ON OPTIMISATION AND DESIGN OF GEODETIC NETWORKS

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ABSTRACT

Optimisation of a geodetic network is performed to provide its pre-set quality requirements. Today, this procedure is almost run with the aid of developed analytical approaches, where the human intervention in the process cycle is limited to defining the criteria. The existing complication of optimisation problem was terminated by classifying it into several stages. By performing these steps, we aim to design a network with the best datum, configuration and the observation weights, which meets the precision, reliability and cost criteria.

In this thesis, which is a compilation of four papers in scientific journals, we investigate the optimisation problem by developing some new methods in simulated and real applications.

On the first attempt, the impact of different constraints in using a bi-objective optimisation model is investigated in a simulated network. It is particularly prevalent among surveyors to encounter inconsistencies between the controlling constraints, such as precision, reliability and cost. To overcome this issue in optimisation, one can develop bi-objective or multi-objective models, where more criteria are considered in the object function. We found out that despite restricting the bi-objective model with precision and reliability constraints in this study, there is no significant difference in results compared to the unconstrained model. Nevertheless, the constrained models have strict controls on the precision of net points and observation reliabilities.

The importance of optimisation techniques in optimal design of displacement monitoring networks leads to the development of a new idea, where all the observations of two epochs are considered in the optimisation procedure. Traditionally, an observation plan is designed for a displacement network and repeated for the second epoch. In the alternative method, by using the Gauss-Helmert method, the variances of all observations are estimated instead of their weights to perform the optimisation. This method delivers two observation plans for the two epochs and provides the same displacement precision as the former approach, while it totally removes more observations from the plan.

To optimise a displacement monitoring network by considering a sensitivity criterion as a main factor in defining the capacity of a network in detecting displacements, a real case study is chosen. A GPS displacement monitoring network is established in the Lilla Edet municipality in the southwest of Sweden to investigate possible landslides. We optimised the existing monitoring network by considering all quality criteria, i.e. precision, reliability and cost to enable the network for detecting 5 mm displacement at the net points. The different optimisation models are performed on the network by assuming single baseline observations in each measurement session. A decrease of 17% in the number of observed baselines is yielded by the multi-objective model. The observation plan with fewer baselines saves cost, time and effort on the project, while it provides the demanded quality requirements.

The Lilla Edet monitoring network is also used to investigate the idea, where we assume more precise instruments in the second of two sequential epochs. In this study, we use a single-objective model of precision, and constrained it to reliability. The precision criterion is defined such that it provides the sensitivity of the network in detecting displacements and has a better variance-covariance matrix than at the first epoch. As the observations are GPS baselines, we assumed longer observation time in the second epoch to obtain higher precision. The results show that improving the observation precision in the second epoch yields an observation plan with less number of baselines in that epoch. In other words, separate observation plans with different configurations are designed for the monitoring network, considering better observation precision for the latter epoch.

SAMMANFATTNING

Optimering av ett geodetiskt nät genomförs för att uppfylla ställda kvalitetskrav. Idag görs detta mer eller mindre med hjälp av en utvecklad analytisk metod, där den mänskliga inblandningen i beräkningen är begränsad till att definiera de optimala kriterierna. Optimeringsproblem klassificeras i flera steg, som förhoppningsvis leder till ett optimalt nät vad avser datum, konfiguration, observationers precision och tillförlitlighet samt kostnader.

I detta avhandlingsarbete, som är en sammanställning av fyra artiklar i vetenskapliga tidskrifter, utvecklar vi några nya metoder för nätoptimering i simulerade och verkliga applikationer. Det är välkänt att ställda bivillkor i optimeringsprocessen ofta leder till motstridiga lösningar, och detta problem kan ofta lösas med en bi-eller multi-objektiv modell, där mer än ett kriterium beaktas i objektfunktionen. I det första försöket undersöks olika begränsningar/restriktioner i en bi-objektiv optimeringsmodell med hjälp av simulerade nät. I lösningarna för dessa designproblem fann vi inga påtagliga skillnader i försöken med eller utan bivillkor för precision och tillförlitlighet. Eftersom precisions- och tillförlitlighets-villkoren redan finns med i objektfunktionen, har de ingen betydelse som tilläggsvillkor.

Deformationsnät designas vanligen en gång för alla före mätprojektets början, och denna observationsplan upprepas sedan vid varje mät-epok. Här har vi utvecklat en ny idé att låta alla observationerna från två epoker ingå i optimeringsproceduren, vilket kan minska antalet nödvändiga observationer i den senare epoken.

I Lilla Edets kommun i Västra Götaland har övervakningsnät designats och mätts med GPS-teknik av kommunen anlitad konsult. Området är väl känt för jordskred vid Göta älv. För att optimera ett deformationsnät bör ett kriterium för nätets *känslighet* för deformationer beaktas. Med utgångspunkt från det befintliga nätets konfiguration genomförde vi en optimal design med beaktande av villkoren för precision, tillförlitlighet och kostnad för detektering av rörelser om minst 5 mm mellan nätets punkter. Mätta koordinatskillnader mellan samtliga punkter (baslinjer) betraktades som oberoende. Bland olika lösningar visade sig en multi-objektiv lösning ge 17 % reduktion i antalet ursprungliga observationer, vilket medger arbets- och kostnadsbesparingar med bibehållna kvalitetskrav.

Vi har också använt Lilla Edets övervakningsnät för att undersöka idén, där vi antar att mer precisa instrument finns tillgängliga i den andra av två sekventiella epoker. I studien har vi endast använt en objekt-modell för precision, och begränsas till tillförlitlighet. Precisions-kriteriet definieras så att det ger önskad känslighet för detektering av deformationer och nätet har bättre varians-kovariansmatris än vid den första epoken. Eftersom observationerna är GPS baslinjer, antog vi längre observationstid i den andra epoken för att få högre precision. Denna observationsplan resulterade i en observationsplan med färre antal baslinjer i den senare epoken med bibehållen känslighet för deformationer. Med andra ord har vi utformat separata observationsplaner för nätverksövervakning i de båda epokerna med tanke på bättre observations- precision i den senare epoken.

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Paper II:

Eshagh, M. & Alizadeh-Khameneh, M. A., 2015. Two-Epoch Optimal Design of Displacement Monitoring Networks. *Boletim de Ciências Geodésicas*. Accepted.

Paper III:

Alizadeh-Khameneh, M. A., Eshagh, M. & Sjöberg, L. E., 2015. Optimisation of Lilla Edet Landslide GPS Monitoring Network. *Journal of Geodetic Science.* Accepted.

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LIST OF ABBREVIATIONS

ZOD	Zero Order Design
FOD	First Order design
SOD	Second Order Design
THOD	Third Order Design
OF	Object Function
SOOM	Single-Objective Optimisation Method
BOOM	Bi-Objective Optimisation Method
MOOM	Multi-Objective Optimisation Method
VC	Variance-Covariance
GPS	Global Positioning System
GNSS	Global Navigations System
SWEREF	Swedish Reference Frame
ITRS	International Terrestrial Reference System

1 INTRODUCTION

The main goal of geodetic network optimisation is to determine the optimal observing plan that fulfils all required criteria such as precision, reliability and sensitivity. Since the cost of the project is of interest in managing level, the optimisation procedure is performed to remove unnecessary observations, which do not affect the quality criteria of the network, to minimise the project cost.

In order to design an optimal geodetic network, Grafarend (1974) proposed a 4-order solution to the optimisation problem. Zero-Order Design (ZOD) to seek an optimum datum for the network; First-Order Design (FOD) to find an optimum configuration to the network by choosing the best possible positions for the net points; Second-Order Design (SOD) to deal with observations weight and choose the optimal one, and THird-Order Design (THOD) to improve an existing network by establishing a new net point and performing new observations.

Based on our intention of designing an optimal network, we try to minimise or maximise one of the network quality criteria as an Object Function (OF) within an optimisation procedure. On occasion, it is needed to consider only one criterion in the OF, namely form a Single-Objective Optimisation Model (SOOM) subject to a number of constraints to place more stress on that criterion. To consider more criteria simultaneously in one OF, one can use a Bi-Objective Optimisation Model (BOOM) with two criteria or a Multi-Objective Optimisation Model (MOOM) with all criteria (Xu 1989).

