Deformation analysis of a GNSS network through global congruence test based on theory of generalized likelihood ratio test

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Introduction

- GCT (Global Congruence Test)
  - In deformation analysis the identification of stable point take key role
  - The global congruency test is more widely accepted.
  - Algorithm of global congruency test are complex (programing)
  - Nowel 2018, presents *"A rigorous and, at the same time, a user-friendly algorithm for GCT"*
  - The algorithm is based on the theory of generalized likelihood ratio testing

Objetive

- Implement the Nowel algorithm for GCT (Nowell 2018)

Deformation analysis - Global Congruence Test (GCT)

Deformation vector:

$$\hat{d} = x_2 - x_1$$

Hypothesis testing

$$H_0: \hat{d} = 0$$
$$H_A: \hat{d} \neq 0$$

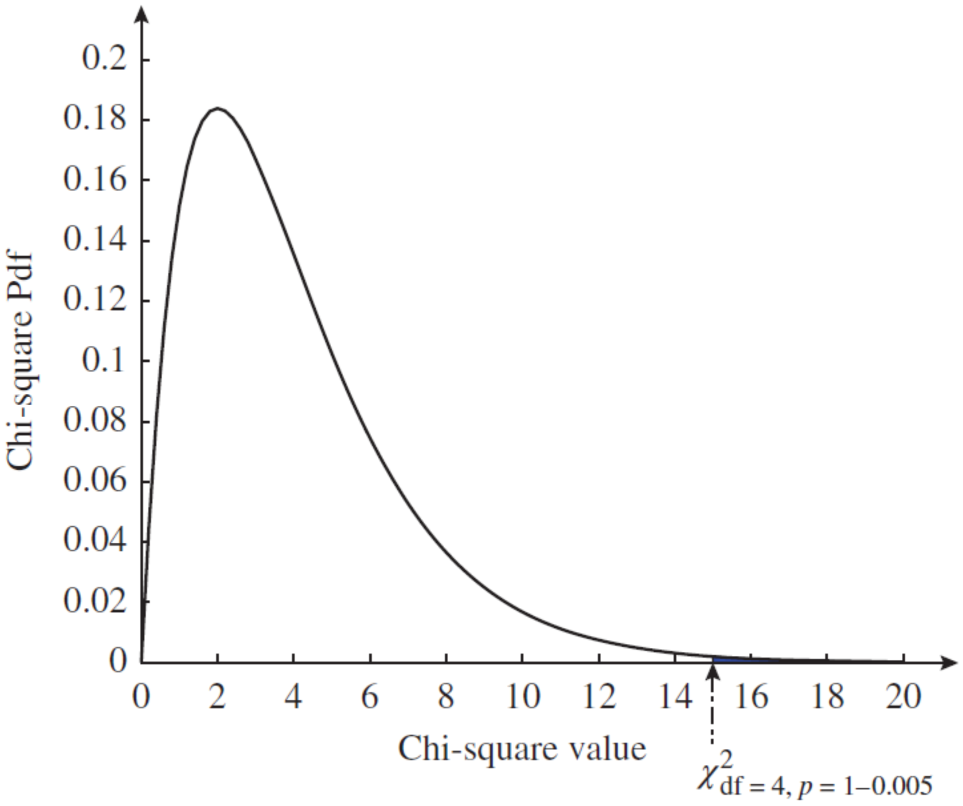
Test statistic

$$\Phi = \frac{\hat{d}^t \cdot Q_{\hat{d}}{}^{+} \cdot \hat{d}}{\sigma_0^2} \sim (\chi^2{}_{h,1-\alpha})$$

$$Q_{\hat{d}} = Q_{\hat{x}1} + Q_{\hat{x}2}$$

Localization

$$\Phi_i = \frac{\hat{d}_i^t \cdot Q_{\hat{d}_i}{}^{+} \cdot \hat{d}_i}{\sigma_0^2} \sim (\chi^2{}_{h,1-\alpha}\ )$$



Significance level α – Type I error probability – *"rejecting the null hypothesis when it is in fact true"*

