



**ROYAL INSTITUTE  
OF TECHNOLOGY**

# • **Optimal Design of Network for Control of Total Station Instruments** •

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# Abstract

This thesis uses the Minimum Norm Quadratic Unbiased Estimation (MINQUE) to estimate standard deviation of observations of a total station. Different setups are created by altering the number of stations and targets and their relative position in the network to study the effect that different setups have to the estimation and define what are important to minimize the effect of the setup to the estimation.

A lot of research has been done around methods for estimation of variance and covariance components, since it is useful in many fields. Various approaches exist to solve the problem of variance components estimation. Geodesy is a special case, where there often is a priori knowledge of how well an instrument is able to record measurements. There is an ISO-standard for testing and verification of geodetic instrument but also an alternative approach the KTH-Total Station Check.

For the estimation three main types of setups were defined and used in the simulation. These main types were then altered to see how different changes to the setup effect the overall estimation. The alterations were changes in distance between station and targets, changes in vertical distance between stations and targets and the amount of observations carried out by adding more stations and targets to the setups.

The result of the simulations shows that the tested changes in the setups do effect the estimation. It was not possible to determine by how much for each change, because a change in vertical displacement also meant a change in angles and distance between the station and the target. Increasing the amount of stations and targets or one of them shows that standard deviation of the estimation becomes smaller. The effect can be seen independent of which type of setup that is used. The most important factor to how good the estimation will be is the amount of observations.

# Sammanfattning

I det här examensarbetet används ”Minimum Norm Quadratic Unbiased Estimation”-metoden (MINQUE) för att skatta standardavvikelsen för de olika observationerna i en totalstation. Olika geometrier av uppställningar skapades genom att förändra antalet stationer och bakobjekt och deras relativa position i förhållande till varandra i nätet för att studera effekten som de olika geometrierna hade på skattningen och definierar vilka parametrar som är viktiga för att minimera effekten av geometrins utformning på skattningen.

Det har utförts mycket forskning på och runt skattning av varians- och kovarianskomponenter, då det är användbart inom många olika områden. Flera olika metoder existerar för att lösa problemet med skattning av varianskomponenter. Geodesi är ett specifikt område som det används inom, här är ofta ett apriori-värde känt för skattningen eftersom information om hur bra instrumentet borde mäta är tillgängligt. Det finns en ISO-standard för test och verifiering av geodetiska instrument men det finns också en alternativ metod KTH-Total Station Check.

För skattningen definierades och användes tre huvudtyper utav geometrier. Dessa tre huvudtyper ändrades sedan för att se hur förändringen påverkade skattningen. Förändringarna var i avståndet mellan stationen och bakobjektet, den vertikala skillnaden mellan stationerna och bakobjekten och antalet observationer som gjordes genom att öka antalet stationer och punkter i geometrin.

Resultatet av simuleringarna visar att de testade förändringarna i geometrierna påverkade skattningen. Det var inte möjligt att bestämma hur mycket det påverkade för varje typ av förändring, då en förändring i t.ex. vertikal skillnad också innebar en förändring i avstånd och vinkel mellan stationen och bakobjektet. Genom att öka antalet stationer och bakobjekt eller endast stationer eller bakobjekt så förbättrades standardavvikelsen. Effekten sågs oavsett vilken huvudtyp av geometri som användes. Detta visar att den viktigaste faktorn för hur bra skattningen blir är hur många observationer som utförs.

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# Chapter 1

## Introduction

### 1.1 Background

Geodetic surveying have many different applications today, cadastral surveying for determination and marking parcels, map production, civil works and for construction works to mention a few. A great toolbox of methods and equipment is available to users working in the field of surveying, from different remote sensing applications to classic geodetic surveying with leveling instruments and total stations. The types of object surveyed differs for the different purposes, from deformation monitoring, construction parts for as built documentation, markers for parcel boundaries, terrain points for creation of terrain models, to simple data collection for point-of-interest in GIS-applications. All these different types of surveyed objects are presented in some sort of coordinate system, globally defined, a local system on the construction site, etc. The required accuracy vary between the purposes, from millimeter accuracy to a few meters.

The use of GNSS receivers has increased during the recent years, one reason may be because of accessibility of receivers. For very simple applications such as tracking the distance covered by a runner a simple sport watch or smartphone with a GNSS-chipset will be sufficient. But the use is also increasing in higher accuracy applications as ditch and cable laying for construction work. This is due to the accuracy of the positioning solutions is improving with the better reference stations and reference networks available and applications for GNSS receivers are still growing. But despite this it is still limited due to the need of free line of sight between the satellites and the receiver and by precision and accuracy and therefore there is still a need for geodetic surveying with total station. Fields where the total station would be the preferred choice (not including the obvious, situations where the line of sight between satellite and receiver is blocked) is when sub-millimeter precision is needed. Examples of this can be for deformation monitoring.

Independent of what surveying method chosen there are tolerances to be met and therefore in all practical applications of surveying methods the same questions are still valid. What precision and accuracy can be expected from the chosen method? Will it be enough to be within the limit of tolerances. The error of the result has a minimum theoretical size depending on the chosen method but in practical use a few more error sources are introduced, for example errors introduced by the user by incorrect handling of the instrument, the quality and calibration of the instrument and the environmental conditions on the surveying site. There is a need to reduce these errors to a minimum or at least determine the size of them.

Depending on what type of instrument or method was used to collect the measurements, there are different error sources and therefore different methods and practices to reduce them. In this thesis the focus will be on the instrument total station which provides three kind of observations, sloping length, vertical and horizontal angle. This type of instrument

contains different sensors that work together to measure these three observations and the sources of errors are complex when the different components errors influence the overall error.

For the user it becomes complicated to verify that the instruments work according to the specified accuracy in practical use. An approach to a practical verification of a total station is described by Horemuz & Kampmann. A Free station positioning approach. By measuring a number of targets from a minimum of two station set-ups an estimation of the accuracy of measured horizontal direction, vertical direction and slope distances can be derived with a variance-covariance component estimation. (Horemuz & Kampmann 2005) Such approach will give an indication of how well the user is measuring with the current station set-up and a verification of, from the manufactured specified, accuracy. The geometry of the backsights and their spread around the station will affect the quality of free station setup. The geometry of the setups for the above described approach to verify the total stations accuracy will effect the result of variance-covariance component estimation. The effect of the geometry of the station set-up to the estimation will be investigated in this thesis.

## 1.2 Aim of the research

The purpose is to perform an analytic investigation of the geometry of station set-ups and targets to find which will give the best result for the estimation of variance components with the MINQUE-method. This will be put to test in simulations and used to give suggestions for a optimal design and procedures of set-ups and targets to use with the KTH Total Station Check for estimation of the accuracy of a certain total station and the deviations of a station set up.

## 1.3 Previous work

### 1.3.1 Variance components estimation

The estimation of variance components is a field where a lot of research has been done, since it's not only of interest for geodetic applications but also for applied mathematics and statistical inferences. Various different approaches exist to solve the problem of variance components estimation. The common approach in the geodetic field and the one used in this thesis is the Minimum Norm Quadratic Unbiased Estimator (MINQUE) first published by Radhakrishna Rao (1971). Further there is the classic Helmert's method from the beginning of the 20th century, described by Kelm (1978). Another method is the Best Quadratic Unbiased Estimates (BQUE). With the BQUE method an optimal estimate is defined as the quadratic unbiased estimate, with the minimum variance, in MINQUE the quadratic unbiased estimate is found by minimizing the Euclidean norm, for the Helmert's method this estimate is not optimized (Fan 1997). There are more methods for estimations, like Maximum likelihood estimation variance components described by Kubik (1970) and Koch (1986), most of these estimations solve the problem by using a least squares analysis of the residuals. Fan (1997) points out a few differences between Helmert's method, BQUE and MINQUE, that are of interest. The Helmert's method, as mentioned previously, does not have any optimization of the estimate and also requires that the observation errors are normally distributed. The BQUE also requires normal distribution of observation errors. With MINQUE a priori information of variance covariance components is not needed as in BQUE, though it is possible to include them in the calculation. A problem with using these estimations applied to geodetic problems is that the variance components can get negative values. Sjöberg have proposed method for estimating non-negative vari-

ance components when a non-negative solution is not found with other methods(Sjöberg 1984)(Sjöberg 1985)(Sjöberg 2011). For a more detailed reading on MINQU estimation, the paper by Radhakrishna Rao (1971) on the subject is recommended. Persson (1980) review of the theory can also be of interest.

### **1.3.2 Geometry of station setup**

The geometry of the backsight relative to the free station setup and how it will effect the setup has been investigated previously. Lithen presented a new method for calculating a free station setup but also noted that it works best when the geometry of the backsights are spread around the station in a way so that the station is interpolated rather than extrapolated (Lithen 1986). This is followed up by Svensson (1987) with a simulation of different geometry of setups where it is suggested that larger over-determinations will increase the possibility to check for gross errors. In Broberg & Johansson (2014) study it is suggested that a geometry where the free station is interpolated will give the lowest uncertainty.

### **1.3.3 Calibration and verification of total station**

There is an ISO-standard for testing and verification for geodetic instruments, ISO 17123, it is divided into 7-parts where part 3 describes a field test for a theodolite, part 4 describes field test for a EDM and part 5 describes a field test for a total station. The standard contains procedures for how the measurements should be done, calculations for measurement data and examples of numerical calculations. For a test of horizontal and vertical angles according to ISO 17123-3, two test courses needs to be constructed one for horizontal angles and a second one for vertical angles. The EDM can be tested according to ISO 17123-4 on a constructed baseline with known length between different marked points. ISO 17123-5 describes a test for standard deviation of measured coordinates X,Y and height. There are number of papers that describe a actual field test carried out according to this standard. In the paper "Verification of Selected Precision Parameters of the Trimble S8 DR Plus Robotic Total Station " are all three field test methods conducted with a total station Trimble S8 DR Plus and the result of the test is presented (Sokol et al. 2014). The test of horizontal and vertical angles where done according to ISO-standard 17123-3 and two test courses where setup. The EDM was tested according to ISO-standard 17123-4 and a test according to ISO17123-5 for measuring and calculating coordinates was also conducted. And in the paper by Pawlowski & Aksamitauskas (2008) is also a field test for testing according to ISO 17123-5 with procedures conducted and the results of the test is presented.

# Chapter 2

## Theory

In this chapter the principle of the total station (the instrument of interest for the simulations), the fundamentals of how it works, which types of measurements it provides and the errors it can have are presented. A short definition of a cartesian coordinate systems will be defined and different methods for adjustment will be presented leading to the main theorem of MINQUE method.

### 2.1 The total station

In short, total station is an instrument used to measure angles and distances. It is a combination of two types of instrument, a theodolite and an EDM (Electronic Distance Meter). The EDM measures the distance and the theodolite measures horizontal and vertical angles, in total that are three observations. To easier understand how the instrument work, we look at these two parts individually.

The principal of a theodolite can be described as a telescope attached on two axis, one vertical and one horizontal. On both these axis a graded scale is attached and when the telescope is moved around one or both of the axis it is possible to read of the change on the corresponding graded scale. The vertical angle is measured as the angle between the zenith line in the instrument and the line of sight towards the point. The horizontal direction cannot be directly measured instead the directional difference between two points is measured, where the horizontal direction already is known towards one of these points.(Egeltoft 2003)

The directional difference is the angle different between two directions, measured from the instruments zero-orientation clockwise. If, as specified above, the horizontal angle is known towards one of the points, this direction can be used to oriented the total station and determine the horizontal direction towards the other point. See figure 2.1.

An EDM can work in two ways but both imply that a signal is sent away reflected and then returned to the source. The first principal uses the lights travel time to determine the distance, time of flight. A short light beam is sent away and the time for it to hit the target and return to the EDM is measured. With the travel time known the distance traveled can be computed which should be twice the distance to the target.(Egeltoft 2003)

$$\begin{aligned} 2 * D_1 &= c * (t_b - t_a) \\ D_1 &= \frac{c * (t_b - t_a)}{2} \end{aligned} \tag{2.1}$$

From equation (2.1), where D is the distance between the transmitter and the reflector,  $t_a$  is the time when the light is sent,  $t_b$  is when it received back and c is the velocity of

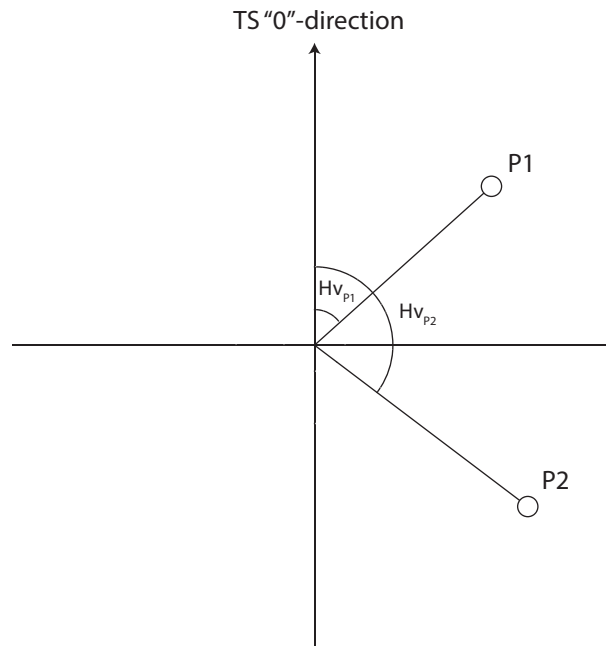


Figure 2.1: Principal of horizontal angle measurement (Egeltoft 2003)

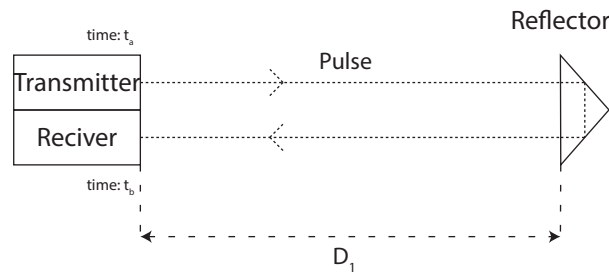


Figure 2.2: Principal for the time of flight method (Egeltoft 2003)

light, it can be seen that the distance is dependent on how accurate the travel time can be measured and the velocity of light in the medium.

The second solution is phase measurement. Where instead of measuring the travel time, the knowledge of the wavelength of the transmitted signal is used. A transmitted signal will after being reflected at the target return to the transmitter. The distance between the transmitter and the reflector can be described as the number of waves times the wavelength divided by two. However it is very unlikely that the measured distance is an exact number of wavelengths. It is more likely that it will be a number of whole wavelengths and a fraction of a wavelength (2.2). To determine the fraction, the phase shift is determined. the difference between two waves phase angle. (Ghilani & Wolf 2002)

$$D_1 = \frac{n * \lambda + d}{2} \quad (2.2)$$

To determine the distance with this method the number of wavelengths needs to be determined. This is not possible to do without transmitting more signals with longer wavelengths.(Ghilani & Wolf 2002)

In both these methods for observing the distance the sender and reflector needs to be at the same height otherwise will the measurement not be a straight line between the points but instead a leaning line.

By combining these two instruments into one, the total station are not only performing

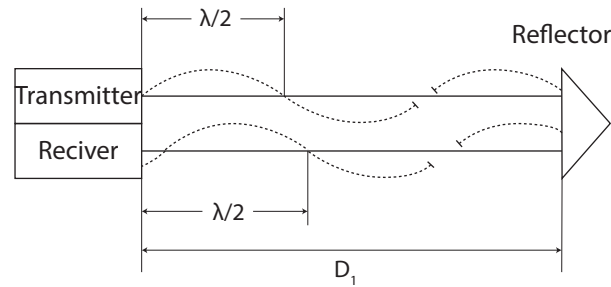


Figure 2.3: Principal for the phase method (Egeltoft 2003)

more observations at once but also solves the problem with leaning lines for the observation of the horizontal length. The EDM is attached at the aim and one can read of the vertical angle between the total station and the target and it is possible to calculate the straight line (distance) between them. For vertical angles the formula is (2.3).  $Hd_1$  is the horizontal distance,  $D_1$  is the slope distance and  $Zv_1$  is the vertical angle.

$$Hd_1 = D_1 * \sin(Zv_1) \quad (2.3)$$

Automatic target recognition (ATR) denotes, when it comes to total station, the possibility for the total station to automatically find and sight the prism. This removes the need for the surveyor to manually aim the total station towards the prism and therefore removes the human error but instead introduces a new error source from automatic target recognition.

### 2.1.1 Instrumental coordinate system

An internal coordinate system can be defined for the total station such as the origin for the system coincide with origin in the instrument, the xy-plane is the horizontal plane and y-axis is the zero direction for the instrument. The z-axis is the normal vector from the horizontal plane, which coincides with the instruments vertical axis. The coordinates are expressed as a distance towards the point, horizontal angle towards it from the y-axis, and a zenith angle from the z-axis ( $D$ ,  $Hv$ ,  $Zv$ ).