Geodetic networks are designed for different purposes, but one important application of such networks is to monitor various deformation processes, such as large man-made structures, crustal movements and landslides. The epoch-wise monitoring procedure is usually performed to decrease the risk of probable deformation consequences. For instance, the Vasa warship in Stockholm has been monitored by epoch-wise measurements to detect possible deformations and shape changes since the year 2000 (Horemuž 2003).

To design an optimum monitoring network, it is of interest to acquire some a priori information about the magnitude and rate of displacements and/or deformations. Displacements are important for monitoring networks as the deformation parameters are estimated from them, if the monitored object can be considered as a continuum medium. However, not all objects are of continuum nature, and in such a case displacements in different parts of the object are considered instead of deformation parameters. So far, geodetic networks have been designed based on the precision and/or reliability of the network for one epoch of observations. In contrast, in displacement monitoring networks the goal is to design a network sensitive to detect the displacements or the coordinate differences between two or more epochs of observations. The Sensitivity of a monitoring network is an important criterion, which is usually considered in the optimisation of deformation networks in addition to the accuracy and reliability criteria. It expresses the capacity of the network in detecting and revealing the possible displacement or deformation of the network.

1.1 RESEARCH OBJECTIVES

Today, designing the geodetic networks are scrutinised from quality control and economical points of view to utilise the designed plan in performing the flawless surveying. Although previously the networks were designed by the surveyors own experience and intuition, nowadays, engineers use developed analytical methods to do so.

The objective of this thesis work is to develop some new methods in optimal design of geodetic networks based on the existing concepts of analytical optimisation solutions. The outcomes will be optimised networks that fulfil all defined quality criteria with less project costs.

Investigating the impact of optimisation techniques in designing an optimum displacement monitoring network is another goal to achieve in this research. It is aimed to implement the developed methods in the optimisation of a displacement monitoring network in a real application to present the feasibility and capability of the derived approaches in delivering an optimum observation plan for epoch-wise measurements.

1.2 THESIS STRUCTURE

The licentiate thesis is written in 6 chapters. Starting from Introduction, where we briefly introduce the optimisation subject in a geodetic network, we explain the importance of utilizing this technique in surveying projects. In the literature review chapter, a number of outstanding and remarkable works that have been carried out in the field of optimisation are mentioned. Also, some of the new approaches that have been used for optimising geodetic networks will be indicated. The study area chapter illustrates the chosen area for performing the optimisation technique and testing the new ideas. In Chapter 4, the methodology of the thesis work is explained in three sections to outline the basic concept of network quality criteria, optimisation of geodetic network and optimisation of monitoring networks. Thereafter, in Chapter 5, our results obtained in the four attached papers are explained. Chapter 6 concludes the whole work and at the end, we explain our future intentions in a few words. Finally, the four accepted or submitted papers are attached in the Appendix.

1.3 AUTHOR’S CONTRIBUTIONS

In the first paper, the roles of the constraints in the BOOM of precision and reliability was investigated. The idea was performed on a simulated network, and it was numerically shown that the unconstrained BOOM can be as efficient as the constrained ones because of using both precision and reliability criteria in the OF.

In the second paper, two optimisation methods were developed based on two different adjustment models to perform a simulated monitoring network. The first method was derived from the Gauss-Markov adjustment model and it optimised the observation weights of one epoch measurement. The Gauss-Helmert model helped us to develop the second method, where we could design the observation plans for two epochs at the same time. It was numerically concluded that the second method acted better from an economic point of view.

In the third and fourth papers, we implemented the optimisation techniques in a real application The existing GPS landslide monitoring network of the Lilla Edet municipality was the subject of our study. The idea in the third paper was to investigate different optimisation models for finding out the best model to redesign the network. The study was ended by the superiority of the MOOM in fulfilling the demanded quality criteria. The optimisation procedure was programmed to solve the problem analytically and redesign the network based on the initial data from Lilla Edet.

In the last paper, another idea was tested on the network over Lilla Edet, where we assumed precision improvements in the second epoch of observations. The improvements were applied incrementally to the optimisation procedure and it was observed that the possibility of using more precise instruments can decrease the number of baseline observations in the subsequent epochs.

2 LITERATURE REVIEW

Generally, an optimal geodetic network is a network with high precision and reliability, which fulfils the economic considerations. Using the method of Kuang (1991) and (1996), one can obtain optimal weights and configuration of the network in one step by different optimisation algorithms and OFs. In this method, the best configuration and observation precisions are designed simultaneously in an optimal way.

A precision criterion matrix is a representative of an ideal Variance-Covariance (VC) matrix for a geodetic network, and since it represents an ideal situation, there is no need for the network VC matrix to be equal to the criterion (Koch 1985). The criterion matrix is introduced to the optimisation procedure to push the current precision of the network towards the desired or required ones. This matrix can either be defined based on theoretical concepts such as Taylor-Karman structured VC matrix introduced by Grafarend (1972), or according to empirical studies on real applications (Cross 1985).

Furthermore, an optimal network should have the capability to detect gross errors in the observations and minimise the effect of the undetected ones on the adjustment results (Fan 2010). Baarda (1968) proposed a global test for outlier detection and data snooping for the localisation of gross errors and introduced the concept of reliability. The reliability criterion is widely used as a requirement in designing optimum networks. As an example, Amiri-Simkooei (2004) optimally designed a geodetic network at the SOD stage to meet maximum reliability. In his work, the weights of the observations were improved so that the redundancy numbers for all observations became the same. In another study, the effect of less reliable observations on a deformation monitoring network of a dam was inspected by Amiri-Simkooei (2001). To confront the probable distortions in the network due to weak observations from a reliability point of view, he came up with a solution to decrease their weights in the SOD stage to reach a reasonable range of reliability.

The cost of a project is another criterion to be considered in optimising a geodetic network. Using the Global Navigation Satellite System (GNSS) measurements in establishing a monitoring network, one can reduce the cost by considering a transportation distance between net points and the length of observation times. Generally, the monitoring projects are carried out either using continuous GNSS stations (e.g. Naito et al. 1998) or by establishing the GNSS receivers temporarily at pre-set net points. Dare and Saleh (2000) performed an epoch-wise survey consisting of many observation sessions in which the Global Positioning System (GPS) receivers were moved between the net points. In their work, the instrument shifts were addressed as more costly than the observation time of each session. The difficulty of finding an optimal solution for large networks induced Dare and Saleh (2000) to use a simulated annealing solution based on a heuristic approach to define the session schedule.

Two different categories for network design are mentioned by Cross (1985), namely computer simulation and analytical design. The analytical method is a solution to a design problem by using a unique series of mathematical steps, while the computer solution requires human intervention to alter repeatedly the originally postulated solution for the problem until a satisfactory network is found. Cross (1985) also defined linear and quadratic programming approaches as sub-categories for the analytical method. The concept of quadratic programming is formerly explained by Lemke (1962) by a numerical example. He described a method of minimising the quadratic functional of variables, which are constrained by a number of linear inequalities.

However, considering multiple criteria in one OF is used widely for different purposes. The multi-objective optimisation technique was the tool that Xu and Grafarend (1995) took the benefit from and designed an optimal deforming network, and Bagherbandi et al. (2009) implemented the single- and multi-objective optimisation models in a simulated geodetic network and concluded that the SOOM of reliability is the best model in providing high reliability and precision. A simple comparison between different SOOMs has been carried out in Eshagh and Kiamehr (2007). This comparison shows that reliability is a much better criterion than the other criteria in SOOMs due to its ability in providing highly reliable observations in addition to the precise net points. The capability of the BOOM versus SOOM was presented in Eshagh (2005) in which the possible inconsistency between constraints in the SOOM is eliminated by the bi-objective model.