Sensitivity analysis

$$H_A: E(d) \neq 0 = \Delta$$

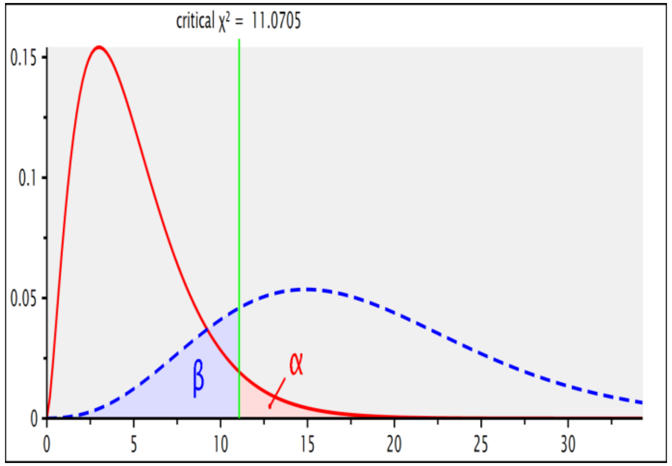
Δ expected deformation vector

Test statistic (Global sensitivity)

$$\Phi = \frac{\hat{d}^t \cdot C_{\hat{d}}^{\ +} \cdot \hat{d}}{\sigma_0^2} \sim (\chi^2_{\lambda_{0(h,\alpha_0,\beta_0)}})$$

Localization (Local sensitivity or MDD)

$$\Phi_i = \frac{\hat{d}_i^t \cdot C_{\hat{d}_i}^{\ +} \cdot \hat{d}_i}{\sigma_0^2} \sim (\chi^2_{\lambda_{0(h,\alpha_0,\beta_0)}})$$



h	$\gamma_0 = 80\%$			$\gamma_0 = 90\%$		
	$\alpha_0 = 5\%$	$\alpha_0 = 1\%$	$\alpha_0 = 0.1\%$	$\alpha_0 = 5\%$	$\alpha_0 = 1\%$	$\alpha_0 = 0.1\%$
1	7.849	11.679	17.075	10.507	14.879	20.904
2	9.635	13.881	19.662	12.654	17.427	23.817
3	10.903	15.458	21.545	14.172	19.247	25.935
4	11.935	16.749	23.100	15.405	20.737	27.683
5	12.828	17.869	24.456	16.470	22.028	29.206
10	16.241	22.177	29.711	20.532	26.982	35.102
20	20.961	28.162	37.073	26.132	33.852	43.347
30	24.547	32.720	42.700	30.379	39.074	49.641
40	27.557	36.550	47.436	33.940	43.459	54.934
50	30.204	39.919	51.605	37.069	47.312	59.590
100	40.556	53.103	67.942	49.293	62.378	77.817

Significance level α_0 –Type I error probability

β_0 - the pre-set type II error probability (the testing fails to reject a null hypothesis which is really false)

Power of test, given by $\gamma_0 = 1 - \beta_0$, probability that the test correctly rejects the null hypothesis

Global sensitivity

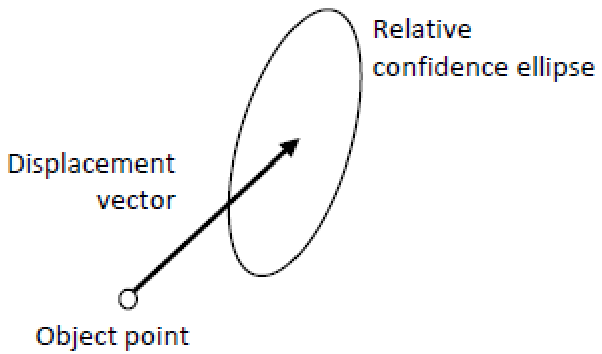
Displacement vector:

$$\Delta = b \cdot g \quad \begin{cases} b\text{: Scalar} \\ g\text{: Direction vector} \end{cases}$$

Minimum detectable displacement vector

$$\Delta_{min} = b_{min} \cdot g$$

$$b_{min} = \sqrt{\frac{\lambda_{0(h,\alpha_0,\beta_0)}}{\lambda_{max}}} \qquad b_{max} = \sqrt{\frac{\lambda_{0(h,\alpha_0,\beta_0)}}{\lambda_{min}}}$$



