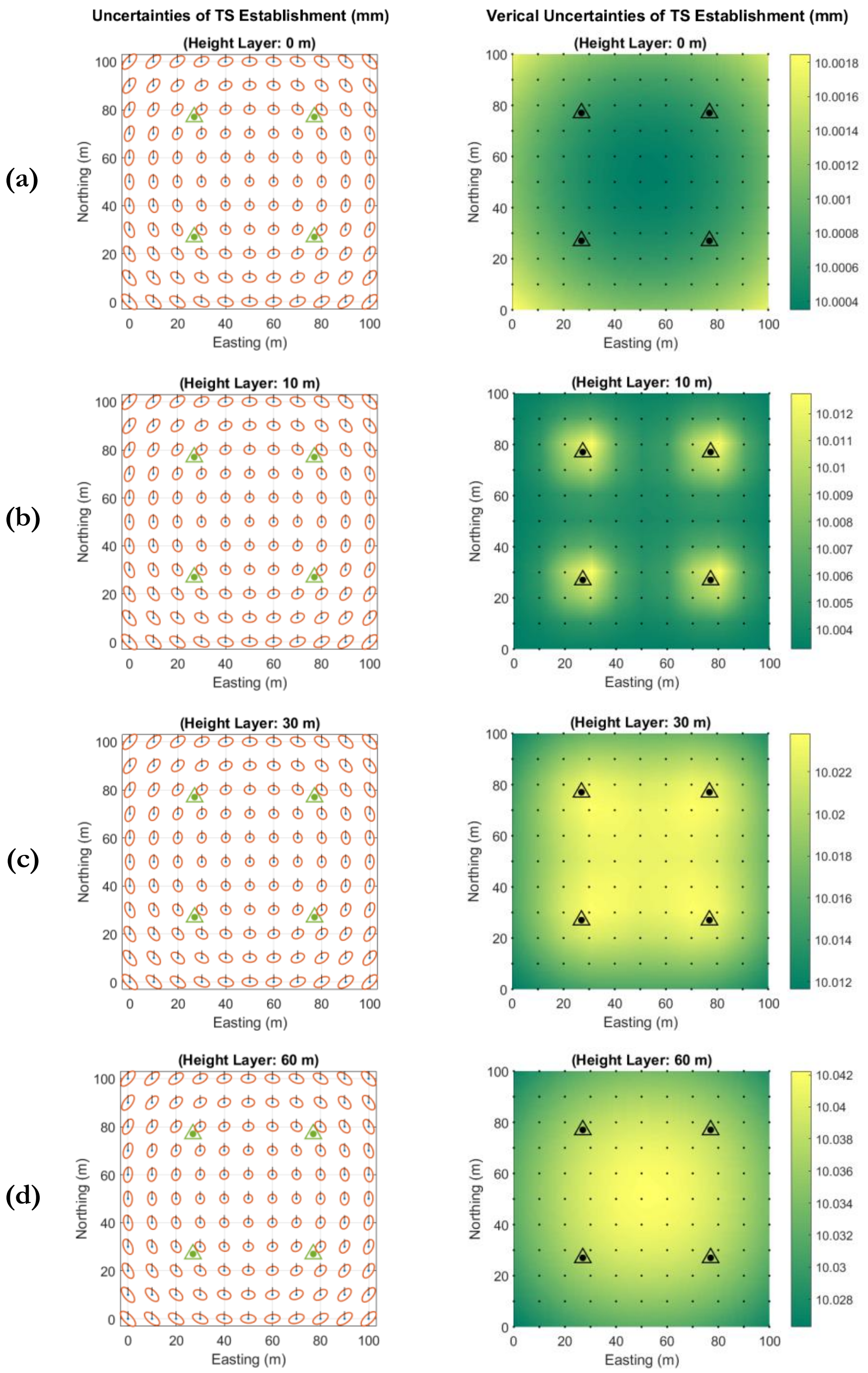

***Figure 12**. Horizontal and vertical standard uncertainties of TS establishment by measuring 4 CPs, where the CPs are distributed in different height levels.*

6 Conclusion and Future work

6.1 Conclusion

This thesis is written on the basis of six peer-reviewed papers. The core idea of the thesis is to design optimal geodetic networks. Four out of the six papers focus on design of optimal GNSS monitoring networks, while the remaining two papers discuss the optimal design and efficiency of TS establishment with RTK-GNSS.