Amiri-Simkooei et al. (2012) presented some basic concepts related to the optimisation and design of geodetic networks. Amongst the different solution methods for the FOD problem, the work by Berné and Baselga (2004) can be mentioned in which they achieved to a centimetre level accuracy in designing the best geodetic network by simulated annealing method. Alzubaidy et al. (2012) discussed the problems of the FOD and SOD in a micro-geodetic network. Kuang (1992) proposed an approach to solve the SOD problem, where an optimal solution for weights is obtained by the best approximation of a defined criterion matrix. Grafarend (1975) and Schmitt (1980, 1985) presented different approaches to the SOD, where the observable weights and types are determined. Kuang (1993) presented another approach to the SOD leading to maximum reliability using linear programming (see e.g. Bazaraa 1974, Smith et al. 1983). To fulfil the postulated precision of a GPS network, Mehrabi and Voosoghi (2014) performed an analytical method to find a solution for the SOD problem in their network. They figured out that by using optimisation techniques it is possible to cut down the number of observed baselines by 36% and save the measurement cost, but at the same time achieve the defined precision.

Since a few decades ago, GNSS measurements are commonly used in geodetic monitoring networks. Such networks are established to investigate and detect possible deformations that can take place on the Earth due to crustal movements, landslides, etc., which in some cases threaten human life. Through many studies that have been carried out in this field, we can mention a work by Sjöberg et al. (2004), where they studied the possibility of using the GPS technology in detecting very small crustal deformations at the nuclear power plant station of the Äspö region in the southeast of Sweden. The GPS landslide monitoring network of the Maçka County in the northeast of Turkey is also optimised by Teke et al. (2008) considering a method, where the direct approximation of the inverse criterion matrix is used. They succeeded in designing a monitoring network that provides a homogeneous and isotropic network.

It is of importance to design and perform an optimal monitoring network in order to achieve high precision and reliability with low cost, as well as a high sensitivity in detecting deformations (see e.g. Schmitt 1982, Gerasimenko et al. 2000, and Shestakov et al. 2005). The FOD problem is solved analytically by Blewitt (2000) to find the best station locations in a fault monitoring network that optimises the precision of geophysical parameters. In his method, the optimum configuration is yielded by maximising the determinant of the design matrix. Even-Tzur (2002) designed a GPS vector configuration to monitor a deformation network based on the effective contribution of each baseline in providing the sensitivity of the network. Two different methods for presenting the precision criterion are compared in the optimisation of a simulated deformation monitoring network by Doma (2014). He performed the SOD in the network by implementing the precision criterion in the form of scalar functions, or criterion matrices. Furthermore, he concluded that, although there are no significant differences in the precision results, one can consider the correlations among the deformation parameters by using the initial VC matrix of them as the criterion.

A key requirement in designing an efficient network in detecting possible deformations is to collect prior information and data about the physics and geology of the deformable object. The importance of data evaluation for estimating the precise deformation models is studied by Setan and Singh (2001). They performed trend analysis of the net points in a displacement field to acquire its deformation model.

Recently, modern metaheuristic algorithms have been used in optimising geodetic monitoring networks. Well-known examples are genetic algorithms, ant colonies and particle swarm optimisation. A kind of nature-inspired method called the shuffled frog leaping algorithm was used by Yetkin and Inal (2015) to design an optimal deformation monitoring network. They used this method to find the optimal reference point positions such that the reliability of the network becomes a maximum. They found this algorithm easier to perform than traditional methods as it does not need either linearisation or differentiation of the OF. In another study, Doma (2013) used the particle swarm optimisation method to optimise a similar simulated GPS network as Kuang (1996, p. 338), concluding that his method is more efficient in optimising the network due to the elimination of more baselines by fulfilling the precision criterion.

3 STUDY AREA

It is of great interest to experiment the developed methodologies on a real case study. Despite the efficiency of simulated networks in delivering acceptable results, we prefer to confirm the ideas of Papers III and IV in reality. For this purpose, the Lilla Edet village, located in the Västra Götaland County, Sweden, is chosen where the area is well-known for its landslides and subductions. According to the previous studies, there are numerous areas with high risk of landslides along the Göta River in the south-west of Sweden, mostly between Lilla Edet and Trollhättan, and the risks will increase with climate changes. Annually several landslides of different sizes occur along the river, and the area belongs to those with the most frequent landslides in the country. Due to many residential areas within this municipality, the study about the landslides is of high interest for the community.

Since the year 2000, the risky area has been settled under monitoring controls in different time intervals (epochs). Based on the size of the area and purpose of the survey, GPS measurements were chosen to create a geodetic network for this region. The existing network, which was established by a consultant for the municipality, has 35 stations, where 6 stations are assumed as fixed points around the village, and the rest are set up inside the landslide area. Totally, 245 independent baselines are observed epoch-wise in this network by neglecting the correlation influence amongst the GPS receivers. The net points are projected to SWEREFF 99 12 00 coordinate system, which is a realisation of the International Terrestrial Reference System (ITRS) at epoch 1989.0, and the height reference system is RH 2000. The purpose of the network is to monitor the landslide displacements within the area. The consultant has measured the points in 8 epochs in different time intervals from 2000 to 2013. During these years, a fixed structure of the observation plan has been followed to monitor the movements.

The study area and the established monitoring stations as well as risky landslide areas are illustrated in Fig. 1. All data for this study area and ancillary information are provided by the municipality of Lilla Edet (Nordqvist, 2012) or downloaded from the Geodata[1] database portal and geo database of Swedish University of Agricultural Sciences[2].

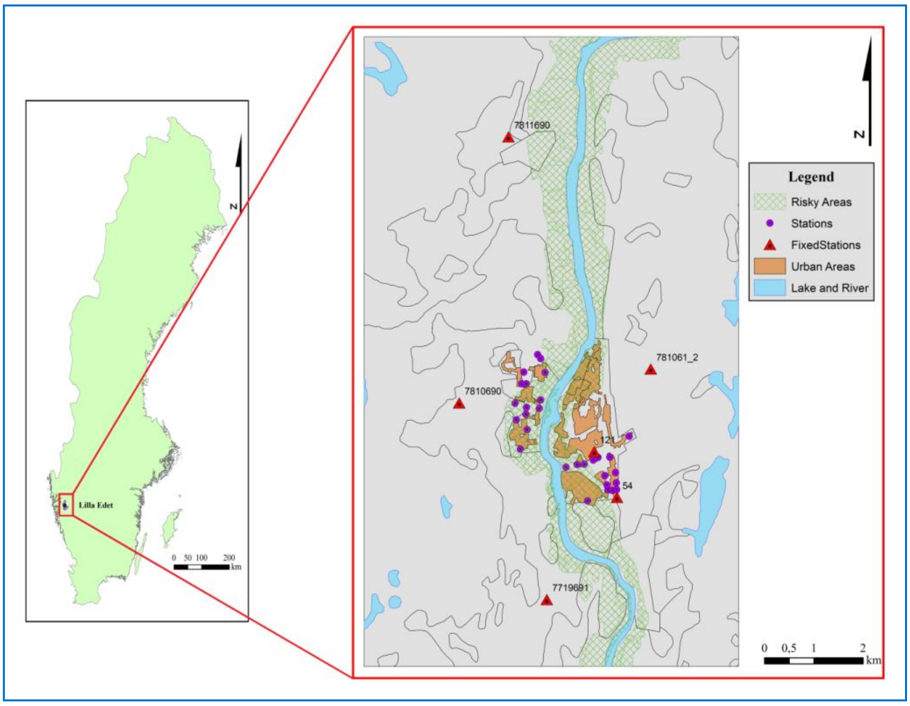

Figure 1 – Study area and the established GPS stations for monitoring purpose. It is also overtly clear in the figure that some residential areas are located within the risky zone.

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[1] https://www.geodata.se/GeodataExplorer/index.jsp?loc=en
[2] https://maps.slu.se/get/

4 METHODOLOGY

It is a big concern for survey engineers to perform a project in an optimal way, where they can fulfil many pre-set and pre-defined criteria by a client. Practically, before starting any measurement campaign, engineers should come up with a designed plan. The cost of the project, the location of net points, the type of observations (e.g. distances, angles, etc.) as well as their quality and thereby the type of measuring instruments (e.g. Total station, GNSS receivers, etc.) are part of the issues that should be decided beforehand. The missing link to connect and bring all these requirements together in a network is an optimisation technique. By the help of this technique, one can design a geodetic network with optimum configuration and observation plan.