The maximum eigen value λ_{max} (g) of $C_{\hat{\Delta}}{}^{+}$ and their eigen-vector Λ_{max} → b_{min}

The minimum eigen value λ_{min} (g) da matriz $C_{\hat{\Delta}}{}^{+}$ and their eigen-vector Λ_{min} → b_{max}

Global Congruence Test (GCT) – Nowel Approach (implicit)

- The H-method → Hanover University
- The K-method → Karlsruhe method
- Testing Whole Network-Method I
- **A new algorithm → based on theory of generalized likelihood ratio test (GLRT)**
  - Some discrepancies between the observations and their functional model are considered.
    - These discrepancies can have many different causes,
      - Could be caused by mistakes made in the functional model.
    - **Such mistakes can be also considered in geodetic deformation analysis**

Nowel Global Congruence Test (GCT) approach

Hypothesis testing

$$H_0: E(y^{obs}) = \mathrm{A} \cdot \mathrm{x}$$

$$H_\mathrm{A}: E(y^{obs}) = \mathrm{A} \cdot \mathrm{x} + \mathrm{B} \cdot \nabla$$

Alternative hypothesis

$$H_\mathrm{A}: E\left(\begin{bmatrix} y_1^{obs} \\ y_2^{obs} \end{bmatrix}\right) = \left[\begin{bmatrix} A_1 & 0 \\ \underbrace{A_2}_{A} & \underbrace{A_2}_{B} \end{bmatrix}\right] \begin{bmatrix} x \\ d \end{bmatrix}$$

Test Statistics

$$T = \frac{q_\Delta}{r\hat{\sigma}_0^2} \leq F(\alpha, r, f)$$

$$q_\Delta = q_0 - q_\Delta \qquad e_0 = y^{obs} - \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \cdot x$$

$$q_0 = \hat{e}_0^T Q_{y^{obs}}^{-1} \hat{e}_0 \qquad e_A = y^{obs} - \begin{bmatrix} A_1 & \vdots & 0 \\ A_2 & \vdots & A_2 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix}$$

$$q_A = \hat{e}_A^T Q_{y^{obs}}^{-1} \hat{e}_A$$

$$C_{y^{obs}} = \sigma_0^2 \cdot Q_{y^{obs}}$$

$$E\left(\begin{bmatrix} y_1^{obs} \\ y_2^{obs} \end{bmatrix}\right) = \left[\begin{bmatrix} A_1 & 0 \\ A_2 & \boxed{A_2^i} \end{bmatrix}\right] \begin{bmatrix} x \\ d_i \end{bmatrix}, \forall i$$

$$q_{A_i} = \frac{\hat{e}_{A_i}^T Q_{y^{obs}}^{-1} \hat{e}_{A_i}}{\sigma_0^2} \qquad \Delta_i = q_0 - q_{A_i}$$

$$H_0: E\left(\begin{bmatrix} y_1^{obs} \\ y_2^{obs} \end{bmatrix}\right) = \left[\begin{bmatrix} A_1 & 0 \\ A_2 & A_2^{i,max} \end{bmatrix}\right] \begin{bmatrix} x \\ d_{i,max} \end{bmatrix}$$