Designing a network with possible maximum precision and reliability that has minimum expenses will be ideal. However, the execution of such network is perhaps not feasible in reality due to inconsistencies between the desired criteria. An optimisation procedure is performed in a geodetic network to overcome the possible inconsistencies, satisfy all the defined criteria (i.e. precision, reliability, and cost), and eventually propose an optimal design.

A short summary and the main outcomes of each paper is provided here:

The first paper concerns the efficiency of different optimisation models in fulfilling the demanded quality of networks. To investigate the models, the GNSS monitoring network of a village in the Southwest of Sweden – Lilla Edet – was opted. We developed and implemented different optimisation models by combining precision, reliability and cost criteria. It is of importance to mention that the precision criterion was built upon the sensitivity requirement of the network. The numerical results confirmed the superiority of the model, where all criteria were involved, over other models that had only single criterion or two criteria. However, it was concluded that each of the single criterion models (precision and reliability) could fulfil the requested requirements of the network, but were not actually beneficial from an economical point of view.

The effect of instrumental precision in designing a monitoring network was investigated in the second paper. The same study area was used here. Considering epoch-wise observations for a monitoring network, it could be assumed that more precise instruments would be used in subsequent epochs. Therefore, we applied incremental improvements to the assumed precision of the GNSS observations in the second epoch to see the possible changes of the observation plan in this epoch. It was shown that the increase of instrument/observation precision yielded a plan with fewer observations, of course, if the quality requirements remained unchanged.

An investigation into the effect of correlations between GNSS baselines was added to the optimisation procedure in the third paper. In the previous two papers, single baselines were subject to optimisation because of the assumption of only two operating receivers in the field. However, by increasing the number of receivers and performing the double-difference observations, sessions of observations, where each session contains a number of correlated baselines, should be considered. A methodology was developed in this study to optimise the variance factor of the sessions instead of baseline weights. The methodology was tested on the Skåne GPS network in southern Sweden by implementing a BOOM of precision and reliability.

A feasibility study was conducted in the fourth paper to analyse the precision of the RUFRIS method in determination of height. The method involves free station computation of a TS developed on the basis of RTK-GNSS measurements for determining coordinates of the common points. The major advantage of this method is its independency from physically marked known points. A RUFRIS dataset was available from the East Link project in Sweden. Data from traditional levelling were also used in parallel to determine the height of the points where RUFRIS was tested. Comparing the results obtained by each method showed that RUFRIS is a good choice for height determination when an uncertainty of about 7 mm is required. However, this value is obtained due to a tailored geoid model for the test area. Achieving an uncertainty of about 11 mm is reasonable if the national geoid model is considered.

An optimal configuration – FOD – for a free station network was sought in the fifth paper. This was including an optimal location for a TS both in the horizontal and vertical planes. However, the focus of this research was on the vertical component. Two different methods were used to solve the optimisation problem: the analytical method, assuming equal uncertainties for all observations, and the trial and error method, applying realistic uncertainties for the observations. The results from both methods are in agreement with each other. The optimal horizontal location of the TS is in the centre of gravity of all control points, while its optimal vertical location does not depend on height.

The precision of deformation parameters, i.e. strains and differential rotations, was used in the sixth paper to define a precision criterion matrix. In order to carefully inspect the deformation behaviour of an area, it is possible to split the area into a number of two- or three-dimensional elements and investigate the deformation of each element individually. This idea was developed in the study, and tested on the Skåne GPS monitoring network due to the deformation history of the region. A displacement VC matrix was defined as a criterion based on the precision of the deformation parameters of each element. This criterion enabled the network to be sensitive to displacements of minimum 3 mm at each network point.

6.2 Recommendations for use in practice

- The superiority of the MOOM is shown in this study as it could fulfil all precision, reliability and cost criteria. However, the BOOM of precision and reliability can be considered as efficient as the MOOM; especially because it could remove more observations. In case that only the precision criterion of a network is of interest, we recommend using the SOOM of precision, which is constrained to reliability.
- For monitoring networks in which the measurements are repeated epoch wisely; if the precision of observations could be enhanced in subsequent epochs, this would affect the design of the observation plan for those epochs.
- Operating more than two GNSS receivers yields between-baseline correlations. In optimisation of such networks, the mathematical correlation can be taken into account through a fully populated weight matrix. Hence, the sessions of observations are optimised, instead of the single baselines.
- Areas that are subject to deformation can be split into smaller elements for a more careful inspection. To design a monitoring network over the area, the precision of the deformation parameters of each element can be used to define an optimisation criterion for detecting displacements.
- We tested two of the developed methodologies of this thesis on the Skåne GPS monitoring network (active in 1989-1998 to monitor the displacements in the Tornquist zone) and were successful to design an

optimal network. However, no funding has been provided to follow up and densify these promising results with further GPS campaigns after 1998, despite a moderately strong earthquake that affected the southern part of this region in 2008. Hopefully, our results could bring back the interests to this network.