In this chapter, first the quality criteria that should be fulfilled in an optimal network are explained, and then we mention the optimisation steps and methods. Finally, the optimisation procedure on a monitoring deformation network is discussed.

4.1 NETWORK QUALITY CRITERIA

Before starting to design an optimal geodetic network, it is needed to describe the quality criteria of a network that should be fulfilled through an optimisation procedure. The quality of a network is specifically presented by precision, reliability and economy as well as sensitivity in case of deformation networks. Based on the purpose of a surveying network, different criteria can be chosen to implement in the optimisation procedure. In other words, a network can be designed to fulfil either one or any combination of these criteria at the same time, in spite of possible inconsistencies between them.

Precision Criterion

In order to design an optimal geodetic network, the precision criterion is the most demanded factor in this process. Rationally, it is of great importance for surveyors to design or perform precise enough observations in a network. The precision of net points in a network is affected by the observational precision and the network geometry. Generally, the VC matrix of a network coordinates, $\mathbf{C_x}$, is the best form of representing the network precision, where its diagonal elements are the variances of the coordinate components, and the off-diagonal elements show the covariances amongst them. As the VC matrix is a datum dependant, so by assuming the minimum constraint datum for a network, it can be written as (Kuang 1996, p. 221):

$$\mathbf{C_x} = \sigma_0^2 \left[\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA} + \mathbf{DD}^{\mathrm{T}} \right)^{-1} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{DD}^{\mathrm{T}}\mathbf{H} \right)^{-1} \mathbf{H}^{\mathrm{T}} \right] \tag{1}$$

where σ_0^2 is the a priori variance factor, which is usually set to 1 at the design stage. The design and weight matrices of observations are represented by **A** and **P**, respectively. **D** and **H** are the matrices with the minimum and inner constraint datum information for the network, respectively. In this thesis, the defined network precision in the first two papers is the same as Eq. (1), while in the last two papers, the precision is defined by the inner constraint datum.

In case of working on large networks, where it is not practically feasible to go through all precision elements in the VC matrix separately, a scalar precision function can be considered as a reasonable alternative. Some widespread examples are the norm, trace, determinant, etc. of the VC matrix of the net points, which can be applied according to the user requirements.

Reliability Criterion

In addition to the precision criterion, an optimal network is supposed to be reliable enough in order to detect gross errors and minimise the effects of undetected ones on the net point coordinates. Baarda (1968) introduced the concept of reliability to perform a quality control for geodetic networks. He used a statistical hypothesis to test if the outliers are detectable or not. The reliability matrix, **R**, has the following structure (Kuang 1996, p. 122):

$$\mathbf{R} = \mathbf{I}_n - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A} + \mathbf{D}\mathbf{D}^{\mathrm{T}}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P} \tag{2}$$

where $\mathbf{I}_n$ is an $n \times n$ identity matrix with n observations, and the other parameters are the same as introduced in Eq. (1).

The i-th diagonal element of the reliability matrix is called the redundancy number of the i-th observation. The redundancy number r_i will be in the range between 0 and 1. The network has no capability in detecting gross errors, if the redundancy numbers become zero, while they are all detectable in case of $r_i = 1$, $(i = 1,2,\ldots, n)$. The ideal situation from a reliability point of view is the latter case, where $\mathbf{R} = \mathbf{I}$, which means that the residual of an observable is not affected by the error of other observables (Amiri-Simkooei 2012). It can be seen in Eq. (2) that the reliability is dependant on the configuration and weight matrices of the observations, so it should be considered in a network design problem as a criterion to guarantee the detection of gross errors.

Cost Criterion

The third criterion that ensures the optimality of a network is the minimum cost of a project. Based on the type of a project, different factors are subjected to cost. For example, the cost of a GNSS monitoring network is related to transportation, the observation length, labour and equipment costs, etc. Practically, it is difficult to come up with a solution that provides the highest precision and reliability as well as the lowest cost for a project. The more precise and reliable is the network, the more expensive is the project. Theoretically, fewer observations, fewer observation iterations, or observations with lower accuracy result in lower project expenses. However, it can be accepted that a network with reasonable precision and reliability is economically an optimum one (Teunissen 1985).

Sensitivity Criterion

Dealing with deformation or displacement monitoring networks, the sensitivity criterion can be introduced in addition to the mentioned ones. The sensitivity of a network describes the ability of the network in detecting the possible displacements or deformations. Therefore, we need to implement the sensitivity criterion within the optimisation procedure to enable the network to detect the specified magnitude of deformation. It is of importance to investigate whether an assumed displacement is detectable in the network or not. For this purpose, we follow the statistical model presented by Kuang (1996, p. 302) as:

$$\lambda_j = \hat{\mathbf{d}}_j^T \mathbf{C}_{\hat{\mathrm{d}}_j}^{-1} \hat{\mathbf{d}}_j \sim \chi_{1-\alpha}^2 \left(df\right) \tag{3}$$

where λ_j denotes a test variable, $\hat{\mathbf{d}}_j$ is the adjusted displacement vector at net point *j*, $\mathbf{C}_{\hat{\mathbf{d}}_j}$ represents the corresponding displacement VC matrix, and $\chi_{1-\alpha}^2 \left(df\right)$ is the chi-square distribution with the significance level of 1-α, where typically α = 0.05 and *df* is the degrees of freedom.

The statistical hypotheses for this experiment can be defined as:

$$\begin{cases} \mathrm{H}_0 : \mathrm{E}\left\{\hat{\mathbf{d}}_j\right\} = 0 \\ \mathrm{H}_1 : \mathrm{E}\left\{\hat{\mathbf{d}}_j\right\} \neq 0 \end{cases} \tag{4}$$

where $\mathrm{E}\left\{\bullet\right\}$ is the statistical expectation operator. The null hypothesis will be rejected if $\lambda_j \geq \chi_{0.95}^2 \left(df\right)$, which means that a detectable displacement occurred at point *j*. Hence, $\hat{\mathbf{d}}_j$ cannot be considered as a random error.

4.2 OPTIMISATION OF GEODETIC NETWORKS

The basic idea of optimising geodetic networks is to come up with a network, which fulfils the mentioned network quality criteria. In other words, here optimisation means minimising or maximising an OF, which represents precision, reliability and economy of a network (Schmitt 1985). As mentioned in the introduction, the different optimisation stages help us to build up an optimal network.

As a first step in optimisation of a geodetic network, the datum problem is addressed in the ZOD stage to be solved. The datum definition is of importance due to its role in fixing the network and preserving it from any rotation, translation and expansion (Eshagh 2005). Using the minimum constraint method is one of the ways to determine the best datum for a network, but the question is which points and directions should be fixed to reach to an optimum datum. As the datum problem is dependant on the precision of a network, the precision optimality criterion is used in this process. The best datum is the one that fulfils the following OF:

$$\left\| \mathbf{C}_{\mathbf{x}} - \mathbf{C}_{\mathrm{s}} \right\|_2 \to \min \tag{5}$$

where $\mathbf{C}_{\mathbf{x}}$ and $\mathbf{C}_{\mathrm{s}}$ are the VC and criterion matrices of the network, respectively, and $\left\| \bullet \right\|_2$ stands for the L_2-norm. Equation (5) tries to minimise the difference between the precision and its desired value in a least-square sense (due to L_2-norm).

It should be noted that a criterion matrix is a representative of an ideal VC matrix for a geodetic network, and since it represents an ideal situation, there is no need for the VC matrix of the network to be equal to the criterion (Koch 1985). The criterion matrix is introduced to the optimisation procedure to push the current precision of the network towards the desired or required ones. Moreover, to be able to compare the criterion matrix with the VC matrix of network, which has a defined datum via Eq. (1), the criterion should also be transformed to the same datum (Kuang 1996, p. 214). However, in this thesis, the best datum for the network in Papers I and II is defined by the minimum constraint method through a number of trial and error tests to find out which point and direction should be fixed to fulfil Eq. (5).