## 2.2 Coordinate system

The coordinate system described above is a type of coordinate system, where the points are expressed with polar coordinates. It should be considered that this system is only a relative system towards the position of the station where the origin is. If a new station is established a new coordinate system will be defined. It is therefore not a convenient way to express a points position. A better way is to define a coordinate system that is valid for all the points and expresses the points position in it. There are different types of coordinate system for different purpose. They are designed with their usage in mind, for example a coordinate system just for a small building site does not have to consider the same problems as a coordinate system designed for a large country. To solve the problem in this thesis a three-dimensional cartesian coordinate system will be used (see section 3.1.1). Such a coordinate system is defined by three axis, any two perpendicular to each other, and creating a plane through these two axis. Each axis should have the same scale, and an orientation, a unit of length and the origin is picked as the point where the three axis meet. If such a coordinate system is defined it is possible to describe all points in the system with a x-, y- and z-coordinate ( $x,y,z$ ). It is also possible to describe all points with spherical coordinates (as a distance from origin and two angles at origin, one on the

xy-plane and one on the plane that goes through z-axis and the point, towards the point). The transformation between polar coordinates and Cartesian coordinates can be expressed as (2.4), (2.5) and (2.6).

$$D_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} \quad (2.4)$$

$$Hv_1 = \arctan\left(\frac{x_1}{y_1}\right) \quad (2.5)$$

$$Zv_1 = \frac{\sqrt{x_1^2 + y_1^2}}{z_1} \quad (2.6)$$

As mentioned above these types of coordinate system are described with six parameters, origin (x,y,z), orientation (e.g. rotation around each axis), and scale. This implies that to be able to fixate a coordinate system of this type in space, six coordinates needs to be defined in space. It is important that this fixation of coordinate does not create a tension in the system.

### 2.3 Basic survey method

Consider the three observations that can be received from a station set up with a total station and assume that a points coordinates (x,y,z) in a rectangular Cartesian coordinate system is known and the horizontal zero direction on that point is also known. This gives an simple solution to how to determine a unknown points coordinates in this coordinate system. The total station is set up above the known point (A) and measures the distance (D), the vertical angle (Zv) and the horizontal angle (Hv) towards the unknown point P. The horizontal angle can be measured because the horizontal zero direction is known. From these observations the coordinates of P can be calculated (Anderson & Mikhail 1998).

The x-coordinate for point P is calculated by first reducing the distance down to the horizontal plane, and then is the horizontal distance (Hd) and the horizontal angle used to calculate the distance along the x-axis (2.8). The y-coordinate is calculated in a similar way but instead the horizontal distance and horizontal angle is used to calculate the distance along the y-axis (2.9), and the z-coordinate is a simple reduction of the horizontal distance directly on the z-axis with the vertical angle (2.10).

$$Hd_{AP} = D_{AP} * \sin Zv \quad (2.7)$$

$$x_P = x_A + Hd_{AP} * \cos \varphi_{AP} \quad (2.8)$$

$$y_P = y_A + Hd_{AP} * \sin \varphi_{AP} \quad (2.9)$$

$$Z_P = z_A + Hd_{AP} * \cos Zv \quad (2.10)$$

For the example described above the horizontal zero direction is assumed to be known, this is rarely the case or never the case in practice. To solve this for the example above another known point is needed, point B. This known point is used as the zero direction for the horizontal angle measurement and instead a difference in horizontal direction is measured between the direction towards the known point and the direction towards the unknown point P. This approach is known as the polar method. See Figure 2.4 for an example in a 2-dimensional coordinate system.



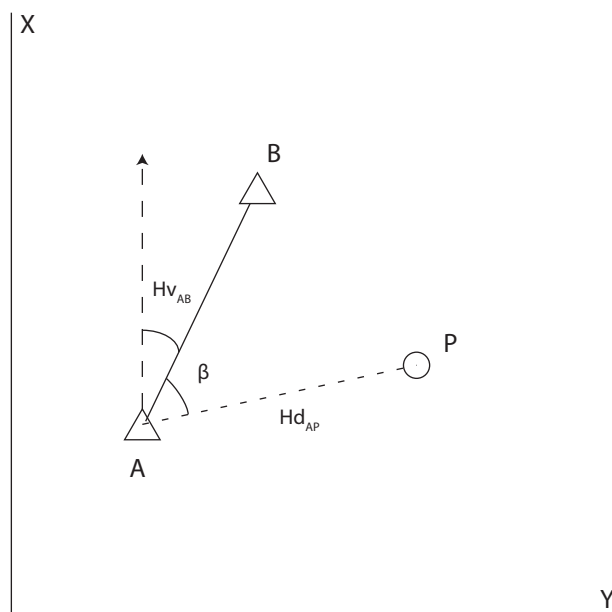


Figure 2.4: The polar method, in a 2-dimensional space (Egeltoft 2003)

This approach changes the equations for the x- and y-coordinate from the example above, because the horizontal angle now needs to be calculated. The angle between direction towards B and towards P is measured and the horizontal angle towards B can be said to be known since it can be derived from the known coordinates. Therefore the horizontal angle towards P can be expressed as the sum of the horizontal angle towards B and the angle between B and P (2.11) (Egeltoft 2003).

$$\varphi_{AB} = \arctan\left(\frac{x_B - x_A}{y_B - y_A}\right) \quad (2.11)$$

$$\varphi_{AP} = \varphi_{AB} + \beta$$

## 2.4 Errors and their adjustments

In geodetic application one often divides the errors that may occur into three different categories, systematic, gross and random errors. The systematic errors affect the result as one would guess in a systematic way. The sources can be a badly calibrated instrument, the environment the measures are carried out in or because of the observer. It could for example be that the scale of a leveling rod is wrong and all length measured with it will be three centimeters too long (Fan 1997). It is possible to correct the observation if the systematic error is known or if it is possible to observe the error in the observations. Another way to treat it, is to try and reduce the error by avoiding badly calibrated instruments, etc.

The gross error is often an error that occurs because of the human factor of failures in instruments. It could be that you read the observations wrongly and/or write it down incorrectly or the observation is not being measured from the correct point. These errors are because of their form not easy to remove by statistical means, but can be discovered by over determine an observation (e.g. measure more than one complete round) (Fan 1997).

Then there is the third type of errors, random errors, and they are as one can guess random, the errors sources could be of any kind. To avoid random errors the best solution is to improve the condition the measurements are taking place in, better instruments and suitable routines (Fan 1997).

In the case of the total stations's precision, there are the internal systematic errors of the total station, the external systematic errors and also the random errors that occur because of the environment the observations are carried out in. The types of errors to consider for the total station are axis errors, eccentricity errors, circle graduation errors, EDM errors (refraction, propagation) and the correlation between them. These errors are systematic.

### 2.4.1 Horizontal collimation error

This error is a result of a mechanical error in the instrument when the line of sight produced by crosshair is not perpendicular to the horizontal axis. The horizontal collimation error can be detected and determined by measuring in both telescopic positions. Consider a point P at the same height as the instrument and two horizontal angle measurements towards it ( $Hv_1, Hv_2$ ), and then the error ( $C_0$ ) can be expressed as (2.12).

$$C_0 = \frac{Hv_1 - Hv_2 - 200\text{gon}}{2} \quad (2.12)$$

It should be noted that the size of the error on the horizontal reading ( $C$ ) depends on the zenith angle ( $Zv$ ) as in (2.13), but the collimation error ( $C_0$ ) does not. Consider figure 2.5 where the total station is placed at "O" and reading the horizontal angle towards "A". But because of the collimation error instead the horizontal angle towards "B" is read on the horizontal plane. It then can be seen that the size of the error on the reading will be dependent on the zenith angle  $Zv$ . (Bjerhammar 1967)

$$\sin C = \frac{\sin C_0}{\sin Zv} \quad (2.13)$$

For small values on  $C$  the dependency could be shortened to (2.14)

$$C = \frac{C_0}{\sin Zv} \quad (2.14)$$

The error can be corrected for in two ways, the common way is to measure in two telescope positions, because then the collimation error will diverge in different direction if the two positions are compared. Therefore using the mean of the two measured angles will remove the error. The other way is to determine the ( $C_0$ ) and calculate the error ( $C$ ) which then is used to correct the horizontal angle.

Even if the collimation error can be corrected or eliminated, by the above described methods, the size of the error is always of interest to determine how big the influence is at different horizontal readings. The collimation error can be determined as in (2.12) by measuring the angle in two circle position and considering that the size on zenith angle effects the influence. This gives us the following (2.15).

$$C = \frac{1}{\sin Zv} \left( \frac{Hv_1 - Hv_2 - 200\text{gon}}{2} \right) \quad (2.15)$$

### 2.4.2 Vertical collimation error

The error occurs when the instrument is leveled but the vertical angle reading diverges from 100 gon at the horizon. The size of the error ( $C_{Zv}$ ) can be detected and removed from the observations by measuring the vertical angle in two telescope positions ( $Zv_1, Zv_2$ ). The sum of the readings should be 400 gon (2.16). (Bjerhammar 1967)

$$C_{Zv} = \frac{Zv_2 + Zv_1 - 400\text{gon}}{2} \quad (2.16)$$



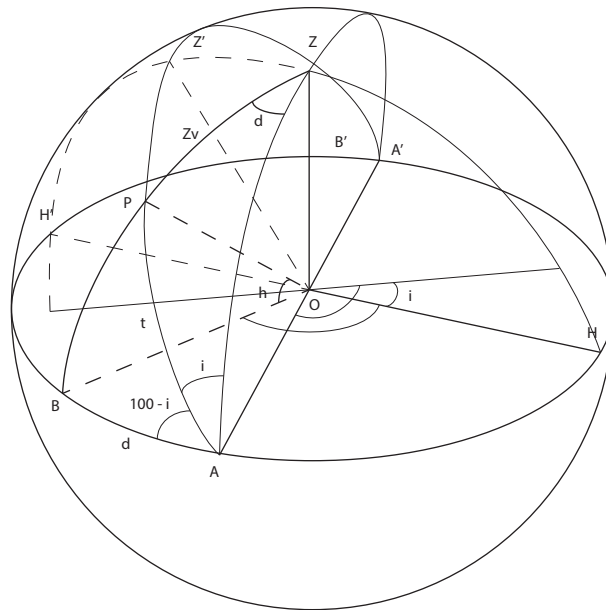


Figure 2.6: Horizontal axis error (Bjerhammar 1967)

$$d = i * \sin \alpha * \cot Zv \quad (2.18)$$

The influence on the zenith distance reading occurs instead because the angle is measured from an incorrect zenith direction, the difference between the two zenith directions is the angle ( $i$ ) and then the error can be calculated as the difference between the zenith distance ( $v$ ) from the true zenith direction ( $Zv_0$ ) and the zenith distance from the actual zenith direction ( $Zv$ ) (2.19).

$$v = Zv_0 - Zv \quad (2.19)$$

The vertical axis error can not be eliminated by measuring in two telescopic positions.

### 2.4.5 Compensator index error

Modern total stations are equipped with compensators for leveling the instrument, to avoid horizontal axis and vertical axis errors. But there exists an error with compensators that correspond to the displacement of the compensator center compared to the instrument's origin. This error can be eliminated by measuring in two telescopic positions.

### 2.4.6 Automatic target recognition collimation error

This error is in principle the same as the horizontal collimation error in section 2.4.1, but instead that the ATR:s (automatic target recognition) line of sight is not parallel with the optical axis of the telescope. The error can be calculated in the same way as for the horizontal collimation error and it can also be eliminated by measuring in two telescopic positions.

### 2.4.7 EDM instrument errors

There are three types of instrumental errors, zero error, scale error and cyclic error. The zero error is deviation between the center of the EDM and the center of the origin of the total station. This error is constant. The scale error is an error proportional to measured

distance and is caused by unexpected deviations to signals frequencies. Cyclic errors occurs from deviations in phase shift measurements.

### 2.4.8 EDM atmospheric errors

Independent of which method used in the EDM, errors will occur that are dependent on the atmospheric variations. The velocity of light and the wavelength in a specific atmosphere can be determined by measuring the temperature and pressure in the environment the measurement is carried out in. The longer the distance is, the larger are the corrections. Therefore it is normally expressed as parts per millions (ppm), meaning for every kilometre is a correction in millimetre added. Most modern total station can correct for this. Another atmospheric variation that affect the observation is refraction of light. This is because there might exist small variations of the climate along the observation. They normally occur above surfaces (for example the ground) and create a change of the index of refraction. It is a good practice to avoid performing measurements close to the ground.

## 2.5 Instrument calibration and verification

All these potential sources of error influence the overall accuracy and precision. By measuring in two telescopic positions the collimation errors, horizontal axis error and compensator index error can be canceled out, however to always measure in two telescopic positions is not seen as an efficient way to perform surveys. It is preferred to be able to measure in only one telescopic position to save time in field. The errors also influence each other and makes it difficult to determine the magnitude of the combined error, it is therefore important to have a calibrated and adjusted instrument to maintain the high accuracy of the instrument. Different methods exists for this, but can be divided into two types, calibrations carried out in field or calibrations in a laboratory environment. Because total station is a high accuracy instrument it is sensitive to mechanical shock and temperature changes and should be calibrated and controlled after such events.

A field calibration has the advantage of being able to calibrate the instrument in the environment it is being used (correct temperature). A calibration according to the instrument manufacturers instructions is one option. Normally it is possible to calibrate the instruments angle measurements with this method, but for errors on the EDM a pre built calibration area is needed. One should note that a calibration in field will only add a constant to correct for the error. Problems with performing a field calibration is that it can be affected by environment as well. It could be vibrations from construction work or, instrument/reflector moves during the calibration.

The other option is to have it calibrated and adjusted in a laboratory environment. This is often performed by the manufacturers service technician and instead of only performing an electronic calibration of the instrument as in the field, the possibility to mechanically adjust the instrument exists. Since the environment is controlled therefore the atmospheric effects such as pressure, temperature and refraction can be measured and be considered well-known. But the instrument will be calibrated for a certain temperature, a normal indoor temperature, which might differ significantly from the temperature on the site where the survey will be carried out. It is therefore recommended to perform an instrument calibration in field after a calibration in a laboratory environment to adjust for the changes in temperature.

It can be clearly seen that it is difficult even with good calibration procedures to be able to document that the instruments overall accuracy is according to manufacturers specifications. A method for verifying the total station accuracy can solve this problem. There is as mentioned in section 1.3.3 a ISO-standard (ISO17123) that describes methods

for verifying measuring equipment of different sorts, including the total station. However there also exists another method, the KTH-TSC, presented by Horemuz & Kampmann.

### 2.5.1 KTH-TSC

The KTH Total Station Check is a method for verifying the total stations accuracy. In the paper by Horemuz & Kampmann two methods are described, one for free station positioning and one for three-dimensional spatial co-ordinate-transformation. The method of free station positioning is of interest for this thesis hence the idea here is to find a optimal design for such verification. The method works as of a first station set-up is made and all targets are measured then the instrument is moved to a second position and the same targets are measured from the second position. See figure 2.7. With a method for adjustment of the network can then the coordinates of the stations be calculated as well as the accuracy for each observation type.(Horemuz & Kampmann 2005)

If the method of KTH-TSC is compared to the tests from the ISO-standard, the KTH-TSC would only need one test course for calculating the accuracy and in principal only one station, but with the ISO-standard two tests will be needed one for the angle measurements and another one for the distance measurements to be able to get verification for all three observations. This will require more station setups. The other method in the ISO-standard for calculations of coordinates only requires one test course but requires three station setups.

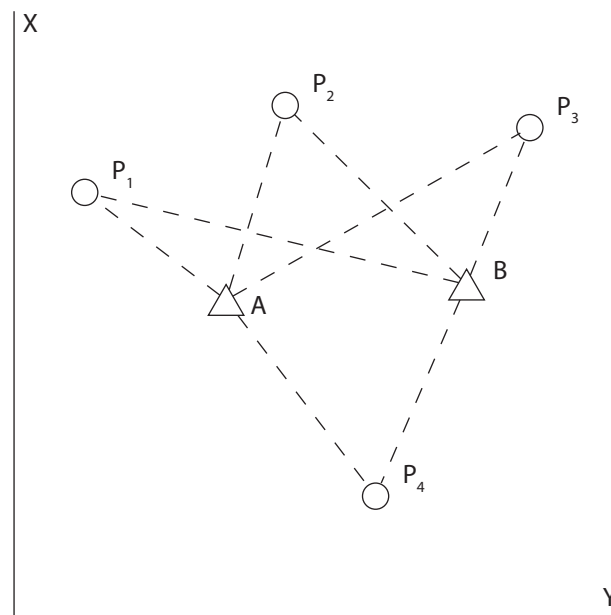


Figure 2.7: Spatial free station positioning

## 2.6 Network adjustment

When measuring a network of points errors in the measurement leads to inconsistency, it is therefore important to correct these errors in the network to remove the inconsistency. There are different approaches for performing this adjustment and the most common is according to the least square principal (Fan 1997). An adjustment of a geodetic network depends on both a functional model and a priori statistical model. The functional model is a number of either condition or observation equations. Observations equations describe the relation between the observation and the parameters to be estimated. The number of

unknown parameters should be as many as the necessary observations. It is important to choose independent parameters. The priori statistical model is simply the expectation and variance-covariance matrix of the measurement errors.