- The efficiency of RUFRIS in height determination is verified in levelling projects with low accessibility to benchmarks. The role of a precise geoid model should also be addressed here as it has a large influence on the uncertainty of the RUFRIS method. However, the method is suggested as an alternative method to traditional levelling, when requirements for uncertainty are in line with results as provided in this thesis.
- An optimal horizontal location of a TS with respect to CPs in its free station establishment is the centre of gravity of the CPs (Horemuž & Jansson, 2016). Even the vertical location of the TS or CPs is of importance in its optimal establishment. Our study shows that the distribution of CPs in different heights or moving the TS in the vertical plane has a negligible effect on the uncertainties of an establishment. Therefore, the TS can be optimally set up in free stationing with respect to CPs located in any reasonable heights.

6.3 Future Work

During this study, we have had an opportunity to delve into optimisation methodology and conceive some new ideas of developing it. There are, however, a number of concerns that still remain untouched and hopefully will be accomplished in the future. For instance, the correlation effect on optimisation could have been investigated in a larger context with assuming many simultaneously operating GNSS receivers, which is more realistic in larger networks nowadays. Performing a THOD in GNSS networks and combining them with newly developed technologies, i.e. dInSAR measurements, is another idea that could be followed in the future. Moreover, implementing and testing the new generation of analytical optimisation methods, such as particle swarm optimisation or genetic algorithms, is of great interest. Generally, developing a program package equipped with a graphical user interface for performing all required design steps, with an option of choosing between different optimisation methods, for any input network is the desire of the author.

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Other Publications and Contributions during Ph.D. Studies

Peer-reviewed Papers:

Eshagh, M. & Alizadeh-Khameneh, M. A., 2015. The Effect of Constraints on Bi-Objective Optimisation of Geodetic Networks. *Acta Geodaetica et Geophysica,* 50(4), pp. 449-459.

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Non-peer-reviewed Paper:

Alizadeh-Khameneh, M. A., Sjöberg, L. E. & Jensen, A. B. O., 2016. *Optimization of GNSS Deformation Monitoring Networks by Considering Baseline Correlations.* Christchurch, New Zealand, 2-6 May, FIG Working Week 2016.

Oral Presentations:

Alizadeh-Khameneh, M. A*, Korrelationer mellan baslinjer vid design och optimering av geodetiska GNSS-nät*, Kartdagarna 2016, 26-28 April, Gävle, Sweden, [in Swedish].

Alizadeh-Khameneh, M. A*, Optimization of GNSS Deformation Monitoring Networks by Considering Baseline Correlations*, FIG Working Week 2016, 2-6 May, Christchurch, New Zealand.

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Poster Presentations:

Alizadeh-Khameneh, M. A, Eshagh, M. & Sjöberg, L. E., *Optimization of Lilla Edet Landslide GPS Monitoring Network*, Nordic Geodetic Commission (NKG) General Assembly 2014, 1-4 September, Gothenburg, Sweden.

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Contributions made by the Ph.D. student to the six papers of this thesis

Paper I: The Ph.D. student developed the methodology in cooperation with other authors. He received data from the Lilla Edet municipality and implemented the methodology using the dataset. He discussed the results with other authors and eventually wrote the article considering their comments.

Paper II: The initial idea was provided by the second author. The Ph.D. student formulated the idea and implemented it using the same dataset as the Paper I. The numerical computations were also performed by him. Finally, he wrote the article and then improved it by considering the other authors' comments.

Paper III: The original idea was provided by the second author. The Ph.D. student developed the idea and tested it on a dataset available at KTH. The numerical results were obtained by him and were discussed with all authors. The text was composed by the student based on the comments from other authors.

Paper IV: The initial idea was provided by the fourth author in compliance with conducting a surveying project. The numerical computations were performed by the Ph.D. student on a dataset available at WSP. The results were discussed with the other authors, and the final paper was prepared by the student considering the other authors' comments.

Paper V: The idea was suggested by the third author, and developed further by the Ph.D. student. The numerical results were obtained by the student and were discussed with other authors. The article was written by the student and improved according to the comments from the other authors.

Paper VI: The idea was originally provided by the second author and developed in discussions with all authors. The numerical computations were performed by the Ph.D. student. The article was also written by him considering the other authors' comments.