As the second step, to obtain an optimum configuration of a network, we usually encounter with some limitations in finding the best station locations. Some exterior conditions, such as visibility, natural features, private properties, etc., are the main determinant factors to decide where the stations should be established (Berné and Baselga 2004). By solving the FOD problem in an optimisation procedure, one can find the best configuration of the network through applying some changes to the positions of the net points. Mathematically, the procedure of finding the appropriate configuration of a network is performed by changes in the design matrix **A** of the network. Generally, the approximate shape of a network is known by the topography of the area that covers the network or by the shape of the construction (Koch 1985). Moreover, it is not the whole shape of the network to be optimised, but the optimisation is performed to optimally locate the net points by small changes in their coordinates to reach to a best optimum shape. Demanding for an optimality criterion such that the optimisation procedure can deliver smaller net errors leads to definition of different types of optimality criteria for precision. Usually, the A-optimality (minimising the trace of the VC matrix), D-optimality (VC matrix determinant is being minimised) and E-optimality (minimising the maximum eigenvalue of the VC matrix) are commonly used in the classical FOD (Berné and Baselga 2004).

The third step in optimisation of a geodetic network is to find the optimum solution for the SOD problem. The goal is to find optimum weights for the observations. It is also probable to encounter zero weights after the optimisation procedure, which implies that there are observations that have no significant effects on the network's quality. SOD is performed to provide the network with any pre-set quality criterion of precision, reliability or cost.

Both the FOD and SOD stages are performed in an optimisation procedure considering any of the quality criteria as an OF. The precision optimality requirement is explained in Eq. (5), where the users ideal precision considerations are implemented to the optimisation procedure in the form of a criterion matrix. The FOD and SOD steps should change the configuration and observation weights of the network, respectively, to fulfil that condition. Another demand for an optimal network concerns its reliability, which is ideally a maximum:

$$\min\left(r_i\right) \rightarrow \max . \tag{6}$$

Equation (6) means that the minimum redundancy number of observations should be maximised to guarantee the network reliability. Another OF concerns the cost optimality, which is defined based on the existing relation between the precision of observations and their costs. One can write:

$$\|\mathbf{P}\|_{\infty} = \min \tag{7}$$

where $\|\bullet\|_{\infty}$ represents the uniform norm and **P** is the matrix including the observation weights. The goal of Eq. (7) is to minimise the maximum weight of the observations. It is obvious that performing the FOD or SOD with the only cost OF does not sound reasonable due the fact that Eq. (7) struggles to discard as many observations as possible, which totally diminishes the quality of the network from precision and reliability point of view.

Generally, a conventional approach is performed to solve the optimisation problem. Kuang (1996, pp. 217-220) divided the whole procedure into several steps, where, based on the preliminary information about a surveying network, one can construct the initial observation plan. Then, to achieve the pre-set quality criteria, observation weight improvements are introduced to the initially designed network. This procedure should be iterated until the optimisation fulfils all the requirements. He also remarked on the field reconnaissance to assure the practicability of the designed plan. The improvements should be optimally solved by either trial and error or analytical approaches, and added to the initial plan. As the station positions are rarely moved in the trial and error method, the SOD problem is the main subject of this solution. In contrary, using the introduced fully analytical and mathematical procedure by Kuang (1996), which is also used in this thesis, the optimal improvements for both observation weights and point positions are applied to the initial plan. In other words, through this method we can perform the FOD and SOD stages to obtain optimum configuration and observation plan, while using the trial and error method, it is probable not to accomplish the best network.

Based on the optimisation goal, the mathematical OF is formulated to include one or many of optimality criteria. Also, different solutions of the analytical approach are used, where in this study, mainly linear and quadratic programming are involved in solving optimisation problems. In order to solve the quadratic programming problem, one has to transfer it to a linear programming problem, which is called the linear complementary problem, and its solution gives the optimal values for the quadratic programming (Koch 1985).

In Paper III, it has been discussed in details about the SOOMs of precision, reliability and cost, where just one criterion is used for the optimisation process. The methodology of the BOOM of precision and reliability is explained in Paper II as well as the impact of constraints in performing the BOOM. The MOOM of precision, reliability and cost, which uses a quadratic programming solution to attain an optimal network plan, is explained and practically applied in Paper III in addition to other optimisation models.

4.3 OPTIMISATION OF A DISPLACEMENT MONITORING NETWORK

Deformation surveys are one of the efficient ways to determine the changes of a deformable object. When that object is a piece of land or a man-made construction, it becomes of importance to establish a precise monitoring network that covers the risky zone or structure to inspect the exact changes of the net points. In order to detect such changes, it is needed to perform regular measurements at different time intervals, namely epoch-wise measurements, or establishing a continuously operating system of observations. Usually, the choice of monitoring techniques and instruments depend on the type, magnitude and rate of the expected deformations (Kuang 1991, p.160). However, it is the task of the optimisation to determine the best position for the net points, and choice of observation types to perform in an optimal network based on some defined criteria.

There are some differences in performing the design orders of geodetic positioning and deformation monitoring networks. The ZOD step is not generally performed in deformation monitoring networks (Kuang 1996, p. 260). However, in case of displacement networks as in this study, the problem is not to define the optimum datum for the reference network, but stabilising the datum at the reference frame to provide the same position and orientation of the network in subsequent epochs. The FOD step is performed in the same way as in a geodetic positioning network, but usually there are some considerations. As the goal of this type of networks is to monitor the deformable body, it is reasonable to place reference points somewhere outside the deformable body to avoid any possible deformation effects on these points. In the GPS monitoring network of Lilla Edet, the reference stations are located on top of the surrounding hills. It should also be noted that using GPS measurements in monitoring purposes, there is no FOD problem either. In contrary to a geodetic positioning network, we have no limitations in selecting the GPS receiver stations. Moreover, the configuration of the network is defined as the formed geometry of the ground stations and satellites (Kuang 1996, p. 327). Since we cannot change the configuration of the satellite constellation, the concept of FOD in this topic is impractical. Finally, the SOD stage is performed in the same procedure, but there is a slight difference in the definition of optimality criteria.

Here we have concentrated on optimisation of displacement networks. Thus the quality criteria in such networks are the subjects of this study. An investigation into the deformation parameters is postponed to future works.

The precision criterion in geodetic networks, which is expressed by the VC matrix, is subjected to station coordinates, while the precision measure in displacement networks is defined by the VC matrix of displacements. Mathematically, we can define the displacement as coordinate changes of a point during two subsequent epochs as:

$$\mathbf{\Delta x} = \mathbf{x}_2 - \mathbf{x}_1 \tag{8}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the coordinate vectors of a point in the first and second epoch. If the VC matrices of $\mathbf{x}_1$ and $\mathbf{x}_2$ are expressed by $\mathbf{C}_{\mathbf{x}_1}$ and $\mathbf{C}_{\mathbf{x}_2}$, respectively, the VC matrix of displacements $\mathbf{C}_{\mathbf{\Delta x}}$ becomes:

$$\mathbf{C}_{\mathbf{\Delta x}} = \mathbf{C}_{\mathbf{x}_1} + \mathbf{C}_{\mathbf{x}_2} . \tag{9}$$

Equation (9) is true if we consider uncorrelated observations. This equation is simplified to $\mathbf{C}_{\mathbf{\Delta x}} = 2\mathbf{C}_{\mathbf{x}_1}$, if we assume the same measurement quality in both epochs. It should be mentioned that we developed the idea of Paper II and III based on this assumption, while the contrariwise assumption is used in Paper IV. Now, referring back to Eq. (1), the VC of the displacement is written as:

$$\mathbf{C}_{\mathbf{\Delta x}} = 2\sigma_0^2 \left[\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A} + \mathbf{D}\mathbf{D}^{\mathrm{T}} \right)^{-1} - \mathbf{H}\left(\mathbf{H}^{\mathrm{T}}\mathbf{D}\mathbf{D}^{\mathrm{T}}\mathbf{H} \right)^{-1} \mathbf{H}^{\mathrm{T}} \right] \tag{10}$$

where its parameters are the same as Eq. (1).

Therefore, the optimisation procedure of a monitoring network, when considering the precision criterion will follow from Eq. (10). The detailed equations for performing the approximation of the VC matrix of displacement for optimising solution are explained in Paper II.