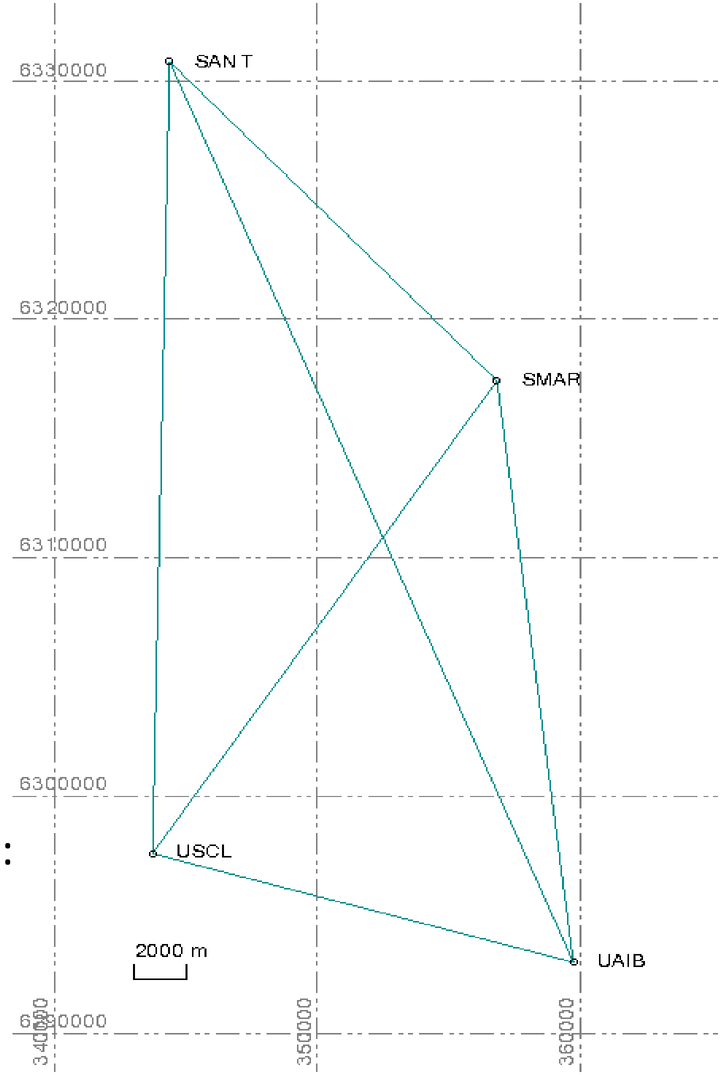
$$H_A: E\left(\begin{bmatrix} y_1^{obs} \\ y_2^{obs} \end{bmatrix}\right) = \left[\begin{bmatrix} A_1: & 0 & 0 \\ A_2: & A_2^{i,max} & \boxed{A_2^i} \end{bmatrix}\right] \begin{bmatrix} x \\ d_{i,max} \\ d_i \end{bmatrix},$$

$$\forall i \neq i, max$$

Experiment: GNSS Processing and adjustment

CORS Stations

- National Seismological Center
  - SMAR, UAIB
- University of Santiago of Chile
  - USCL
- IGS / SIRGAS
  - SANT
- Campaigns
  - January and February of 2022
- Standard GNSS processing (engineering) Trimble business Center:
  - Precise ephemeris
  - Independent vectors
  - **Baselines**
  - **Stochastic model**



Matlab code:

- Free adjustment
  - Inner
  - Pseudo-inverse
- Automation of GCT
  - GCT
  - Localization
  - Adaptation
  - GCT
  - Iterative process

Simulation: Global Sensitivity

$\alpha_0 = 0.05$ and $\beta_0 = 0.20$

$$\Delta_{min} = b_{min} \cdot g \rightarrow b_{min} = \sqrt{\frac{\lambda_{0(h,\alpha_0,\beta_0)}}{\lambda_{max}}} \quad \forall \quad \Lambda_{max}(g)$$

$$T > \chi^2_{h,1-\alpha}$$

Δ_{min}=[-0.0827 0.0040 0.0578 2.3141 -0.0891 -0.5959 -0.2762 -0.4104 0.6734 -1.9552 0.4955 -0.1353]';

Displacements:

- SANT
  - X= 4 mm
- SMAR
  - Y= 4 mm
- USCL
  - Z= 4 mm
- UAIB
  - X= 4 mm

→

```
stab =

     1
     2
     3
     4
     5
     6
     7
     8
     9
    10
    11
    12
```

1–3: SANT (1)
4–6: SMAR (5)
7–9: USCL (9)
10–12: UAIB (10)