### 2.6.1 The variance-covariance matrix

An general model for a variance-covariance model is defined for a general functional model where  $L$  is the observation vector,  $\varepsilon$  is the error vector and  $\tilde{L}$  is the true observation vector.

$$\tilde{L}_{nx1} = L_{nx1} - \varepsilon_{nx1} \quad (2.20)$$

Furthermore the expectations on the error is considered to be zero.

$$E(\varepsilon) = 0 \quad (2.21)$$

The error vector has a variance-covariance matrix defined as in (2.22)

$$Q_{n \times n} = E(\varepsilon \varepsilon^T) = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1k} \\ q_{21} & q_{22} & \cdots & q_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ q_{k1} & q_{k2} & \cdots & q_{kk} \end{bmatrix} \quad (2.22)$$

This matrix can be broken down to represent individual variance components. If one considers the above variance-covariance matrix to be built up by sub matrices on each row/column so  $q = k(k+1)/2$ . Each of these matrices are expressed as the product of a variance component and a matrix (2.24) (Fan 1997).

$$Q_{n \times n} = \begin{bmatrix} \sigma_{11} * Q_{11} & \sigma_{12} * Q_{12} & \cdots & \sigma_{1k} * Q_{1k} \\ \sigma_{21} * Q_{21} & \sigma_{22} * Q_{22} & \cdots & \sigma_{2k} * Q_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{k1} * Q_{k1} & \sigma_{k2} * Q_{k2} & \cdots & \sigma_{kk} * Q_{kk} \end{bmatrix} \quad (2.23)$$

$$Q_{n \times n} = \sum_{i=1}^q (\sigma_j * Q_j) \quad (2.24)$$

Where  $\sigma_j (j = 1, \dots, q)$  are the variance components and  $Q_j (j = 1, \dots, q)$  is a  $n \times n$  matrix corresponding to each variance-component.

### 2.6.2 Estimation of variance-covariance components

A problem that may occur with the least square method is as mentioned above that it is dependent on the relative accuracy more than the absolute accuracy. The relative accuracy works well when using measurements or observations of the same type, because one can then form a weight matrix from the relative accuracy. A sort of empirical weighting is done based on the available information, for example an angle measurement done in two circle positions should have higher weight than another angle measurement only done in one. But how should two observations be weighted if they have correlations to each other or for the case of a total station, the observations are of different kinds. If the absolute accuracy is known for each observation type it can be used as a relative accuracy between the different types of observations (Fan 1997).

Another way to solve the problem is to, instead of using a empirical weighting, estimate the variance-covariance matrix of the observation from the observations and their condition equations or observations equations.

There exists a few different methods for estimation of variance-covariance matrix, but the one described and used here is MINQUE-method (Minimum Norm Quadratic Unbiased Estimate). The Indian mathematician C. Radhakrishna Rao has written articles explaining and deriving the method and for more detailed explanation of it, the article by Radhakrishna Rao (1971) can be of interest.

### 2.6.3 Estimation of the variance components with MINQUE

This derivation of the method follows the one presented by Radhakrishna Rao (1972). First consider the functional model for adjustment by elements as in equation (2.25).

$$L = AX + \epsilon \quad (2.25)$$

Where  $L$  is the observation vector,  $A$  is the coefficient matrix,  $X$  is the parameter vector and  $\epsilon$  is the error vector

This is the functional model used and the error is assumed to have zero expectation and a variance covariance model defined as in section 2.6.1. The expectation and the variance of  $L$  is formulated in (2.26).

$$E(L) = AX \quad (2.26)$$

$$E(\epsilon) = 0$$

$$V(L) = E(\epsilon\epsilon^T) = \sigma_1 Q_1 + \dots + \sigma_q Q_q$$

Where  $Q_i = U_i^T U_i$ , because  $\epsilon = U * \varepsilon E(\epsilon) = U * E(\varepsilon) = 0$ ,  $E(\epsilon\epsilon^T) = U * E(\varepsilon\varepsilon^T) * U^T = \sum_{j=1}^q U_j * E(\varepsilon_j\varepsilon_j^T) * U_j^T = \sum_{j=1}^q \sigma_j * U_j U_j^T = \sum_{j=1}^q \sigma_j * Q_j$  The problem to estimate then becomes the unknown parameters  $X$  and the variance components.

The estimation should be seen as an estimation of the linear function of the variance components in (2.27) by a quadratic form  $L^T M L$  of the random variable  $L$ .

$$p_1 \sigma_1 + \dots + p_q \sigma_q \quad (2.27)$$

The estimation to the above problem (eg. determine the M-matrix) with the MINQUE method can be defined from certain criteria, (1) invariance, (2) unbiasedness and (3) minimum euclidian norm.

#### Invariance

If instead of the unknown parameter  $X$  a approximate value is introduced and the difference between them are expressed as  $\gamma = X - X_0$ . The adjustment model with the approximate value will be written as (2.28). This changes the estimation of the linear function of variance components from  $L^T M L$  to  $(L - AX_0)^T M (L - AX_0)$ . However the solution should still have the same numerical solution so that  $L^T M L$  holds. This implies that  $MA = 0$  must be true. The estimation  $L^T M L$  that is independent of  $X_0$  is called an invariant estimate.

$$L - AX_0 = A\gamma - \epsilon \quad (2.28)$$

#### Unbiadness

Under the above assumption of restriction, the quadratic form could be expressed by means of the vector  $\varepsilon$ .

$$L^T M L = \varepsilon^T U^T M U \varepsilon \quad (2.29)$$

If  $L^T M L$  is unbiased for  $\sum p_j \sigma_j$ , then (2.30) is valid for all  $\sigma$ .



$$\begin{aligned}
E(\varepsilon^T U^T M U \varepsilon) &= \sum_{j=1}^q E(\varepsilon_j^T U_j^T M U_j \varepsilon_j) & (2.30) \\
&= \sum_{j=1}^q (U_j^T M U_j) \\
&= \sum_{j=1}^q p_j \sigma_j \\
&\Rightarrow \text{tr}(U_j^T M U_j) \\
&= \text{tr}(M Q_j) = p_j
\end{aligned}$$

### Minimum euclidian norm

An estimator to  $\sum p_j \sigma_j$ , if  $\varepsilon$  had been known is  $\varepsilon \Delta \varepsilon$ , where  $\Delta$  is (2.31). But the estimator that has been derived is  $(\varepsilon^T U^T M U \varepsilon)$ . The difference between them is then  $(\varepsilon^T (U^T M U - \Delta) \varepsilon)$ .

$$\Delta = \begin{bmatrix} \frac{p_1}{c_1} & & & \\ & \frac{p_2}{c_2} & & \\ & & \dots & \\ & & & \frac{p_q}{c_q} \end{bmatrix} \quad (2.31)$$

This difference could be made smaller by minimizing the norm of it. If the norm is chosen as the euclidian norm and with the determined restriction on M, the following could be derived.

$$\begin{aligned}
(U^T M U - \Delta)^2 &= \text{tr}(U^T M U - \Delta)(U^T M U - \Delta) & (2.32) \\
&= \text{tr}(U^T M U U^T M U) - \text{tr}(\Delta \Delta) \\
&= \text{tr}(M Q M Q) - \text{tr}(\Delta \Delta)
\end{aligned}$$

Note that  $\text{tr}(\Delta \Delta)$  does not involve the matrix M and therefore does not need to be considered. The problem of the MINQUE is then to find a M matrix that is valid under the conditions in (2.33).

$$\begin{aligned}
M A &= 0 & (2.33) \\
\text{tr}(M q_j) &= p_j \\
\text{tr}(M Q M Q) &= \text{minimum}
\end{aligned}$$

One can not expect all  $\varepsilon_j$  to have the same standard deviation, this can be expressed by  $n_j = \frac{1}{\sqrt{\sigma_j}} \varepsilon_j$ , and inserting this term into  $(\varepsilon^T (U^T M U - \Delta) \varepsilon)$  gives us (2.34).

$$n^T \Lambda^{1/2} (U^T M U - \Delta) \Lambda^{1/2} n \quad (2.34)$$

$$\Lambda = \begin{bmatrix} \sigma_1 I_{c_1} & & \\ & \dots & \\ & & \sigma_q I_{c_q} \end{bmatrix} \quad (2.35)$$

Then the solution will be as (2.36), where  $Q_* = \sigma_1 Q_1 + \dots + \sigma_q Q_q$

$$\begin{aligned}
MA &= 0 & (2.36) \\
tr(Mq_j) &= p_j \\
tr(MQ_*MQ_*) &= \text{minimum}
\end{aligned}$$

### 2.6.4 MINQUE of variance components

With the principal idea of MINQUE presented above, now the variance component can be computed with those ideas.

Choose  $Q_j$  such as described in section 2.6.1 with  $\sigma_1^0, \dots, \sigma_q^0$  if there exists a priori ratios of the unknown variance component.

$$Q = Q_1 + \dots + Q_q \quad (2.37)$$

$$Q_0 = \sigma_1^0 Q_1 + \dots + \sigma_q^0 Q_q \quad (2.38)$$

The MINQUE of  $\sum p_j \sigma_j$  is found by (2.39).

$$L^T M L = \sum \lambda_j L^T R Q_j R L \quad (2.39)$$

If  $u^T = (u_1, \dots, u_q)$  where  $u_j = L^T R Q_j R L$  then (2.39) can be expressed as (2.40).

$$\begin{aligned}
L^T M L &= \lambda^T u = p^T S^{-1} u & (2.40) \\
p^T S^{-1} &= p^T \hat{\sigma} \\
S \hat{\sigma} &= u
\end{aligned}$$

This shows how to estimate the variance components with MINQUE, where the parts are defined as,

$$\begin{aligned}
u_j &= L^T R Q_j R L \\
s_{ij} &= tr(R Q_j R Q_i) \\
R &= Q^{-1}(1 - P) \\
P &= A(A^T Q^{-1} A)^{-1} A^T Q^{-1}
\end{aligned}$$

A covariance matrix of the variance components can be computed as (2.41). Where the diagonal elements of  $Q$  represents the squared standard deviations of the estimation for each variance component.

$$Q_{\hat{\sigma}\hat{\sigma}} = (S^T S)^{-1} \quad (2.41)$$

$$Q_{\hat{\sigma}\hat{\sigma}} = \begin{bmatrix} \hat{\sigma}_{1,1} & \hat{\sigma}_{1,2} & \hat{\sigma}_{1,3} \\ \hat{\sigma}_{2,1} & \hat{\sigma}_{2,2} & \hat{\sigma}_{2,3} \\ \hat{\sigma}_{3,1} & \hat{\sigma}_{3,2} & \hat{\sigma}_{3,3} \end{bmatrix} \quad (2.42)$$

# Chapter 3

## Methodology

In this part the above theory will be applied on the experiment, to define the network (e.g. numbers of targets, station set-ups and geometry) of observations for estimating the variance components of a certain total station in certain conditions.

### 3.1 Problem statement

The aim is to find an optimal design of network and a general idea is to reduce the amount of needed station set-ups and targets. Certain criteria should be fulfilled to make this method valid to use in field; it must provide good estimations of the variance components and it should be time effective (no more set ups or targets than necessary for receiving an acceptable estimation). The methodology used will be to try and define a geometry between stations and targets that will have the lowest standard deviations. Then stations and/or targets will be added to see if the standard deviation will increase or decrease.

#### 3.1.1 Datum definition

The coordinate system used for the simulation will be a three dimensional cartesian system and to be able to define the datum for such a system, six parameters are needed (see section 2.2). If the criteria for this simulation is considered, the datum should be defined such as it is not in need of any other points than those measured from the stations in the simulation. The stations should neither need to be on known points. Therefore the parameters for the coordinate system are defined by fixing six of the coordinates of the stations and targets in the network. It is important to consider which coordinates that are fixed to avoid constraints in the network. To define the datum in this simulation the first station set-up (STN1) is chosen as the origin of the system. To define the XY-plane and to lock the rotation around the X-axis, the y- and z-coordinates of station two (STN2) is fixed in the system. To lock the rotation around the Y-axis, the z-coordinate of the first target (P1) is fixed in the coordinate system. Figure 3.1 illustrates this definition.

### 3.2 Observation equation

From the definition of polar coordinates, it can be seen that measurements from a total station can be used to describe a points coordinates, relative to the total stations position. See equations (2.4), (2.5) and (2.6). These measurements can be seen as the observations that should be adjusted. From them the observation equations for the adjustment can be defined. If  $D_{sp}$  is the measured slope distance between Station S and Target P, the observation equation for slope distance will then be as (3.1)

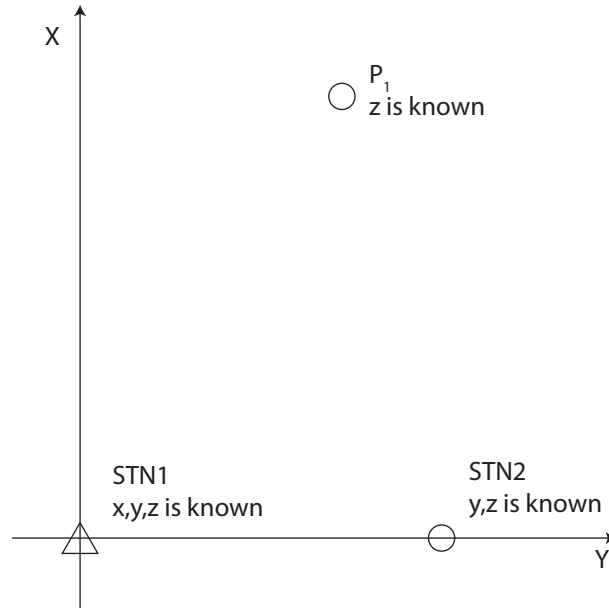


Figure 3.1: The definition of the coordinate system

$$D_{sp} = \sqrt{(x_p - x_s)^2 + (y_p - y_s)^2 + (z_p - z_s)^2} \quad (3.1)$$

This equation is a non-linear equation that needs to be linearized. The linearized equation becomes (3.2) where  $(x_s^0, y_s^0, z_s^0)(x_p^0, y_p^0, z_p^0)$  is the approximate coordinates and  $(\delta x_s, \delta y_s, \delta z_s)(\delta x_p, \delta y_p, \delta z_p)$  are the corrections of them. (Fan 1997)

$$(D_{sp} - D_{sp}^0) - \varepsilon_{sp} = a * \delta x_s + b * \delta y_s + c * \delta z_s - a * \delta x_p - b * \delta y_p - c * \delta z_p \quad (3.2)$$

$$D_{sp}^0 = \sqrt{(x_p^0 - x_s^0)^2 + (y_p^0 - y_s^0)^2 + (z_p^0 - z_s^0)^2} \quad (3.3)$$

$$a = \frac{\partial D_{sp}}{\partial x_s} = -\frac{x_p^0 - x_s^0}{D_{sp}^0}$$

$$b = \frac{\partial D_{sp}}{\partial y_s} = -\frac{y_p^0 - y_s^0}{D_{sp}^0}$$

$$c = \frac{\partial D_{sp}}{\partial z_s} = -\frac{z_p^0 - z_s^0}{D_{sp}^0}$$

For coordinates that are fixed in stations or targets, their corresponding corrections should be removed from the observation equation. For the horizontal direction and the vertical angle we can form observation equation on the same principles. For horizontal direction see (3.4), (3.5) and (3.6).

$$Hv_{sp} = \arctan\left(\frac{y_p - y_s}{x_p - x_s}\right) - \beta_s \quad (3.4)$$

$$(Hv_{sp} - Hv_{sp}^0) - \varepsilon_{sp} = a * \delta x_s + b * \delta y_s - a * \delta x_p - b * \delta y_p - \beta_s \quad (3.5)$$

$$Hd_{sp}^0 = \sqrt{(x_p^0 - x_s^0)^2 + (y_p^0 - y_s^0)^2} \quad (3.6)$$

$$\alpha_{sp}^0 = \arctan \left( \frac{y_p^0 - y_s^0}{x_p^0 - x_s^0} \right)$$

$$a = \frac{\partial D_{sp}}{\partial x_s} = \frac{y_p^0 - y_s^0}{(Hd_{sp}^0)^2}$$

$$b = \frac{\partial D_{sp}}{\partial y_s} = -\frac{x_p^0 - x_s^0}{(Hd_{sp}^0)^2}$$

And for the vertical angle see (3.7), (3.8) and (3.9).

$$Zv_{sp} = \frac{\sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}}{z_p - z_s} \quad (3.7)$$

$$(Zv_{sp} - Zv_{sp}^0) - \varepsilon_{sp} = a * \delta x_s + b * \delta y_s + c * \delta z_s - a * \delta x_p - b * \delta y_p - c * \delta z_p \quad (3.8)$$

$$D_{sp}^0 = \sqrt{(x_p^0 - x_s^0)^2 + (y_p^0 - y_s^0)^2 + (z_p^0 - z_s^0)^2} \quad (3.9)$$

$$Hd_{sp}^0 = \sqrt{(x_p^0 - x_s^0)^2 + (y_p^0 - y_s^0)^2}$$

$$a = \frac{\partial D_{sp}}{\partial x_s} = \frac{(x_p^0 - x_s^0) * (z_p^0 - z_s^0)}{Hd_{sp}^0 * (D_{sp}^0)^2}$$

$$b = \frac{\partial D_{sp}}{\partial y_s} = \frac{(y_p^0 - y_s^0) * (z_p^0 - z_s^0)}{Hd_{sp}^0 * (D_{sp}^0)^2}$$

$$c = \frac{\partial D_{sp}}{\partial z_s} = -\frac{Hd_{sp}^0}{(D_{sp}^0)^2}$$

### 3.3 Computations and implementation

With the unknown parameters defined, the estimation of the variance-covariance components can be done. The calculations have been performed with Matlab and the simplest case is presented here.