In Paper III, we tried to implement all the network quality criteria in optimising a displacement monitoring network.

5 RESULTS

In this chapter, a round-up of the selected results of the attached papers will be discussed. During this licentiate study, we applied the explained methodology of optimisation technique to both simulated and real networks. Our endeavours resulted in the development of four ideas with placing more stress on the displacement monitoring networks. It should be mentioned that the whole procedure of the optimisation was performed analytically by programming in MATLAB software. The obtained results will be explained in the following two sections: first, the optimisation results in a simulated network and second, the results in a real application.

5.1 OPTIMISATION OF A SIMULATED NETWORKS

Working with simulated networks has an advantage of not needing real data and information, but it is easy and straightforward to test any new idea and come up with conclusions. In the first two papers of this thesis, we performed the idea on a simulated trilateration geodetic network, which consists of seven net points. It was also assumed that all distances between the points were measurable, so we initially had 21 length observations in the network.

The BOOM of precision and reliability is one of the efficient models in fulfilling the network quality criteria, where the precision and reliability criteria appear simultaneously in the OF of the model. As previously mentioned, the superiority of this model exists in capability of the model in overcoming possible inconsistencies between two major criteria. This model can be subjected to a number of constraints to apply more controls to the network. The effect of constraints on performing a BOOM of precision and reliability was scrutinised in the first paper. We tested an unconstrained BOOM as well as a model with all possible combinations of precision and reliability constraints in a constructed simulation of a geodetic network. The optimisation stages were performed in this network; the ZOD problem was solved by defining the best datum, the FOD by finding the optimum configuration by assigning a ±2 metre range of relocation for the net points and, finally, the SOD step by searching for the optimum observation weights. The procedure was carried out by assuming 2 and 3 mm errors for the net points after optimisation. For instance, Table 1 represents the accuracy of net points before and after the optimisation process by assuming a desired accuracy of 3 mm. Demanding 2 mm accuracy from the built network did not yield any decrease in the number of observations after optimisation, but it increased the accuracy of the network. However, when this network was optimised to obtain 3 mm precision, it ended up to an observation plan with fewer observations. Referring to the first paper, we see that the unconstrained BOOM succeeded in removing more unnecessary observations, which, from an economic point of view, can be interpreted as the best observation plan. In other words, the unconstrained BOOM provided the requested accuracy with fewer measurements.

Referring to Table 1 anew, it can be observed that all types of constrained BOOMs mostly fulfilled the desired precision criterion. One can refer to Table 2 in Paper I to see that the reliability criterion was met as well. A small rise in error of the last point in the unconstrained and reliability-constrained BOOMs can be seen. The reason is due to lack of precision control in the constraints. Although we have both criteria in the OF, it cannot guarantee the precision for hundred percent without aforementioned constraint. However, the rise is so small (below one mm) that it can be ignored.

Table 1 – Standard errors of net points (σ) in mm before and after optimisation by BOOM: Unconstrained (U), constrained to Precision (P), Reliability (R), Precision and Reliability (PR).

	Before Optimisation	Desired Accuracy: 3 mm			
		U	P	R	PR
σ_{x1}	1.3	1.4	1.4	1.4	1.4
σ_{y1}	1.7	1.8	1.7	1.7	1.7
σ_{x2}	2.3	2.5	2.4	2.3	2.3
σ_{y2}	2.3	2.5	2.4	2.3	2.3
σ_{x3}	2.2	2.7	2.6	2.4	2.4
σ_{y3}	2.5	2.6	2.6	2.6	2.7
σ_{x5}	2.4	2.7	2.5	2.7	2.6
σ_{y5}	2.4	2.9	2.8	2.7	2.7
σ_{x6}	2.3	2.6	2.3	2.4	2.3
σ_{y6}	2.3	2.8	2.6	2.7	2.5
σ_{x7}	1.9	2.0	1.9	1.9	1.9
σ_{y7}	3.0	3.3	3.0	3.2	3.0

In Paper II, where the idea was applied to the simulated network, an optimal two-epoch observation plan was designed for the network as a displacement monitoring network. This new idea was tested against the traditional one-epoch optimal design of such network. According to the methodology of optimisation technique, explained in Section 4.2, the main goal of the SOD stage is delivering the optimum weight matrix, where its elements are the variables. Therefore, it is the weight of each observation after optimisation that reports if this measurement should be performed or not. In contrary to this method, in two-epoch optimisation approach, the variances of all observations rather than their weights are optimised. The difference of these two methods emanates from two adjustment models (Gauss-Markov and Gauss-

Helmert), which are used to derive optimisation formulas. To find out the differences of using these two models in optimisation techniques, we used both approaches to design optimum observation plans for monitoring purposes. By using the Gauss-Markov adjustment model, one can proceed with an optimisation procedure and come up with an optimum observation plan for one epoch. This plan should be exactly repeated in the next epoch to detect displacements. The method deals with observation differences of two epochs and tries to optimise their weights. On the other hand, the Gauss-Helmert adjustment model enables us to consider all observations of both epochs simultaneously in the optimisation procedure. The variances of all observations are being optimised by this model, which leads to the design of two separate observation plans.

In this work, which was presented as Paper II, we assumed a 3 mm error for displacements. Thus, in the one-epoch method we designed one observation plan for two epochs of displacement monitoring, and therefore the error of 3 mm was divided by $\sqrt{2}$, namely the coordinate error of each point should be 2.1 mm. On the other hand, in the two-epoch approach, we used the displacement error of 3 mm itself in the calculations. The optimised observation plans after both methods are shown in Fig. 2, where the top figure (a) represents the result of the one-epoch method and the bottom figures (b, c) are those of the two-epoch approach. It should be mentioned that the sizes of the error ellipses were exaggerated to illustrate the errors efficiently. Following the traditional one-epoch method yielded elimination of 3 baselines of the observation plan in this network, while the new two-epoch method provided an observation plan with the same configuration and precision as the former method, but with less number of observations. Implementing the developed approach to this network removed about 9 observations of initial 21 measurements in each epoch. Comparing these two methods, we can contend that the two-epoch approach is economically efficient enough to be used in designing displacement monitoring networks. If it does not lead to better network configuration or precision than the former method, it becomes economically favourable for the engineers.

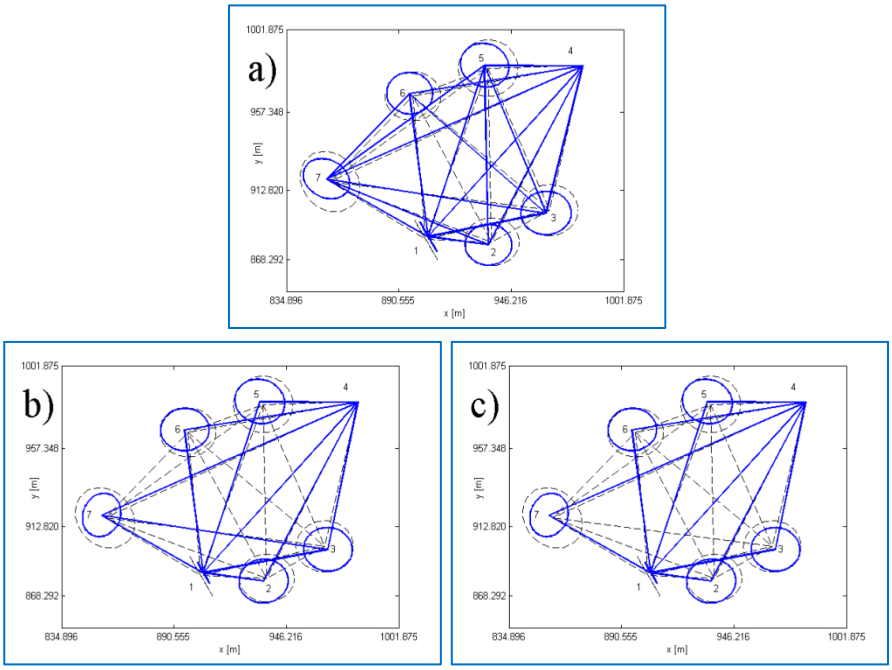

Figure 2 – Network configuration and error ellipses of the displacements before and after optimisation. a) Optimised network based on one-epoch optimisation, b) and c) two-epoch optimisation of the network in Epochs 1 and 2, respectively.