→

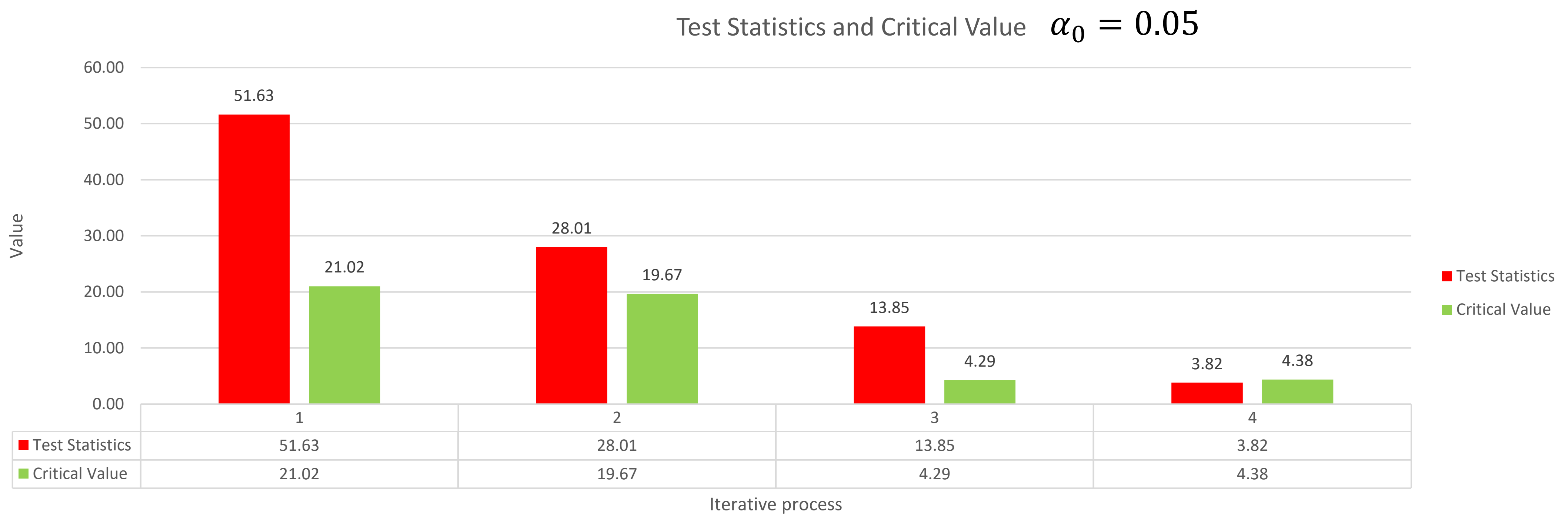
```
stab =

     2
     3
     4
     6
     7
     8
    11
    12
```

Simulation

i	Test Statistics	Critical Value	1	2	3	4	5	6	7	8	9	10	11	12
1	112.00 >	21.02	42.67	2.67	2.67	10.67	24.00	2.67	10.67	2.67	24.00	42.67	2.67	2.67
2	69.33 >	19.67	21.33	2.67	2.67	1.33	24.00	2.67	1.33	2.67	24.00	--	2.67	2.67
3	45.33 >	4.29	21.33	0.00	2.67	1.33	--	2.67	1.33	0.00	24.00	--	0.00	2.67
4	21.33 >	4.38	21.33	0.00	0.00	1.33	--	1.33	0.00	0.00	--	--	0.00	0.00
5	1.E-04 <	4.49												

Application: GNSS network (SANT, USCL, SMAR, UAIB)

i	Test Statistics	Critical Value	1	2	3	4	5	6	7	8	9	10	11	12
1	51.63	21.02	8.65	1.98	0.00	0.08	7.19	8.02	22.49	1.84	2.68	23.62	6.12	0.31
2	28.01	19.67	2.94	0.79	0.10	1.00	4.77	9.56	14.16	0.90	1.97		0.36	0.18
3	13.85	4.29	0.28	0.00	0.41	0.22	7.83	8.11		0.36	0.62		0.38	0.19
4	3.82	4.38												



Conclusions

- GCT (Global Congruence Test)
  - The GCT algorithm of Nowel is easier to use in practice
  - The algorithm gives correct results in comparison with the traditional approaches

- Importance of geodetic monitoring of networks (engineering approach)

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- CSN

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