Consider Figure 3.2, which shows two stations and the observation from the stations towards the target. The coordinate system is defined as in section 3.1.1 and observation equation are defined as (3.2), (3.5) and (3.8) in section 3.2. Then the functional model will be as in (3.10) with the corresponding matrices defined as (3.11), (3.12) and (3.13). The elements of the A-matrix corresponds to the partial derivatives of the corresponding observation equation.

$$\underset{6*1}{\tilde{L}} = \underset{6*31*3}{A} \underset{6*1}{X} + \underset{6*1}{\varepsilon} \quad (3.10)$$

$$L = \begin{bmatrix} D_{STN1-P1} \\ Hv_{STN1-P1} \\ Zv_{STN1-P1} \\ D_{STN2-P1} \\ Hv_{STN2-P1} \\ Zv_{STN2-P1} \end{bmatrix} \quad (3.11)$$

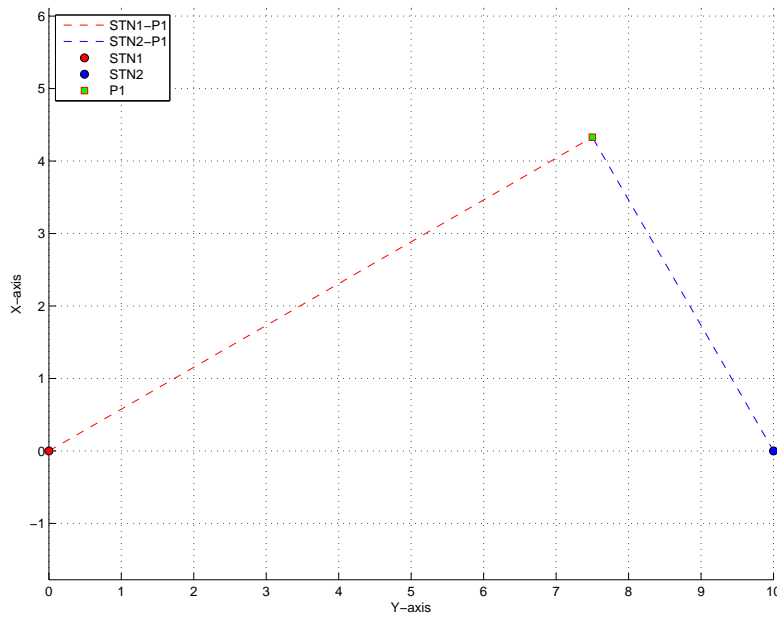


Figure 3.2: Test with two stations and one point

$$A = \begin{bmatrix} 0 & d_{STN1-P1} & e_{STN1-P1} \\ 0 & c_{STN1-P1} & d_{STN1-P1} \\ 0 & d_{STN1-P1} & e_{STN1-P1} \\ a_{STN2-P1} & d_{STN2-P1} & e_{STN2-P1} \\ a_{STN2-P1} & c_{STN2-P1} & d_{STN2-P1} \\ a_{STN2-P1} & d_{STN2-P1} & e_{STN2-P1} \end{bmatrix} \quad (3.12)$$

$$X = \begin{bmatrix} x_{STN2} \\ x_{P1} \\ y_{P1} \end{bmatrix} \quad (3.13)$$

The a variance-covariance matrix for the a priori values of the observations should be defined as in (2.23). In this case there are three variance-components, one for each observation and the a priori value is set to a normal value for a standard total station.

$$\begin{aligned} \sigma_d &= 1mm + 1ppm \\ \sigma_{hv} &= 0,45mgon \\ \sigma_{Zv} &= 0,45mgon \end{aligned} \quad (3.14)$$

The a priori values of the distance observation is divided in two parts a standard error and an error that grows with the distance (parts per million). This means that when calculating the a priori values of a certain distance, the approximate distance of the observation needs to be known. In this case the variance-covariance matrix can be formed as (3.15)(3.16)(3.17)(3.18).

$$Q_1 = \begin{bmatrix} 1 + \frac{D_{STN1-P1}}{1000} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 + \frac{D_{STN2-P1}}{1000} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.15)$$

$$Q_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0,3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0,3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.16)$$

$$Q_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0,3 \end{bmatrix} \quad (3.17)$$

$$Q_0 = Q_1 + Q_2 + Q_3 \quad (3.18)$$

With the definition of MINQUE in (2.40) and (2.41) the standard deviation of the observation for this particular geometry can be calculated.

### 3.3.1 Matlab implementation

For the calculation of the observations equations and the standard deviations for different "stations-geometry" a Matlab code has been written. The principal idea explained in section 3.3 was translated into an algorithm. The main problem is to define the A- and Q-matrix. To do this, first the size of the matrices has to be determined. The minimum is a setup with two stations and one target and with six "known" coordinates, but there is no upper limit of over determinations. The columns in the A-matrix is then  $m = (\text{numberofstations} + \text{numberoftargets}) * 3 - 6$ . The rows of the A-matrix corresponds to the amount of observations  $n = \text{numberofstations} * \text{numberoftargets} * 3$ . The Q-matrix is a  $m * m$ -matrix. To populate the matrices a nested for loop is used. In the first level it goes through the stations and in the second level it goes through the targets. Dependent on the station number there are three cases. In the second level there are two cases. All together this gives us six cases. The first case is if it is station one and target one, then all the partial derivatives corresponding to the stations coordinates and the targets z-coordinate should not be considered because they are not being adjusted and should not be placed in any element of A-matrix. The second case is when it is station one but not target one, then all coordinates corresponding the target should be considered. The third case is when it is station two and target one, then the partial derivatives corresponding to the stations y- and z-coordinate and the targets z-coordinate are not considered. The fourth case is when it is station two but not target one, then the partial derivatives corresponding to the stations y- and z-coordinate and all of the targets coordinates are considered. The fifth case is when is any other station and target one, then all partial derivatives corresponding to the stations coordinates and the targets x- and y-coordinate are part of the observation equation and should be inserted in its corresponding element in the A-matrix. The sixth case is when it is not station one or station two and not target one, then all station and

target coordinates should be adjusted and therefore part of the A-matrix. An algorithm corresponding to this was defined.

Listing 3.1: Algorithm for creating A-matrix and Q-matrix

```

1 Read stations and their coordinates
2 Read targets and their coordinate
3 Define size of A matrix as nXm, n='number of stations'*'number of ...
   targets'*3,
4 m=('number of stations'+-number of targets')*3-6.
5 Define size of Q matrix as nXn.
6 For station i
7   For target j
8     case 1 i == 1, j == 1
9       calculate partial derivates for targets x- and y-coordinate,
10      for all three observations.
11      Insert in corresponding element in A matrix.
12      Calculate q-element for Q_1 for distances.
13      Insert q-element in element in corresponding Q matrix.
14     case 2 i == 1
15       calculate partial derivates for targets x-, y- and ...
16       z-coordinates,
17       for all three observations.
18       Insert in corresponding element in matrix.
19       Calculate q-element for Q_1 for distances.
20       Insert q-element in element in corresponding Q matrix.
21     case 3 i == 2, j == 1
22       Calculate partial derivates for stations x- and targets x-...
23       and y- coordinate, for all three observations.
24       Insert in corresponding element in matrix.
25       Calculate q-element for Q_1 for distances.
26       Insert q-element in element in corresponding Q matrix.
27     case 4 i == 2
28       Calculate partial derivates for stations x- and targets ...
29       x-, y- ...
30       and z-coordinate, for all three observations.
31       Insert in corresponding element in matrix.
32       Calculate q-element for Q_1 for distances.
33       Insert q-element in element in corresponding Q matrix.
34     case 5 j == 1
35       Calculate partial derivates for stations x-,y-,z- and ...
36       targets...
37       x- and y-coordinate, for all three observations.
38       Insert in corresponding element in matrix.
39       Calculate q-element for Q_1 for distances.
40       Insert q-element in element in corresponding Q matrix.
41     case 6
42       Calculate partial derivates for stations x-,y-,z- and ...
43       targets...
44       x-,y- and z-coordinate, for all three observations.
45       Insert in corresponding element in matrix.
46       Calculate q-element for Q_1 for distances.
47       Insert q-element in element in corresponding Q matrix.

```

With the principal algorithm a Matlab code was written. The complete Matlab code is attached in appendix A.



# Chapter 4

## Results

In this chapter the results of the simulation will be presented. Starting with the simplest case and geometry and continuing with more complex networks with an increasing number of targets and stations to find the optimal design. In Appendix B all networks used and mentioned in this chapter are listed.

### 4.1 The simplest case

Starting with considering a network with two stations and one target as in Figure 3.2. In this particular network the angle between the observations at the point is 100 gon, meaning that the lines from the station towards the target are perpendicular to each other at the target. For an estimation with only horizontal angle observations this would be the best case. By comparing different networks of this type, the best solution for an estimation with two stations and one target can be found. In Table 4.1 the result for different networks are presented. Type S1 is as in Figure 3.2 and S2 is as Figure B.1b. These two types can be seen as resulting in similar results and both have an angle at the point that is 100 gon. This can be compared to the other two S3 and S4 as in Figure B.1c and B.1d, where in S3 the target is placed on a line that goes through both points, but in between the stations. In S4 the targets are placed on a line that goes through both lines and the target is on the outside of the stations. In all networks the heights are set to zero for all stations/targets. But in the simulations it has been seen that the impact of changing the height of the target in relation to the stations greatly affect the final result. By only shifting the height of the target by less than one meter the result can shift by a factor ten. This only affect the standard deviation in distances and vertical angle observations because the horizontal angle is not dependent on the z-coordinate. Measuring towards one target from two stations is clearly not enough to get reliable results.

Table 4.1: Result for standard deviation, two stations one target.

Type	Distance (mm)	Horizontal angle (mgon)	Vertical angle (mgon)
S1	0,752	0,479	0,895
S2	0,745	0,477	0,885
S3	1,414	0,707	1,346
S4	0,471	0,707	0,837

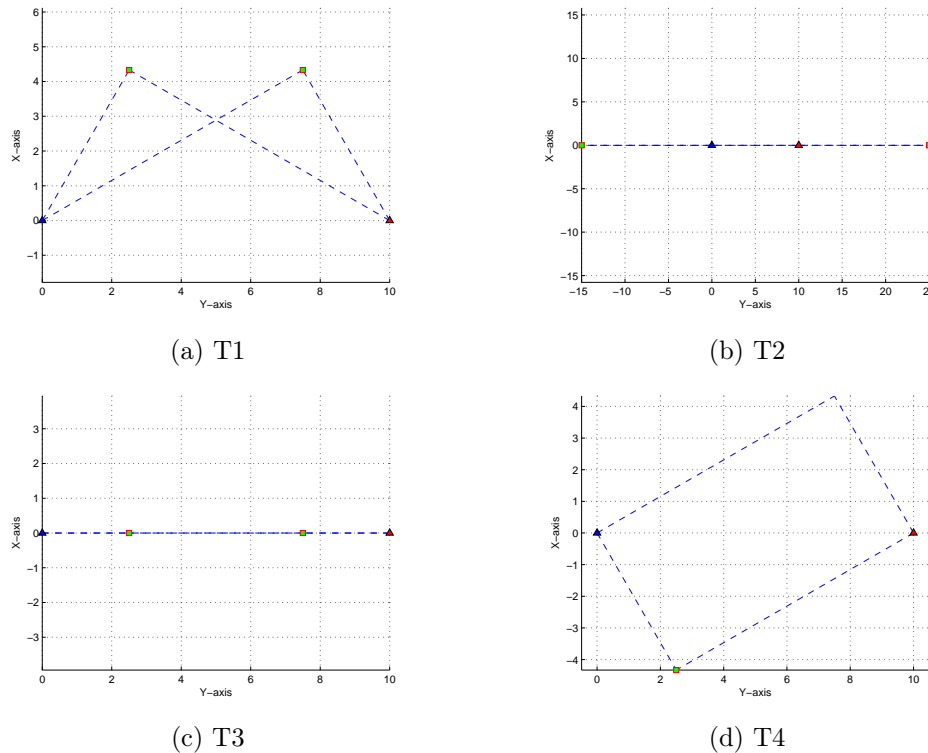


Figure 4.1: Types with two stations, two targets

### 4.1.1 The case with two targets

A network with only one target is not enough for the simulation instead a test with networks containing two targets and two stations is carried out. The result can be seen in Table 4.2. In this simulation four different networks are tested. T1 and T4 are defined on the same rule for the targets as in S1 and S2, the difference between them is that T4 have one target on each side of the stations and T1 on have both targets on the same side. T3 and T2 are defined on the same principle as S3 and S4, for T3 the targets are on a straight line in between the stations and for T2 the targets are on a straight line on either side of the stations.

Table 4.2: Result for standard deviation, two stations, two targets.

Type	Distance (mm)	Horizontal angle (mgon)	Vertical angle (mgon)
T1	0,216	0,144	0,237
T2	0,162	0,184	0,217
T3	4.198	1,487	3,206
T4	0,232	0.146	0.230

The result from this test shows that placing the targets on a straight line in between the stations as in T3 is far worse than the other networks, however the other networks are compared to each other quite similar in the estimation and compared to the test with only one target all networks except T3 show a far better estimation. This raises the question of how much the geometry of the network affect the result compared to the number of targets.

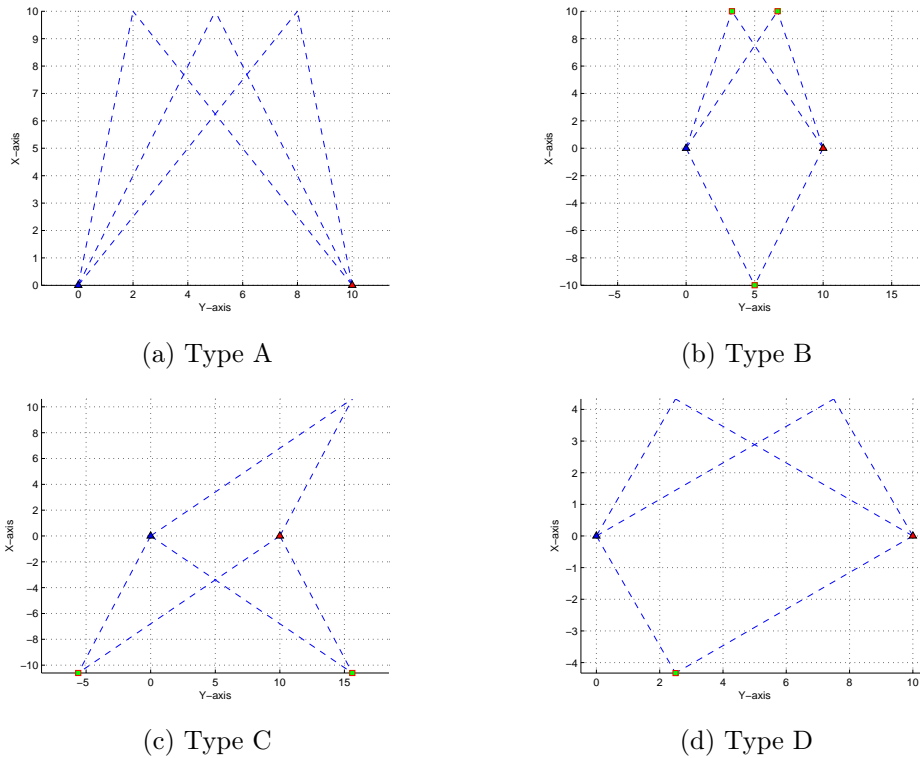


Figure 4.2: Types with two stations, three targets

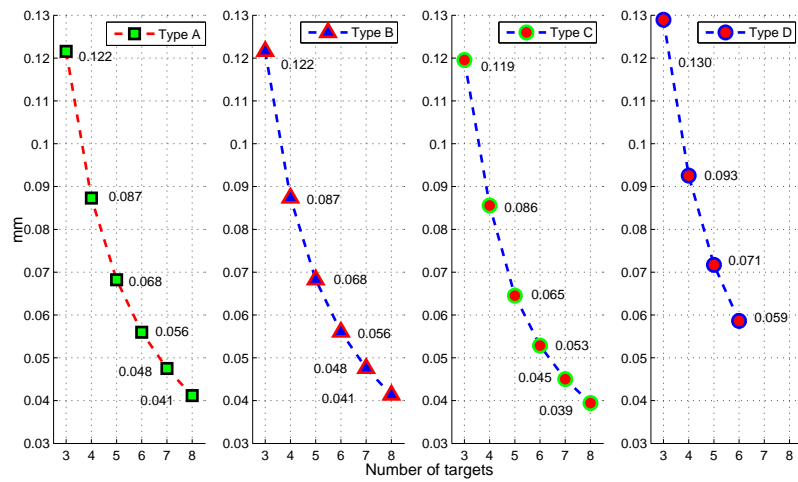
## 4.2 Increasing the number of targets

In the previous section a geometry has been shown to be better than the others for the estimation. To try and further improve the estimation more stations or targets or both can be added. When more targets are added the design is altered for each target that is added. To be able to consider them as different types of design and compare them, three main types are defined.