5.2 Lilla Edet GPS Monitoring Network

As described before, there is no need to perform ZOD or FOD stages in a GNSS monitoring network, so during the optimisation procedure, these two stages were eliminated from our instruction. Instead, we performed a SOD step and determined the optimum weight matrix of observed baselines. Whenever the GNSS baselines become the subject of measurement, it should be noted that for each single baseline observation we use three weight elements in the weight matrix by considering uncorrelated baseline components (cf. Paper III). Namely, considering *n* baseline observations, the dimension of weight matrix is $3n \times 3n$. Since in optimisation of a GNSS network we cannot consider each component of the baseline independent from the other components, we have to define a weight matrix for each baseline rather than each component. In this case, the baseline, including all its three elements, will either be removed or kept after optimisation and not its elements.

Based on the different optimisation methods with the principles explained in Chapter 4, we started from the existing GPS monitoring network of Lilla Edet (shown in Fig 3, left panel) to optimise it with respect to demanded network quality criteria. The idea was to apply the SOOM, BOOM and MOOM to the network and came up with the best solution. Moreover, as the purpose of this monitoring network is to detect possible displacements in the area, we decided to implement also the sensitivity criterion to the optimisation procedure. Defining the sensitivity of the network, we can guarantee the capability of it in detecting and revealing displacements or deformations. In the performed study in Paper III, first we obtained the maximum sensitivity of the network in each net-point as 5 mm, and then we optimised the network in such a way that all the net-points became sensitive to detect 5 mm displacement. Figure 3, illustrates the network before and after optimisation by the MOOM of precision, reliability and cost.

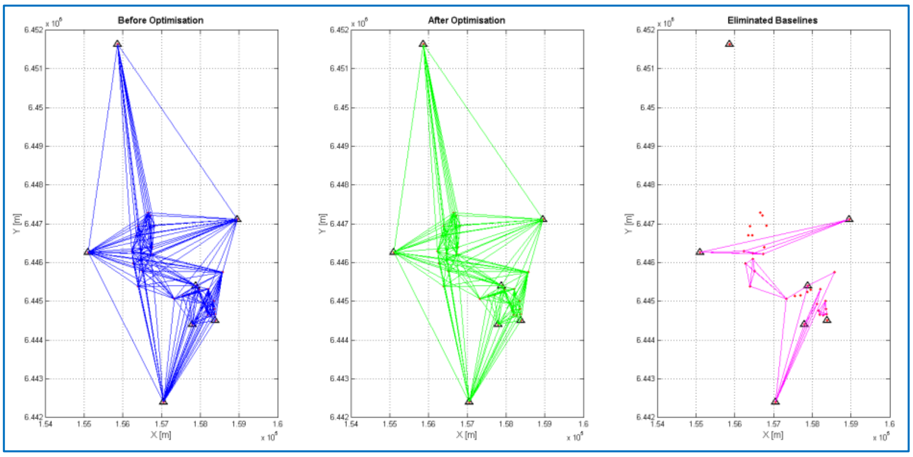

Figure 3 – MOOM of precision, reliability and cost, sensitive to detect 5 mm displacement. The panels from left to right illustrate the original network before optimisation, optimised one and removed baselines, respectively.

As can be seen in Table 2, the most economical optimisation solution for this network is the SOOM of precision, which can eliminate 19 percent of the observables. And in contrary, the worst model from an economic point of view is the SOOM of reliability, which almost keeps all the baselines. Nevertheless, referring to Figs. 4 and 5, one can have different perception.

Table 2 – Results of different optimisation techniques performed on the GPS network.

		After Optimisation			
		Sensitivity for detecting displacements: 5 mm			
	Before optimisation	SOOM of *precision*	SOOM of *reliability* with *precision* constraint	BOOM of *precision* and *reliability*	MOOM of *precision*, *reliability* and *cost*
No. of necessary GPS baselines	245	199	240	206	203
Eliminated baselines in %		19	2	16	17

It is obvious from Figs. 4 and 5 that the SOOM of reliability yields the best result for this network. As mentioned, this model tries to keep all the observations in the plan to be able to provide such precision and reliability for the network. Referring to Fig. 4, one can see that this model is more precise than our demands as a criterion matrix, which requests the detection of 5 mm possible displacements. Thus, if we come up with a model that provides our required precision, it can be the desired one. Here, the SOOM of precision, BOOM, and MOOM models are acceptable for our purpose. The outcome of MOOM and BOOM are not impressive due to the extra cost constraint of the MOOM. In the definition of this constraint, we considered the distances between receivers to place stress on the weight of that observation. Nevertheless, all the distances in this network are shorter than 10 kilometres, which cannot actually affect too much on the result of the MOOM.

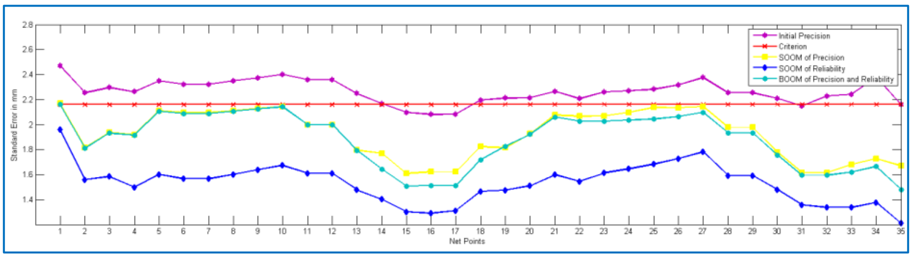

Figure 4 – Precision of the net points after optimisation based on different models. The standard errors of points before optimisation is also drawn for better comparison.

It can be seen in Fig. 4 that the SOOM of precision provides acceptable precision for the network, but at the same time it is clear in Fig. 5 that this model cannot preserve the reliability requirement of the network. As this model is not constrained to the reliability, it delivers an irregular behaviour that decreases the capability of this model in practical use. Unlike this model, The BOOM and MOOM are successful models in retaining the reliability of the network within the specified range.

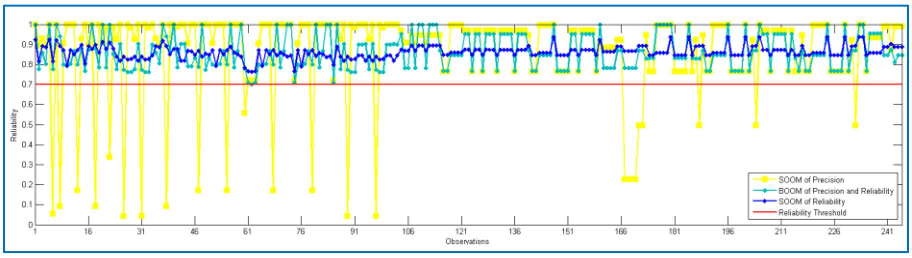

Figure 5 – Redundancy numbers (reliability) of the observations after optimisation procedure. The results of three different optimisation models are compared according to the threshold line.

In the second attempt, we tested another idea on this case study, where the goal was to redesign an observation plan by assuming higher observation precision in the next epoch of two subsequent epochs. The

SOOM of precision, which was constrained to reliability, was used for developing this idea. Based on previously obtained results, it is required to bind the SOOM of precision by the reliability control to ensure the robustness of the network. Since we want to redesign a network that should be able to detect displacements by more precise instruments, the definition of the criterion matrix has an important role in optimising procedure (see Paper IV for more details). The VC matrix of the second epoch was defined by multiplying the variances of the net points in the first epoch by a coefficient $k < 1$, implying a higher precision of the observations for the second epoch.

The results of our test for several values of *k* in the criterion matrix are summarised in Table 3. The assumption of $k = 1$ delivered the optimised observation plan for both epochs, while using $k < 1$, the optimised plan for the second epoch was yielded. It is obvious from the table that the larger the weight matrix in the second epoch becomes, the less number of baselines is required for measuring.