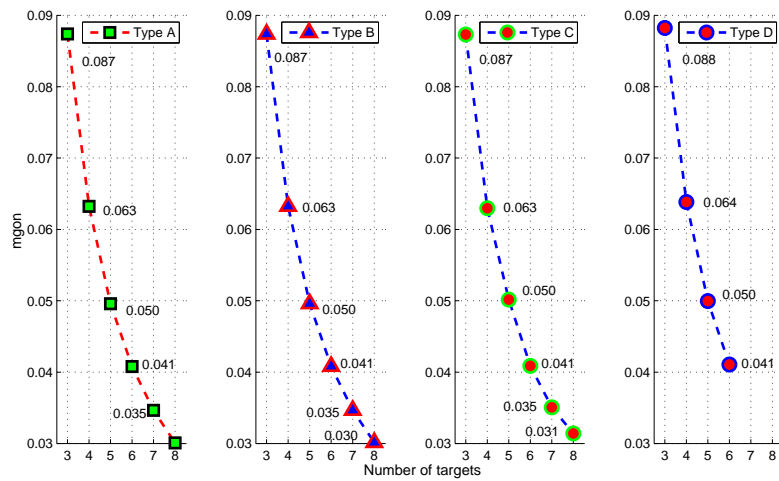
- Type A, all targets are placed on one side of the line that is formed between the stations. Figure 4.2a is an example of a Type A geometry.
- Type B, all targets are placed on two sides of the line that is formed between the stations. Figure 4.2b is an example of a Type B geometry.
- Type C, all targets are spread evenly around the stations on a circle. Figure 4.2c is an example of a Type C geometry.
- A fourth special case to consider Type D, where the design follows the idea from S1/S2/T1/T2. Figure 4.2d is an example of a Type D geometry.

A simulation is made for each type from three targets up to eight targets. For type D there has only been made calculations for up to six targets. And all geometries have the height set to zero for both stations and targets. The result is presented in Figure 4.3a, Figure 4.3b and Figure 4.3c.

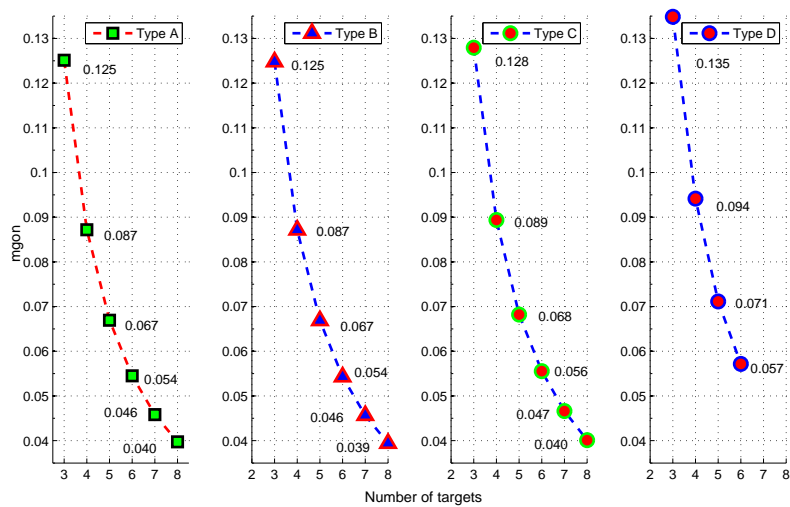
For each type of measurement distance, horizontal direction and zenith direction small differences exists between the different types. For example type C is slightly better than the others for distance and horizontal direction but slightly worse for zenith direction. The general trend of the estimation for each type is similar and the largest influence of the result is the number of targets. More targets gives better estimation for each type and compared to each other. The spread of the targets does not influence as much.



(a) Distance



(b) Horizontal direction



(c) Zenith direction

Figure 4.3: Standard deviation when increasing number of targets

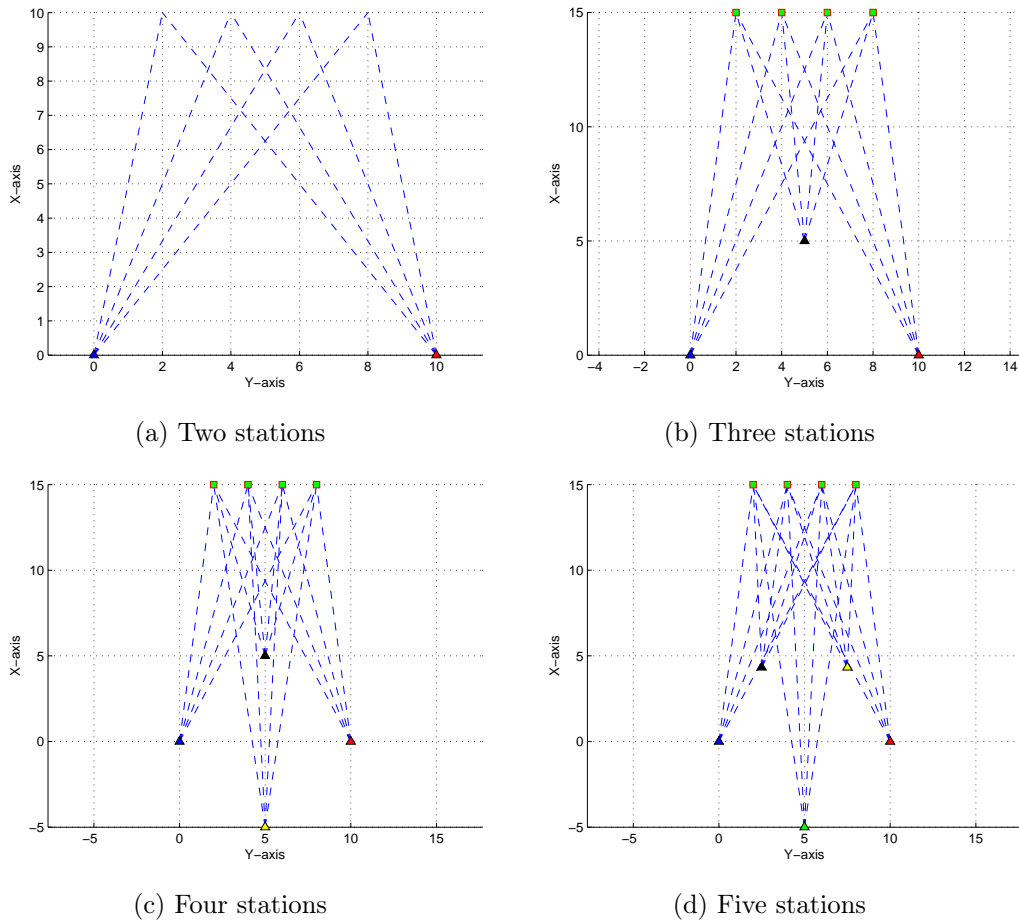
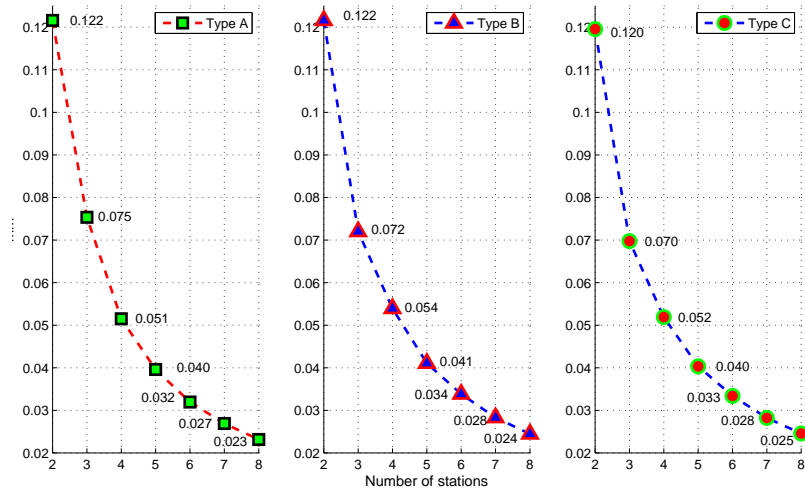


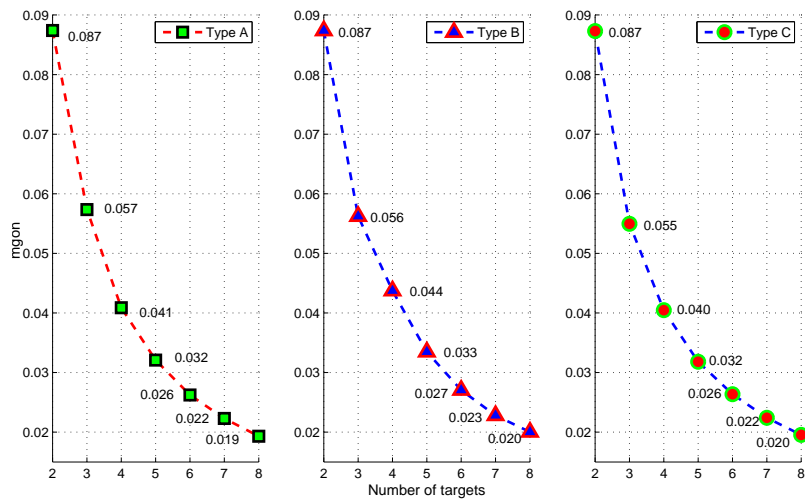
Figure 4.4: Increasing the number of stations

### 4.3 Increasing the number of stations

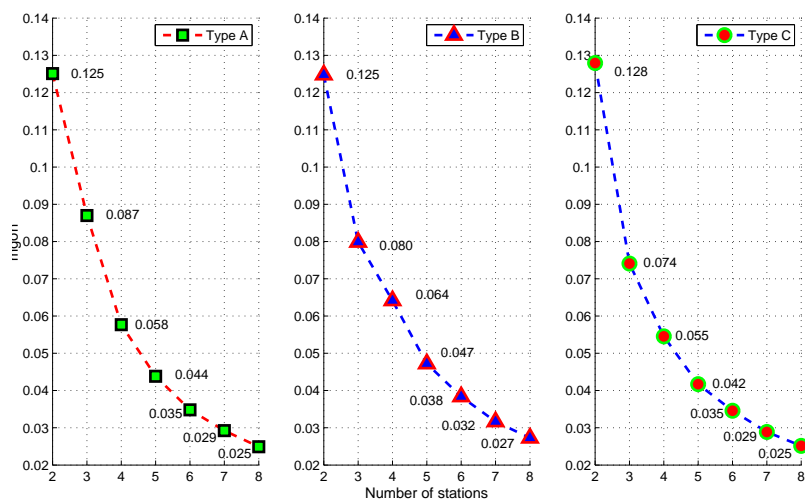
As an alternative to increasing the number of targets, instead the number of stations can be increased. The same design is used with three targets and the stations are placed in the same way for all three designs, they are placed upon the border of a circle with a radius of five. Figure 4.4 shows an example of how stations are added. The result of the calculations is presented in Figure 4.5a, Figure 4.5b and Figure 4.5c. Type D is not considered here because it is not possible to create such geometry with more than two stations. Increasing the number of station set-ups increases the number of observations with the same size as the number of targets since all targets should be observed from the new set-up. This means that for a design with three targets extended with one station set-up, three new observations are carried out. Therefore a large decrement of the size of estimation should be expected. These calculations show a similar trend compared to increasing the number of targets. There exist differences between the types but when the number of observations are increased the differences between the types becomes smaller and the estimation becomes better. As expected a large step between two and three station can be seen for all types and it can be suspected that by adding additional station the estimations standard deviation will decrease but the steps will be smaller and smaller.



(a) Distance



(b) Horizontal direction



(c) Zenith direction

Figure 4.5: Standard deviation when increasing number of stations

## 4.4 Increasing the number of both targets and stations

By comparing the figures for increasing the number of targets with them for increasing the number of stations it can be seen that the result will be similar for those with equal amount of observations. This tells us that the amount of observations is the most important factor. Calculations were done with the possible combinations from three to eight targets with two to eight stations for each of the three types. The result is plotted in a table with the x-axis representing the number of observations in that particular estimation. Examining Figure 4.6a it shows that the amount of observations is the key factor, not if it is stations or targets that are increased. For Type B and type C we get similar results in Figure 4.6b and 4.6c. It also shows that the difference between the types is not of big importance compared to the number of observations. In Appendix C, Figure C.1a, C.1b, C.2a, C.2b, C.3a and C.3b show the result for horizontal direction and vertical direction for each type.

## 4.5 Influence of height difference

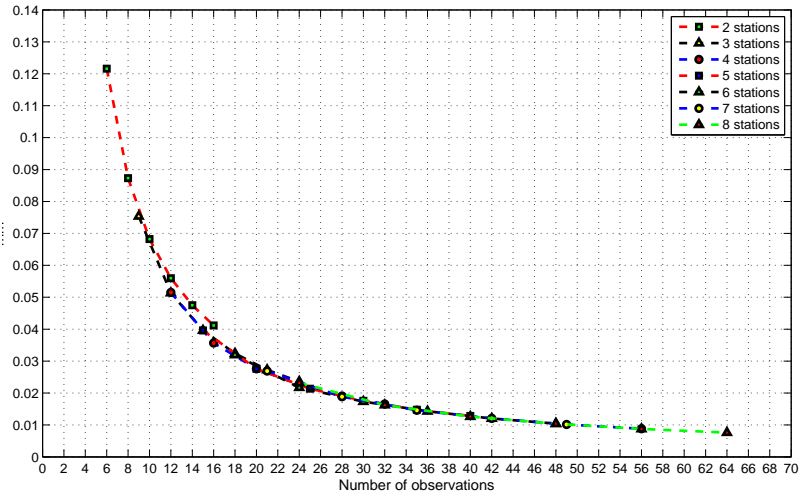
For all the simulations that has been made the spread in height has not been changed. The influence of the Z-coordinate is still of importance, it influences the result of the estimation for both distance and zenith directions. In the calculations carried out all z-coordinates were set to zero. To test the influence of the height difference in between targets and stations, geometries with all z-coordinates set to zero are compared to the same geometry but with changed values on z-coordinates.

For all types with two stations and three targets, in Z1 the height has been set for one of the targets to 2 meters, in Z2 it has been changed to - 2 meter and in Z3 it has been changed to 10 meters. An example of this for type A is shown in Figure 4.7. The result is in Figure 4.8. The effect of different height values on the targets influence the overall estimation.

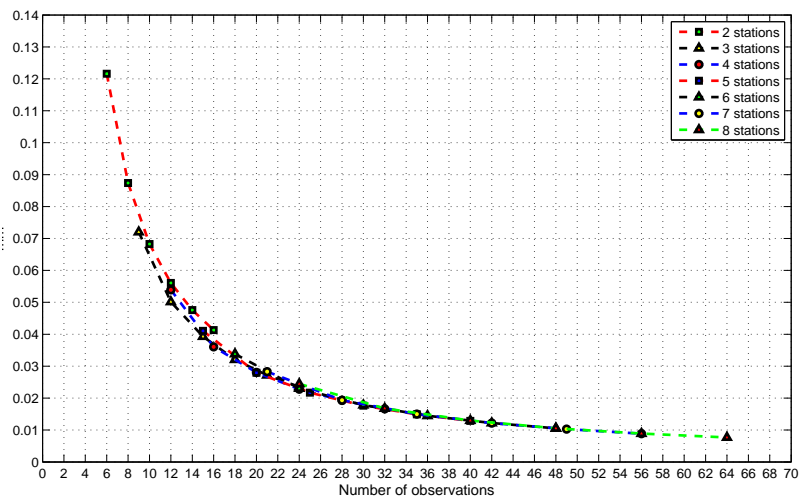
## 4.6 Influence of distance to target

The a priori values of the estimation are defined in such way that we expect the a priori values to become larger when the observed distance becomes larger. This should also effect the estimation, if observed distance is longer the estimation should be larger. This can be tested in the same way as the influence of height difference was tested. The three types with two stations and three targets are altered so that some of the observed distances becomes longer. For D1 the length towards P1 is altered from 10 meter to 50 meters, for D2 the length is altered to 100 meters and for D3 is also P3 altered from 10 meters to 100 meters.