Table 3 – Number of required observations in the first and second epochs after optimisation procedure according to precision improvements. The number of baselines before optimisation is 245.

$k < 1$	$P_2 = \frac{1}{k} P_1$	No. of observations in	
		first epoch	second epoch
1	$P_2 = P_1$	215	215
0.9	$P_2 = 1.1\,P_1$	215	204
0.8	$P_2 = 1.2\,P_1$	215	193
0.7	$P_2 = 1.4\,P_1$	215	175
0.6	$P_2 = 1.7\,P_1$	215	154
0.5	$P_2 = 2\,P_1$	215	143

Figure 6 shows the optimisation results for $k = 0.5$, where we need 215 baselines to be observed in the first epoch and 143 baselines in the second one to be able to build a monitoring network, which is sensitive to detect 5 mm displacement. It should be clarified here that it is improbable these days to double the weight matrix for the next epoch. It is obvious that with available precise measuring devices in the market, it is very rare to be able to increase the precision of the instruments very much within a time interval of some months. However, it has been investigated theoretically in this work to express the idea and bring up the thoughts around this issue. Moreover, the effect of even small precision improvements on the number of baselines is recognisable enough to consider this method as an efficient one.

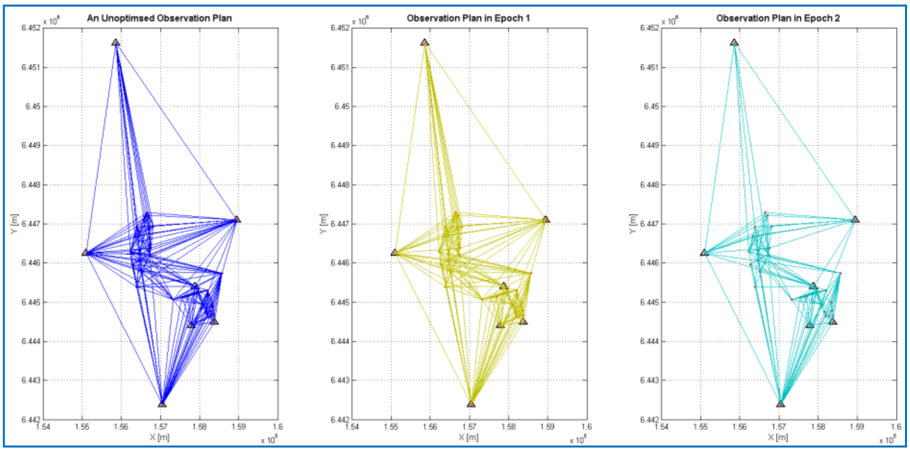

Figure 6 – The panels in the figure display the unoptimised and optimised networks in first epoch and observation plan for second epoch considering *k* = 0.5, respectively, from left to right.

6 CONCLUSION AND FUTURE WORKS

6.1 CONCLUSION

In this licentiate thesis, we studied about optimisation techniques in geodetic positioning networks. We also tried to develop some new ideas in this field and implement them on a real displacement monitoring site, which was established to monitor possible landslides in the Lilla Edet region by GPS measurements.

The optimisation procedure is needed to design an efficient network, which is abundantly precise, reliable and economically feasible. Furthermore, it can be sensitive enough also, if we deal with a displacement network. However, according to the purpose of each paper in this work, a number of these quality measures were used within optimisation process.

The first paper dealt with the BOOM of precision and reliability of a simulated network, where the effect of all possible constraints was investigated in this model. The numerical results showed that although the errors of some net points and the reliabilities of some observables did not match the required accuracy and reliability, the differences were not very large and considerable in practice. According to this simple numerical study, we concluded that the use of unconstrained BOOM was more economical in practical considerations as more observables were deleted from the plan and the accuracy and reliability of the network almost met the network requirements.

The objective of the second paper was to design an optimal displacement monitoring network for two-epoch measurements. Traditionally, the Gauss-Markov adjustment model is used to optimise the observation plan, but here we took the benefit of the Gauss-Helmert model to consider all observations of two epochs in the optimisation process. And instead of seeking for optimum weights, optimum variances of observations were sought. In contrary to the former method, where the designed observation plan should repeat for the second epoch also, two individual observation plans for two epochs were designed by the latter method. Based on the results, both methods delivered the same accuracies of displacements and configuration with more baselines to ignore in the new method.

In the third paper, the network optimisation technique was practically applied to a real case study. We aimed at optimising the existing GPS monitoring network of the Lilla Edet village to build up a network sensitive to displacements on the net points. Different optimisation models were defined and performed on the network as SOOM, BOOM and MOOM. The advantages of BOOM and MOOM were investigated such that the power of combined models on overcoming the constraint inconsistencies in single-objective models. The SOOM of reliability, which was constrained to precision, yielded better results in the sense of optimised precision and reliability compared to the other models. Naturally, it kept most of the observations to create a highly reliable network and consequently provided a network with much less standard error than we requested. BOOM or MOOM provided a network, which completely met the network quality criteria, and by removing unnecessary independent baselines one could save a considerable amount of time, cost and effort in the project. It should be mentioned that the obtained results in this work, affirmed the results attained in the first paper for the BOOM. In other words, the unconstrained BOOM fulfilled the required quality of the network as the constrained BOOMs.

The last paper contained a similar optimisation concept and case study as the third one, but with a different definition of the precision criterion. In this paper, the effect of observation precision was investigated in the optimisation of the Lilla Edet GPS displacement monitoring network. It has been assumed that the precision of GPS observations can be increased in the subsequent epochs. The enhancement of GPS observation precisions is achievable by increasing the observation time, using forced centring pillars and the combined use of GPS and other satellite systems. However, this study was conducted to numerically present the results of such an assumption. The existing monitoring network of Lilla Edet comprises 245 single baselines. We started the optimisation procedure by defining a criterion matrix to fulfil both sensitivity and precision of the network. The sensitivity was introduced as capability of the network in detecting 5 mm displacement at each net point. Considering the similar precisions for both epochs has yielded two optimum observation plans with 215 baselines in each. Increasing the precision for the second epoch was performed within several increments. In the extreme case, we assumed two times larger observation weights in the second epoch in the optimisation procedure. An observation plan with 143 baselines in the second epoch versus 215 in the first one was designed as the process result. Regardless of the fact that the aforementioned assumption is unrealistic, the other slight improvements in the observation weights can efficiently and practically decrease the number of observation demands. In the medium scale networks such as Lilla Edet, removing unnecessary baselines from the observation plan, whilst the quality requirements are preserved, can save a considerable amount of time and cost in the project.

6.2 FUTURE WORKS

During this study we have had the opportunity to conceive and develop the optimisation techniques in geodetic networks. In addition to the general studies in this field, we specifically investigated the optimisation problem in a GNSS monitoring network to detect possible displacements leading to landslides.

It is crystal clear that what takes place in a landslide is beyond the simple displacement of a net point position. To be more realistic, the deformation parameters, strain components, should be taken into account to estimate a more precise deformation model and eventually, optimise a monitoring network based on these parameters. According to previous studies (see e.g. Kuang 1996, pp. 292-300, Shestakov et al. 2005 and Doma 2014), it is quite common to consider the whole study area as one object and estimate the deformation parameters of the whole deformable body. As a future plan, we are interested in splitting the deformable object to small elements by the finite element method and study more precisely the deformation parameters in each of those elements, and consequently optimise the monitoring network more realistically.

The correlation between GNSS observations is an inevitable fact, where in the present study, we ignored it in optimising the GPS monitoring network of Lilla Edet. Similar to many GNSS processing software packages, we assumed single baseline observations between GNSS receivers. In practice, more than two receivers are set up at the net points for collecting data to speed up the measurement process. In this case, by considering the observation sessions and correlations amongst the satellite data more realistic result will be obtained. Therefore, it is also on our to-do list to consider such correlations by assuming double difference observations and investigate its impact on designing an optimal network.

As the FOD of the GNSS network is only meaningful in large extent areas, we did not perform the FOD stage in the optimisation of the case study in the last two papers. However, the geometry of both satellite constellation and network points can be included in the design matrix to enable us to develop an optimisation program that covers both design steps.

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