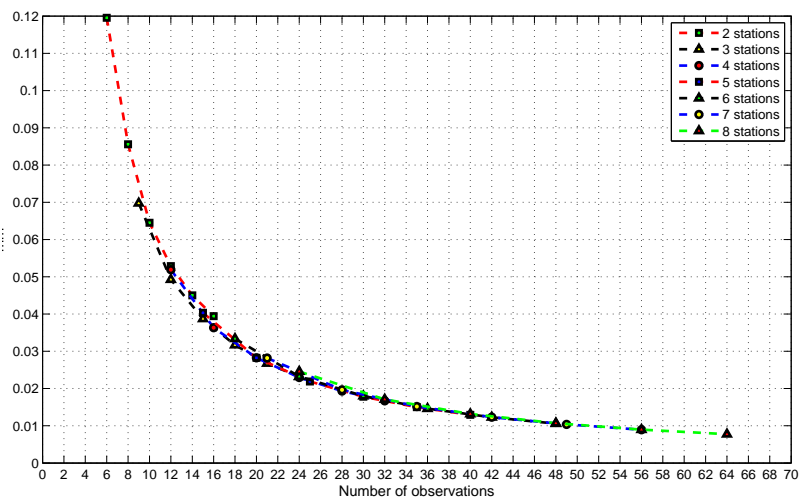
Figure 4.9 compares the different versions for each type. For setups with longer observed distance the estimation for distance is slightly worse than those with a shorter observed distance, except for D3 for type C that actually is slightly better than the original setup. By this alteration of the setups, the angles for both the horizontal direction as well as the zenith direction are affected and that needs to be considered as well.



(a) Distance, for type A



(b) Distance, for type B



(c) Distance, for type C

Figure 4.6: Standard deviation compared to number of observations



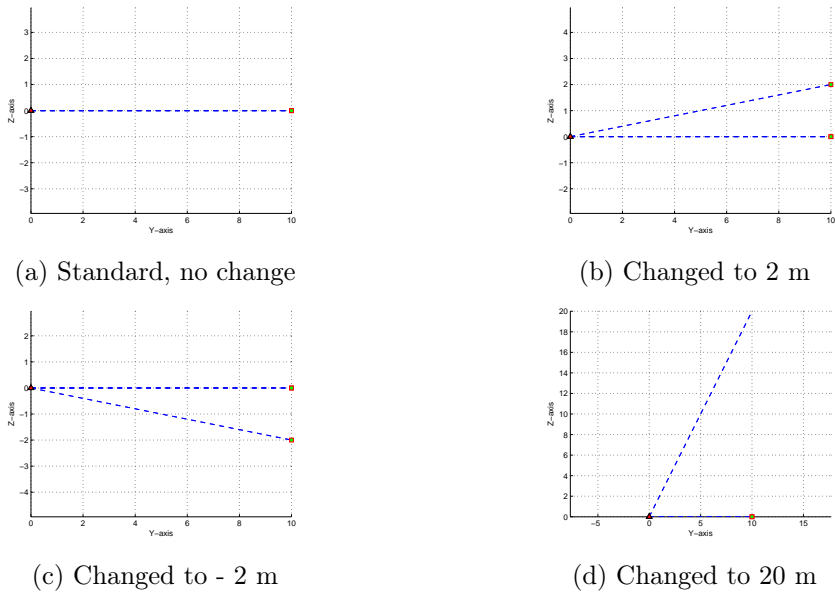


Figure 4.7: Example of change in height of one target

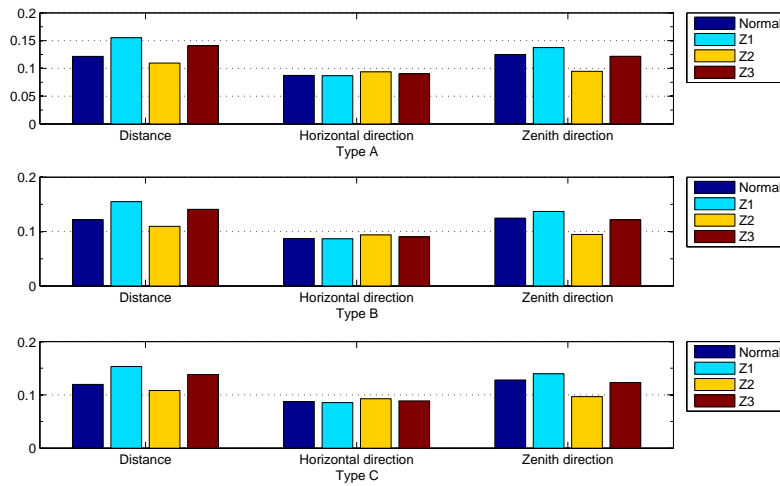


Figure 4.8: Effect of change in height, two stations, three targets

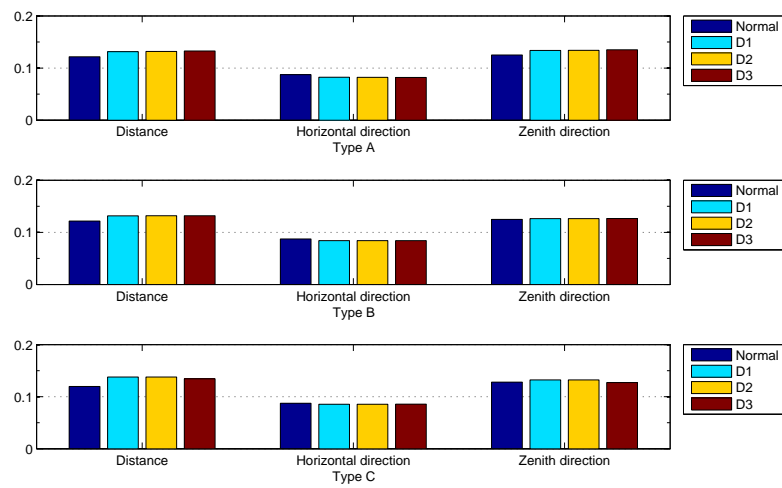


Figure 4.9: Effect of distance change, two stations, three targets

# Chapter 5

## Discussion and analyze

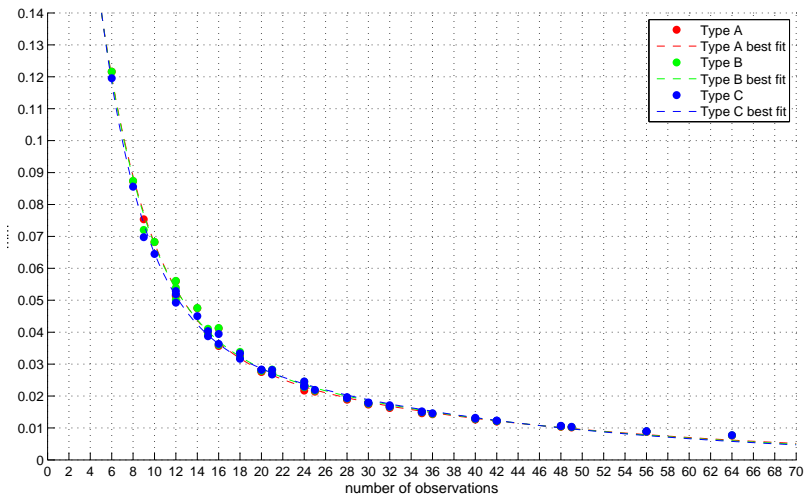
### 5.1 Discussion

There exists infinite numbers of possible setups that can be used for this estimation. In this thesis a few have been simulated to define the importance of certain attributes such as length from station to target, angle between observations, spread of targets, influence of elevation change and number of observations. From the estimations it can be seen that all these attributes to the setup effects the estimation, however most of them are dependent of each other. A change in height will also change the distance between the station and the target, a longer distance from the station to the target will change the angle between the observations from different stations, etc. This dependency makes it difficult to point out a single attributes importance, but it can be seen that for a small amount of estimations, less than three targets and only two stations, the affect of these changes will be larger. As more targets and stations are added the difference between the designs becomes smaller and to distinguish different attributes effect is also harder.

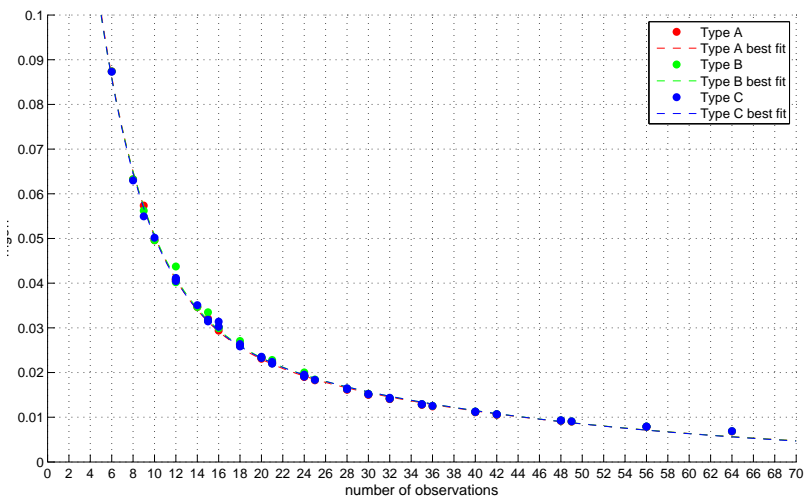
In Figure 5.1a, 5.1b and 5.1c a best fit curve is plotted for standard deviation compared to number of observations for each type (A,B and C) of design. Some alteration in between each type but also within each type due to different number of station and targets, and the effect this have on different attributes, can be seen. It is quite clear that the largest effect on the estimation is the number of observations. More observations gives a better result, independent from the geometry of the setup.

#### 5.1.1 Optimal design

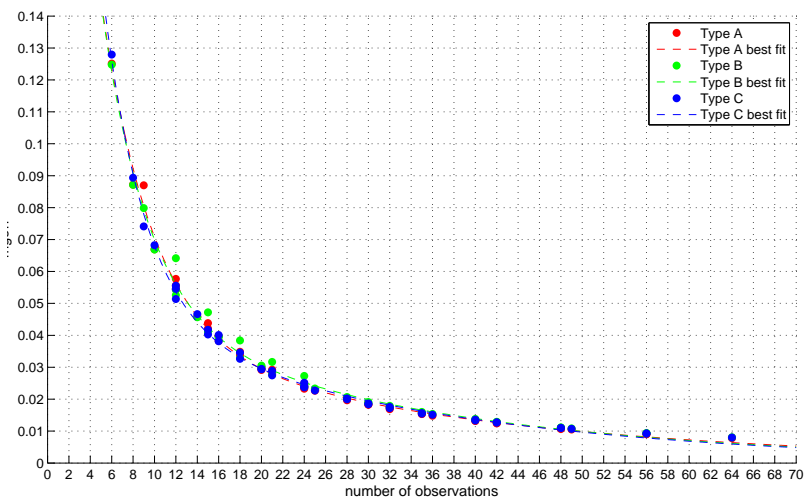
When considering an optimal design for the estimation, the application of the theory to the environment where these measurements will be done needs to be considered. If the work is carried out in a narrow shaft, should the estimation be done in the shaft under those restrictions of possible setups, or should the estimation be carried out before entering the shaft where the possibilities of the setups is less limited, but the environment (light, pressure, temperature) might be different. This is not a question to be answered in this thesis but it is important to point out as something that needs to be considered as well. As stated above the single most important factor to the estimation is the number of observations, more observations gives a better result of the estimation. The geometry of the design is less important. The recommendation for an optimal design is, rather than creating certain angles between observations or a certain minimum distance between stations and targets, to make sure there are a sufficient number of observations done. Another factor to consider is that for each extra measurement needed, more time in the field will be needed to perform the verification. The total station used in the simulations have a priori values as in (3.14), if an estimation should have a standard deviation of one tenth of the expected a priori value then a minimum of 14 observations needs to be measured to have a standard



(a) Distance



(b) Horizontal direction



(c) Zenith direction

Figure 5.1: Standard deviation compared to number of observations for all types

deviation less than 0,045 mgon for the two direction observations. This is equivalent to two stations and seven targets or three stations and four targets. The Figures 5.1a, 5.1b and 5.1c can be used as guides when deciding the number of observations needed for the verification.

## 5.2 Conclusions

For estimation of variance-covariance components with the MINQUE method for verification of standard deviation of a total station the following can be said. The geometry of the network will affect the estimation, the distance between stations and targets, the spread of the targets around the stations and the spread in height both of stations and targets. This is however not as important as the number of observations, with more observations the estimations will be better and in practice it would be easier to increase the number of observations than to try and improve the network in such way that an optimal setup would be created. In practice it would be convenient to use existing backsights, on a construction site for example, there is normally existing backsights with different types of reflectors. The accuracy for reflection of the signal and how it could affect the calculations has not been considered in the calculations. Further work on this topic could be to verify the results in this thesis by field tests and considering the effect of using different types of backsights in the same estimation or a verification of these results by using a different method for estimation of variance components.

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# Appendix A

## Matlab code for calculations

Matlab code used for the calculations.

Listing A.1: Matlab code

```
1 %% Implementation of Minque-estimation
2 % Master Thesis work
3 % Joel Bergkvist
4
5 % clear everything
6 clear all;
7 close all;
8 clc;
9
10 %% Indata
11
12 % Stations coordinats
13 S = [0 0 0;...
14      10 0 -1 ...
15      ];
16
17 % Points coordinats
18 P = [7.5 4.33 2
19      ];
20
21 % A-priorivalues
22 qd = 1;
23 ppm = (1/1000);
24 qhv = 2.5;
25 qvv = 2.5;
26
27 if S < 2
28     error('stationCheck: need more than 1 station')
29 end
30
31 %% A-matrix
32
33 % number of stations
34 nS = size(S,1);
35
36 % number of points
37 nP = size(P,1);
38
39 % locked elements in stations is always 5.
40 lockedElementStation = 5;
41
42 % points is always 1
43 lockedElementPoints = 1;
```



```

44
45 totalLockedElements = lockedElementStation + lockedElementPoints;
46
47 totalFreeElements = (nS + nP)*3 - totalLockedElements;
48
49 % define size A matrix
50 A = zeros(nS*nP*3,totalFreeElements);
51
52 % define size Q matrices
53 Q1 = zeros(nS*nP*3, nS*nP*3);
54 Q2 = zeros(nS*nP*3, nS*nP*3);
55 Q3 = zeros(nS*nP*3, nS*nP*3);
56
57 % calculate all length from stations to points
58 % calculate horizontal distance from stations to points
59 distance = zeros(size(S,1),size(P,1));
60 hd = zeros(size(S,1),size(P,1));
61
62 for i = 1:size(S,1)
63     for j = 1:size(P,1)
64         distance(i,j) = sqrt( ...
65             (P(j,1) - S(i,1))^2 ...
66             + (P(j,2) - S(i,2))^2 ...
67             + (P(j,3) - S(i,3))^2 ...
68             );
69         hd(i,j) = sqrt( ...
70             (P(j,1) - S(i,1))^2 ...
71             + (P(j,2) - S(i,2))^2 ...
72             );
73     end
74 end
75
76 % fill in A matrix
77
78 commutationCount = 0;
79
80 for i = 1:size(S,1)
81     for j = 1:size(P,1)
82
83         % distances
84         distanceRow = 3*commutationCount + 1;
85         a = (P(j,1) - S(i,1))/distance(i,j);
86         b = (P(j,2) - S(i,2))/distance(i,j);
87         c = (P(j,3) - S(i,3))/distance(i,j);
88         d = -a;
89         e = -b;
90         f = -c;
91
92         switch i
93             case 1
94                 if j == 1
95                     % set in distances for points
96                     columnRangeStartP = (size(S,1))*3 - 5 + 1;
97                     columnRangeEndP = columnRangeStartP + 1;
98                     A(distanceRow, columnRangeStartP:columnRangeEndP) = [d e];
99                     Q1(distanceRow, distanceRow) = qd+abs(distance(i,j))*ppm;
100
101                 else
102                     % set in distances for points
103                     columnRangeStartP = 1 + ((size(S,1))*3) + (2 + (j - ...
104                         2)*3) - 5;
104                     columnRangeEndP = columnRangeStartP + 2;
105                     A(distanceRow, columnRangeStartP:columnRangeEndP) = [d e f];

```

```

106         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
107
108     end
109 case 2
110     if j == 1
111
112         % set in distances for stations
113         columnRangeStartStat = 1;
114         columnRangeEndStat = 1;
115         A(distanceRow , columnRangeStartStat:columnRangeEndStat) = ...
116             [a];
117
118         % set in distances for points
119         columnRangeStartP = (size(S,1))*3 - 5 + 1;
120         columnRangeEndP = columnRangeStartP + 1;
121         A(distanceRow , columnRangeStartP:columnRangeEndP) = [d e];
122         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
123
124     else
125         % set in distances for stations
126         columnRangeStartStat = 1;
127         columnRangeEndStat = 1;
128         A(distanceRow , columnRangeStartStat:columnRangeEndStat) = ...
129             [a];
130         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
131
132         % set in distances for points
133         columnRangeStartP = 1 + ((size(S,1))*3) + (2 + (j - ...
134             2)*3) - 5;
135         columnRangeEndP = columnRangeStartP + 2;
136         A(distanceRow , columnRangeStartP:columnRangeEndP) = [d e f];
137         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
138
139     end
140 otherwise
141     if j == 1
142         % set in distances for stations
143         columnRangeStartStat = 1 + (1 + (i - 3)*3);
144         columnRangeEndStat = 2 + columnRangeStartStat;
145         A(distanceRow , columnRangeStartStat:columnRangeEndStat) ..
146             = ...
147             [a b c];
148         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
149
150         % set in distances for points
151         columnRangeStartP = (size(S,1))*3 - 5 + 1;
152         columnRangeEndP = columnRangeStartP + 1;
153         A(distanceRow , columnRangeStartP:columnRangeEndP) = ...
154             [d e];
155         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
156     else
157         % set in distances for stations
158         columnRangeStartStat = 1 + (1 + (i - 3)*3);
159         columnRangeEndStat = 2 + columnRangeStartStat;
160         A(distanceRow , columnRangeStartStat:columnRangeEndStat) ..
161             = ...
162             [a b c];
163         Q1(distanceRow , distanceRow)= qd+abs(distance(i , j))*ppm;
164
165         % set in distances for points
166         columnRangeStartP = ...

```

```

163         1 + ((size(S,1))*3) + (2 + (j - 2)*3) - 5;
164         columnRangeEndP = columnRangeStartP + 2;
165         A(distanceRow , columnRangeStartP : columnRangeEndP) = ...
           [d e f];
166         Q1(distanceRow , distanceRow)= qd+abs(distance(i,j))*ppm;
167         end
168     end
169
170
171     % horizontal angles
172     hangleRow = 3*commutationCount + 2;
173     hangleA = (P(j,2) - S(i,2))/hd(i,j)^2;
174     hangleB = (P(j,1) - S(i,1))/hd(i,j)^2;
175     hangleC = 0;
176     hangleD = -hangleA;
177     hangleE = -hangleB;
178     hangleF = 0;
179
180     switch i
181     case 1
182         if j == 1
183             % set in horizontal angles for points
184             columnRangeStartP = (size(S,1))*3 - 5 + 1;
185             columnRangeEndP = columnRangeStartP + 1;
186             A(hangleRow , columnRangeStartP : columnRangeEndP) = ...
               [hangleD hangleE];
187             Q2(hangleRow , hangleRow)= qhv;
188
189         else
190             % set in horizontal angles for points
191             columnRangeStartP = 1 + ((size(S,1))*3) + (2 + (j - ...
192               2)*3) - 5;
193             columnRangeEndP = columnRangeStartP + 2;
194             A(hangleRow , columnRangeStartP : columnRangeEndP) = ...
               [hangleD hangleE hangleF];
195             Q2(hangleRow , hangleRow)= qhv;
196
197         end
198     case 2
199         if j == 1
200
201             % set in horizontal angles for stations
202             columnRangeStartStat = 1;
203             columnRangeEndStat = 1;
204             A(hangleRow , columnRangeStartStat : columnRangeEndStat) = ...
               [hangleA];
205             Q2(hangleRow , hangleRow)= qhv;
206
207
208
209             % set in horizontal angles for points
210             columnRangeStartP = (size(S,1))*3 - 5 + 1;
211             columnRangeEndP = columnRangeStartP + 1;
212             A(hangleRow , columnRangeStartP : columnRangeEndP) = ...
               [hangleD hangleE];
213             Q2(hangleRow , hangleRow)= qhv;
214
215
216         else
217             % set in horizontal angles for stations
218             columnRangeStartStat = 1;
219             columnRangeEndStat = 1;
220             A(hangleRow , columnRangeStartStat : columnRangeEndStat) = ...
               [hangleA];
221             Q2(hangleRow , hangleRow)= qhv;

```

```

222
223
224     % set in horizontal angles for points
225     columnRangeStartP = 1 + ((size(S,1))*3) + (2 + (j - ...
226         2)*3) - 5;
227     columnRangeEndP = columnRangeStartP + 2;
228     A(hangleRow, columnRangeStartP:columnRangeEndP) = ...
229         [hangleD hangleE hangleF];
230     Q2(hangleRow, hangleRow)= qhv;
231
232     end
233 otherwise
234
235     if j == 1
236         % set in horizontal angles for stations
237         columnRangeStartStat = 1 + (1 + (i - 3)*3);
238         columnRangeEndStat = 2 + columnRangeStartStat;
239         A(hangleRow, columnRangeStartStat:columnRangeEndStat) ...
240             = ...
241             [hangleA hangleB hangleC];
242         Q2(hangleRow, hangleRow)= qhv;
243
244         % set in horizontal angles for points
245         columnRangeStartP = (size(S,1))*3 - 5 + 1;
246         columnRangeEndP = columnRangeStartP + 1;
247         A(hangleRow, columnRangeStartP:columnRangeEndP) = ...
248             [hangleD hangleE];
249         Q2(hangleRow, hangleRow)= qhv;
250     else
251         % set in horizontal angles for stations
252         columnRangeStartStat = 1 + (1 + (i - 3)*3);
253         columnRangeEndStat = 2 + columnRangeStartStat;
254         A(hangleRow, columnRangeStartStat:columnRangeEndStat) ...
255             = ...
256             [hangleA hangleB hangleC];
257         Q2(hangleRow, hangleRow)= qhv;
258
259         % set in horizontal angles for points
260         columnRangeStartP = ...
261             1 + ((size(S,1))*3) + (2 + (j - 2)*3) - 5;
262         columnRangeEndP = columnRangeStartP + 2;
263         A(hangleRow, columnRangeStartP:columnRangeEndP) = ...
264             [hangleD hangleE hangleF];
265         Q2(hangleRow, hangleRow)= qhv;
266     end
267
268     end
269
270     % vertical angles
271     vangleRow = 3*commutationCount + 3;
272     vangleA = (P(j,1) - S(i,1))*(P(j,3) - ...
273         S(i,3))/distance(i,j)^2*hd(i,j);
274     vangleB = (P(j,2) - S(i,2))*(P(j,3) - ...
275         S(i,3))/distance(i,j)^2*hd(i,j);
276     vangleC = hd(i,j)/distance(i,j)^2;
277     vangleD = -vangleA;
278     vangleE = -vangleB;
279     vangleF = -vangleC;
280
281     switch i
282     case 1
283         if j == 1
284             % set in vertical angles for points

```

```

280     columnRangeStartP = (size(S,1))*3 - 5 + 1;
281     columnRangeEndP = columnRangeStartP + 1;
282     A(vangleRow , columnRangeStartP:columnRangeEndP) = ...
283         [vangleD vangleE];
284     Q3(vangleRow , vangleRow)= qvv;
285
286 else
287     % set in vertical angles for points
288     columnRangeStartP = 1 + ((size(S,1))*3) + (2 + (j - ...
289         2)*3) - 5;
290     columnRangeEndP = columnRangeStartP + 2;
291     A(vangleRow , columnRangeStartP:columnRangeEndP) = ...
292         [vangleD vangleE vangleF];
293     Q3(vangleRow , vangleRow)= qvv;
294
295 end
296 case 2
297     if j == 1
298
299         % set in vertical angles for stations
300         columnRangeStartStat = 1;
301         columnRangeEndStat = 1;
302         A(vangleRow , columnRangeStartStat:columnRangeEndStat) = ...
303             [vangleA];
304         Q3(vangleRow , vangleRow)= qvv;
305
306     % set in vertical angles for points
307     columnRangeStartP = (size(S,1))*3 - 5 + 1;
308     columnRangeEndP = columnRangeStartP + 1;
309     A(vangleRow , columnRangeStartP:columnRangeEndP) = ...
310         [vangleD vangleE];
311     Q3(vangleRow , vangleRow)= qvv;
312
313 else
314     % set in vertical angles for stations
315     columnRangeStartStat = 1;
316     columnRangeEndStat = 1;
317     A(vangleRow , columnRangeStartStat:columnRangeEndStat) = ...
318         [vangleA];
319     Q3(vangleRow , vangleRow)= qvv;
320
321     % set in vertical angles for points
322     columnRangeStartP = 1 + ((size(S,1))*3) + (2 + (j - ...
323         2)*3) - 5;
324     columnRangeEndP = columnRangeStartP + 2;
325     A(vangleRow , columnRangeStartP:columnRangeEndP) = ...
326         [vangleD vangleE vangleF];
327     Q3(vangleRow , vangleRow)= qvv;
328
329 end
330 otherwise
331     if j == 1
332         % set in vertical angles for stations
333         columnRangeStartStat = 1 + (1 + (i - 3)*3);
334         columnRangeEndStat = 2 + columnRangeStartStat;
335         A(vangleRow , columnRangeStartStat:columnRangeEndStat) ...
336             = ...
337             [vangleA vangleB vangleC];
338         Q3(vangleRow , vangleRow)= qvv;

```

```

338         % set in vertical angles for points
339         columnRangeStartP = (size(S,1))*3 - 5 + 1;
340         columnRangeEndP = columnRangeStartP + 1;
341         A(vangleRow , columnRangeStartP : columnRangeEndP) = ...
342             [vangleD vangleE];
343         Q3(vangleRow , vangleRow)= qvv;
344     else
345         % set in vertical angles for stations
346         columnRangeStartStat = 1 + (1 + (i - 3)*3);
347         columnRangeEndStat = 2 + columnRangeStartStat;
348         A(vangleRow , columnRangeStartStat : columnRangeEndStat) ...
349             = ...
350             [vangleA vangleB vangleC];
351         Q3(vangleRow , vangleRow)= qvv;
352     % set in vertical angles for points
353     columnRangeStartP = ...
354         1 + ((size(S,1))*3) + (2 + (j - 2)*3) - 5;
355     columnRangeEndP = columnRangeStartP + 2;
356     A(vangleRow , columnRangeStartP : columnRangeEndP) = ...
357         [vangleD vangleE vangleF];
358     Q3(vangleRow , vangleRow)= qvv;
359     end
360 end
361
362     commutationCount = commutationCount + 1;
363 end
364 end
365
366 Q0 = Q1+Q2+Q3;
367 Q = {Q1 Q2 Q3};
368
369 %% Calculating variance-covariance components
370
371 R0=inv(Q0) - inv(Q0)*A*inv(A'*inv(Q0)*A)*A'*inv(Q0);
372
373 Sij = zeros(3, 3);
374
375 for i=1:3
376     for j=1:3
377         Sij(i , j)=trace(R0*Q{i}*R0*Q{j});
378     end
379 end
380
381 %Estimation of standard deviation of variance-components (indipendet ...
382     of observation vector)
383
384 V=inv(Sij '* Sij);
385
386 sdist = sqrt(V(1,1)); %Standard deviations for distance
387 shv = sqrt(V(2,2)); %Standard deviations for horizontal angle
388 svv = sqrt(V(3,3)); %Standard deviations for vertical angle
389
390 %% Output
391
392 sprintf('Standard error of the estimated variance-components \n ...
393         Distance %0.10f, Horizontal Angle %0.10f, Vertical angle %0.10f', ...
394         sdist , shv , svv)
395
396 %% Plot
397 hold on

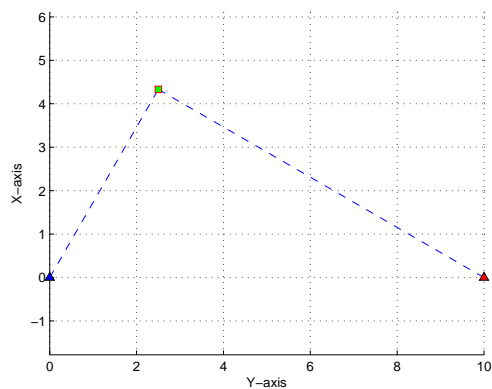
```

```
397 grid on
398 axis equal
399
400 L1X = [S(1,1); P(1,1)];
401 L1Y = [S(1,2); P(1,2)];
402 L2X = [S(2,1); P(1,1)];
403 L2Y = [S(2,2); P(1,2)];
404
405 plot(L1X, L1Y, '--');
406 plot(L2X, L2Y, '--');
407
408 for i=1:nS
409     plot(S(i,1),S(i,2), 'o', 'Markeredgecolor','k','Markerfacecolor','b')
410 end
411 for i=1:nP
412     plot(P(i,1),P(i,2), 's', 'Markeredgecolor','r','Markerfacecolor','g')
413 end
```

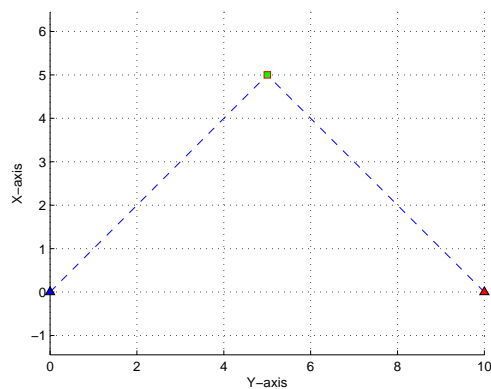
# Appendix B

## Geometry of setups

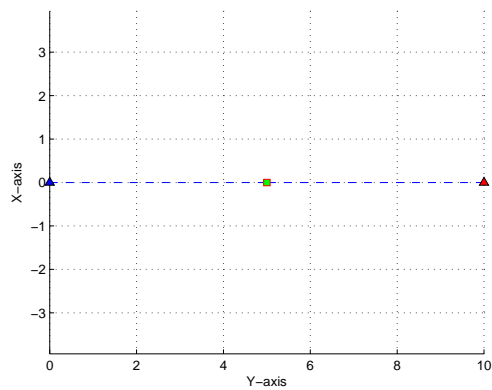
Simulated setups not previously shown. For setups with more than two stations, only the principal for adding more station to the setup is presented here.



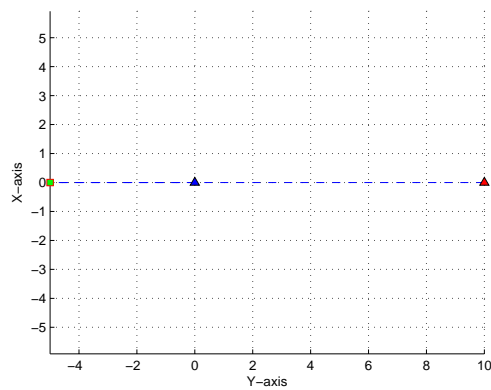
(a) S1



(b) S2



(c) S3



(d) S4

Figure B.1: Types with two stations, one targets



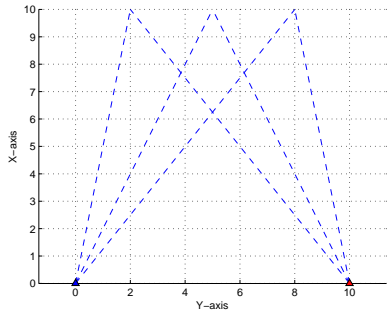


Figure B.2: Type A, two stations, three targets

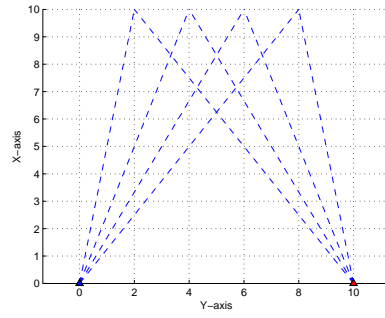


Figure B.3: Type A, two stations, four targets

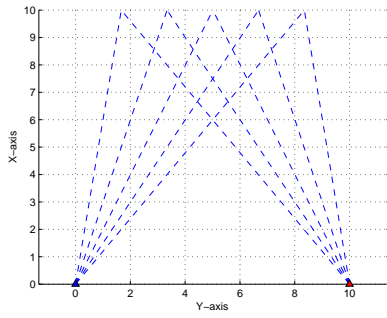


Figure B.4: Type A, two stations, five targets

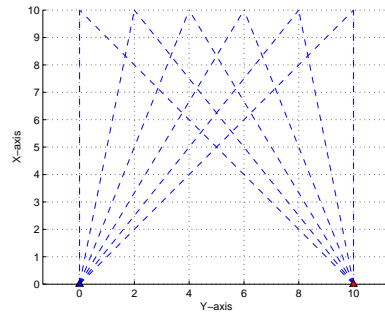


Figure B.5: Type A, two stations, six targets

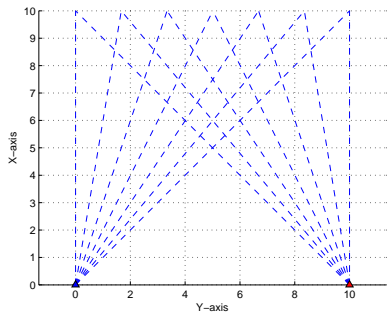


Figure B.6: Type A, two stations, seven targets

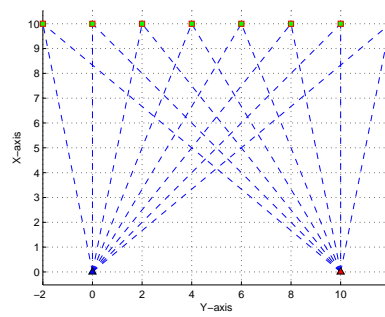


Figure B.7: Type A, two stations, eight targets

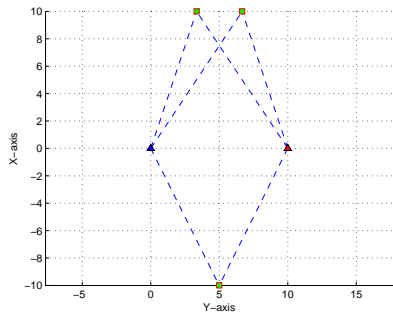


Figure B.8: Type B, two stations, three targets

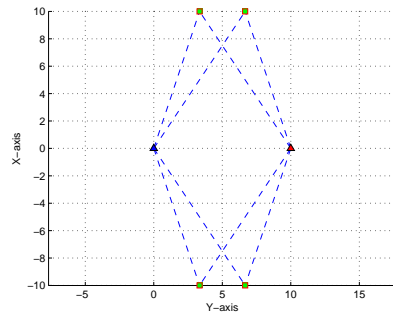


Figure B.9: Type B, two stations, four targets

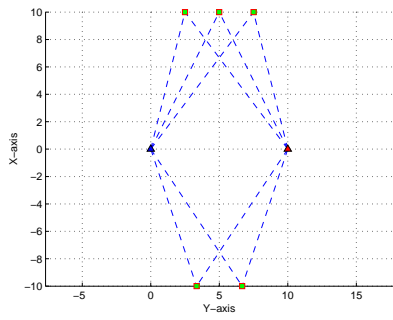


Figure B.10: Type B, two stations, five targets

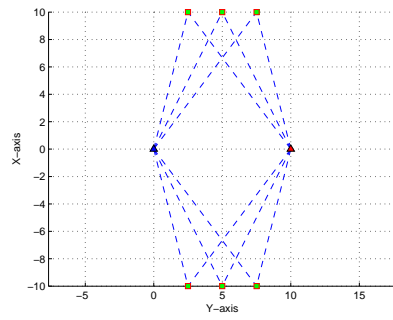


Figure B.11: Type B, two stations, six targets

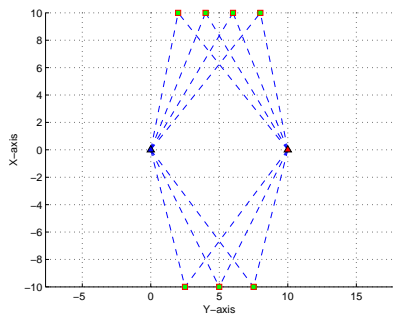


Figure B.12: Type B, two stations, seven targets

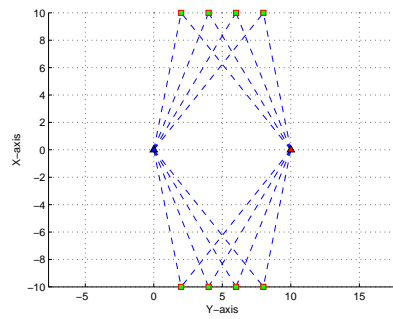


Figure B.13: Type B, two stations, eight targets

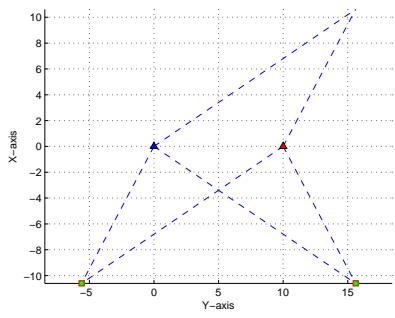


Figure B.14: Type C, two stations, three targets

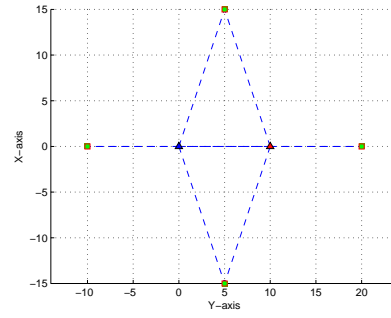


Figure B.15: Type C, two stations, four targets

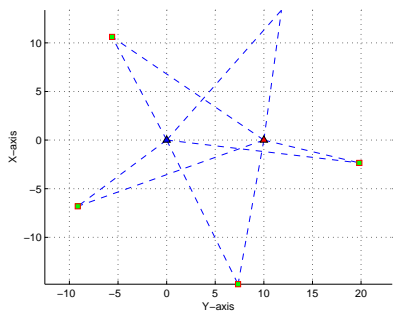


Figure B.16: Type C, two stations, five targets

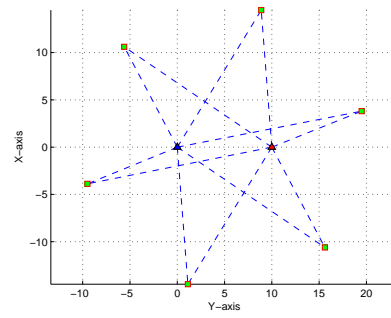


Figure B.17: Type C, two stations, six targets

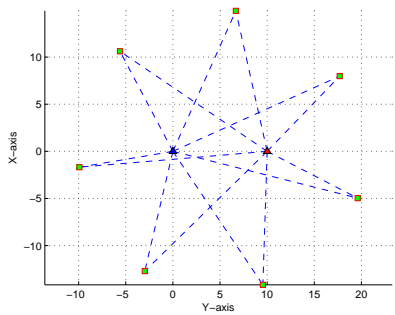


Figure B.18: Type C, two stations, seven targets

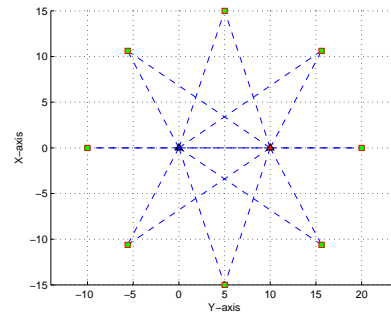


Figure B.19: Type C, two stations, eight targets

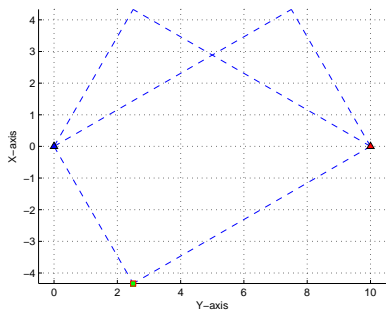


Figure B.20: Type D, two stations, three targets

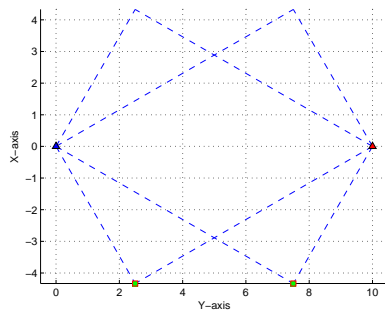


Figure B.21: Type D, two stations, four targets

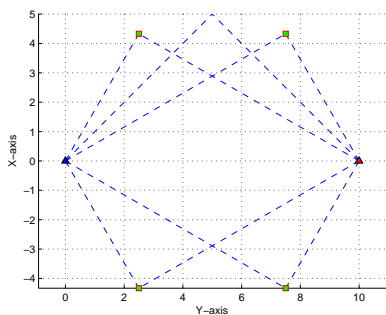


Figure B.22: Type D, two stations, five targets

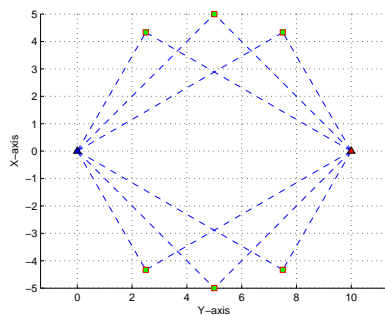


Figure B.23: Type D, two stations, six targets

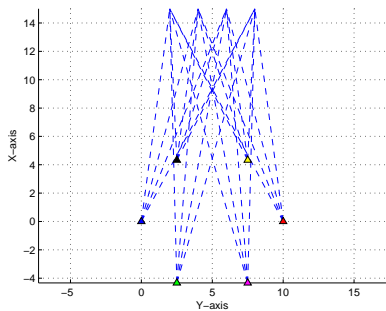


Figure B.24: Type A, six stations, four targets

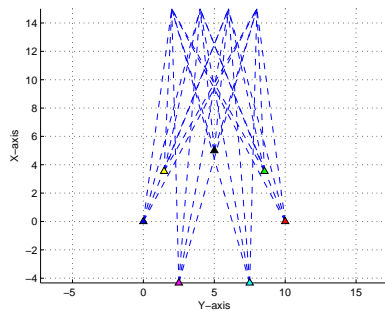


Figure B.25: Type A, seven stations, four targets

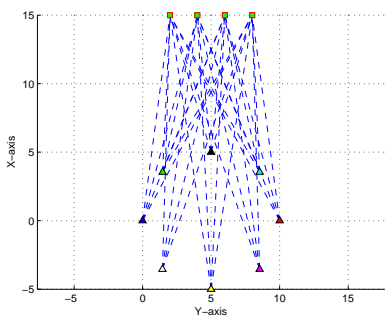
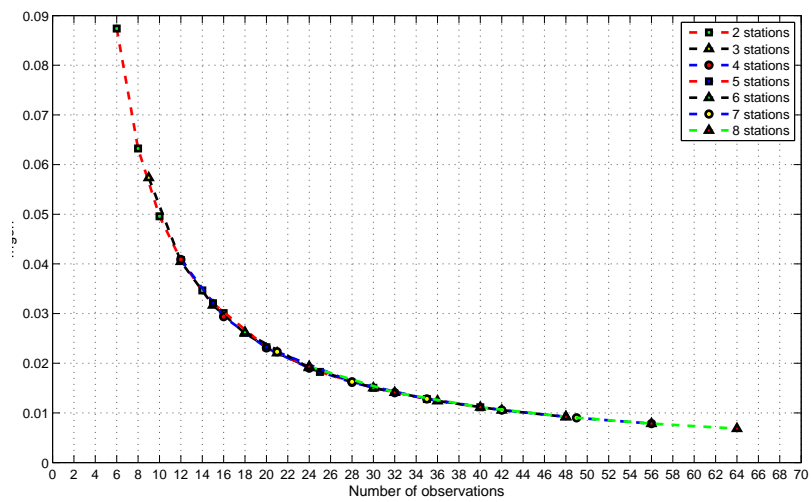


Figure B.26: Type A, eight stations, four targets

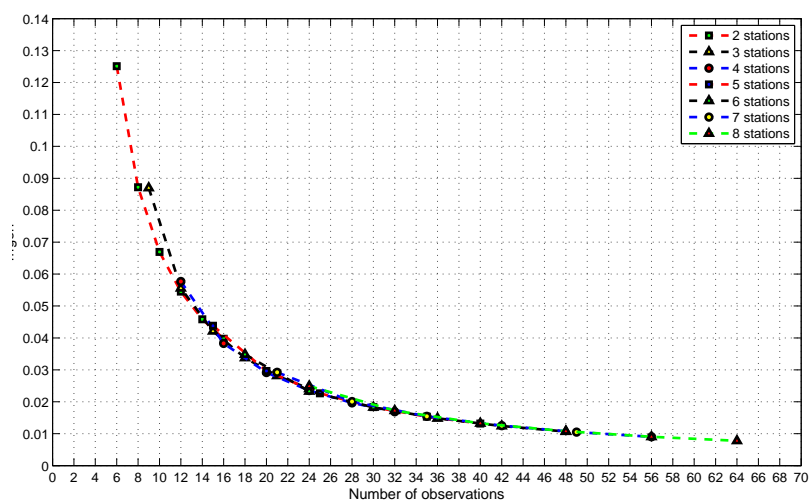
# Appendix C

## Figures

Figures not presented previously.

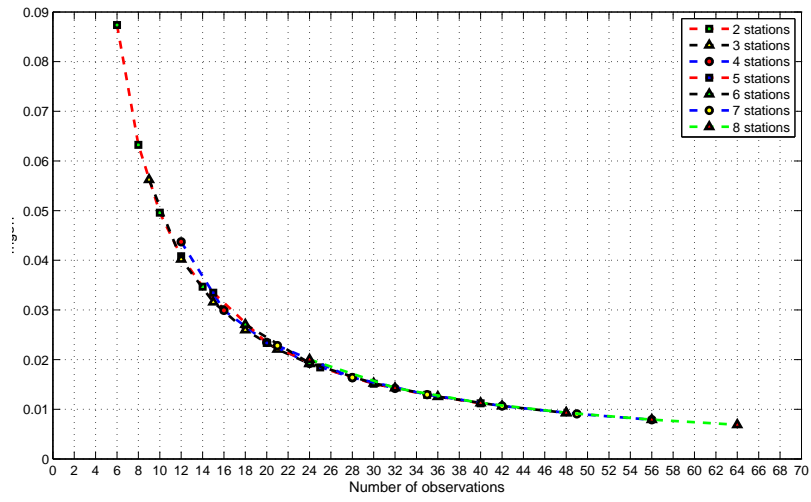


(a) Horizontal direction, for type A

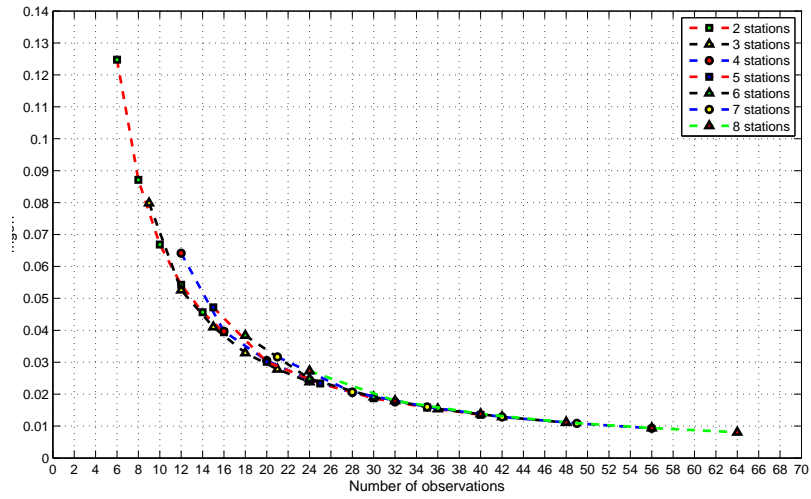


(b) Zenith direction, for type A

Figure C.1: Standard deviation compared to number of observation

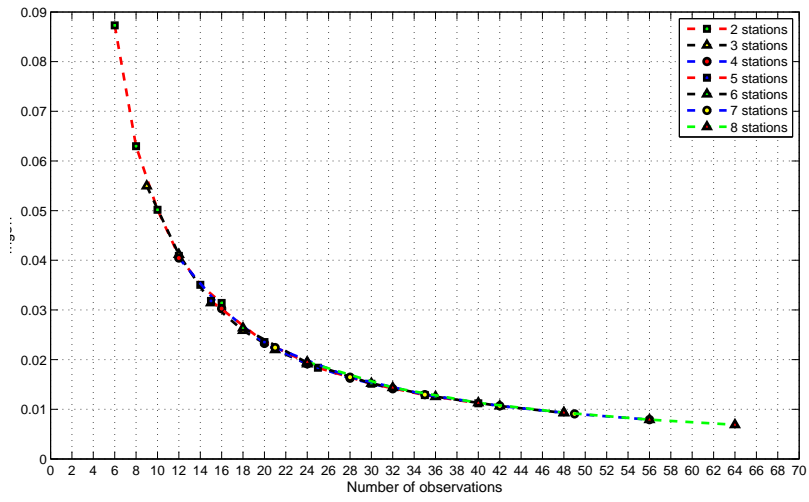


(a) Horizontal direction, for type B

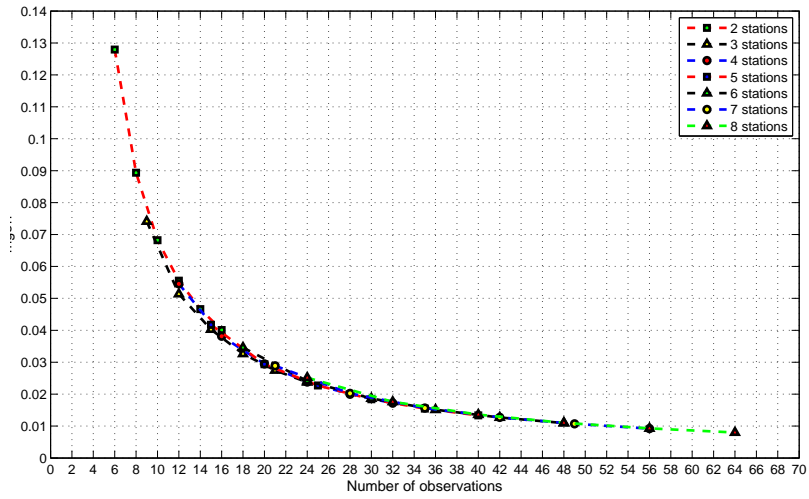


(b) Zenith direction, for type B

Figure C.2: Standard deviation compared to number of observation



(a) Horizontal direction, for type C



(b) Zenith direction, for type C

Figure C.3: Standard deviation compared to number of observation



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