CHAPTER 21

BLUNDER DETECTION IN HORIZONTAL NETWORKS

21.1 INTRODUCTION

Up to this point, data sets are assumed to be free of blunders. However, when adjusting real observations, the data sets are seldom blunder free. Not all blunders are large, but no matter their sizes, it is desirable to remove them from the data set. In this chapter, methods used to detect blunders before and after an adjustment are discussed.

Many examples can be cited that illustrate mishaps that have resulted from undetected blunders in survey data. However, few could have been more costly and embarrassing than a blunder of about 1 mile that occurred in an early nineteenth-century survey of the border between the United States and Canada near the north end of Lake Champlain. Following the survey, construction of a U.S. military fort was begun. The project was abandoned two years later when the blunder was detected and a resurvey showed that the fort was actually located on Canadian soil. The abandoned facility was subsequently named Fort Blunder!

As discussed in previous chapters, observations are normally distributed. This means that occasionally, large random errors will occur. However, in accordance with theory, this should seldom happen. Thus, large errors in data sets are more likely to be blunders than random errors. Common blunders in data sets include number transposition, entry and recording errors, station misidentifications, and others. When blunders are present in a data set, a least squares adjustment may not be possible or will, at a minimum, produce poor or invalid results. To be safe, the results of an adjustment should never be accepted without an analysis of the post-adjustment statistics.

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21.2 A PRIORI METHODS FOR DETECTING BLUNDERS IN OBSERVATIONS

In performing adjustments, it should always be assumed that there are possible observational blunders in the data. Thus, appropriate methods should be used to isolate and remove them. It is especially important to eliminate blunders when the adjustment is nonlinear because they can cause the solution to diverge. In this section, several methods are discussed that can be used to isolate blunders in a horizontal adjustment.

21.2.1 Use of the *K* Matrix

In horizontal surveys, the easiest method available for detecting blunders is to use the redundant observations. When initial approximations for station coordinates are computed using standard surveying methods, they should be *close* to their final adjusted values. Thus, the difference between observations computed from these initial approximations and their observed values (K matrix) are expected to be small in size. If an observational blunder is present, there are two possible situations that can occur with regard to the K-matrix values. If the observation containing a blunder is not used to compute initial coordinates, its corresponding K-matrix value will be relatively large. However, if an observation with a blunder is used in the computation of the initial station coordinates, the remaining redundant observations to that station should have relatively large values.

Figure 21.1 shows the two possible situations. In Figure 21.1(a), a distance blunder is present in line BP and is shown by the length PP'. However, this distance was not used in computing the coordinates of station P, and thus the K-matrix value for $BP' - BP_0$ will suggest the presence of a blunder by its relatively large size. In Figure 21.1(b), the distance blunder in BP was used to compute the initial coordinates of station P'. In this case, the redundant angle and distance observations connecting P with A, C, and D may show

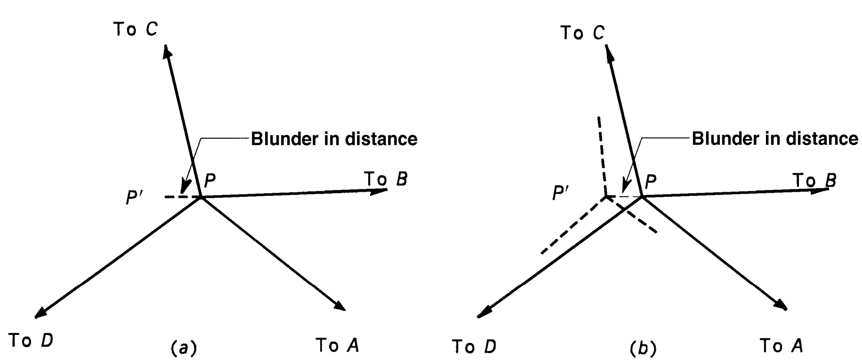

Figure 21.1 Presence of a distance blunder in computations.

large discrepancies in the *K*-matrix. In the latter case, it is possible that some redundant observations may agree reasonably with their computed values since a shift in a station's position can occur along a sight line for an angle or along a radius for a distance. Still, most redundant observations will have large *K*-matrix values and thus raise suspicions that a blunder exists in one of the observations used to compute the coordinates of station *P*.

21.2.2 Traverse Closure Checks

As mentioned in Chapter 8, errors can be propagated throughout a traverse to determine the anticipated closure. Large complex networks can be broken into smaller link and loop traverses to check estimated closures against their actual values. When a loop fails to meet its estimated closure, the observations included in the computations should be checked for blunders.

Figure 21.2(*a*) and (*b*) show a graphical technique to isolate a traverse distance blunder and an angular blunder, respectively. In Figure 21.2(*a*), a blunder in distance *CD* is shown. Notice that the remaining courses, *DE* and *EA*, are translated by the blunder in the direction of course *CD*. Thus, the length of closure line (*A′A*) will be nearly equal to the length of the blunder in *CD* with a direction that is consistent with the azimuth of *CD*. Since other observations contain small random errors, the length and direction of the closure line, *A′A*, will not match the blunder exactly. However, when one blunder is present in a traverse, the misclosure and the blunder will be close in both length and direction.

In the traverse of Figure 21.2(*b*), the effect of an angular blunder at traverse station *D* is illustrated. As shown, the courses *DE*, *EF*, and *FA′* will be rotated about station *D*. Thus, the perpendicular bisector of the closure line *AA′* will point to station *D*. Again, due to small random errors in other observations, the perpendicular bisector may not intersect the blunder precisely, but it should be close enough to identify the angle with the blunder. Since the angle at the initial station is not used in traverse computations, it is possible to

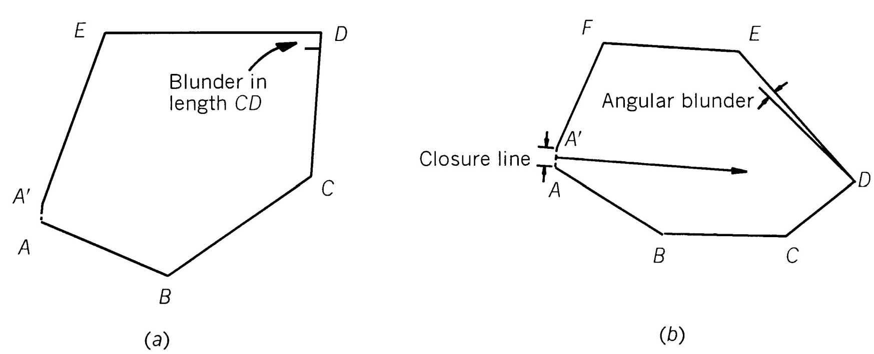

Figure 21.2 Effects of a single blunder on traverse closure.

isolate a single angular blunder by beginning traverse computations at the station with the suspected blunder. In this case, when the blunder is not used in the computations, estimated misclosure errors (see Chapter 8) will be met and the blunder can be isolated to the single unused angle. Thus, in Figure 21.2(*b*), if the traverse computations were started at station *D* and used an assumed azimuth for the course of *CD*, the traverse misclosure when returning to *D* would be within estimated tolerance since the angle at *D* is not used in the traverse computations.

21.3 A POSTERIORI BLUNDER DETECTION

When doing a least squares adjustment involving more than the minimum amount of control, both a *minimally* and *fully constrained* adjustment should be performed. In a *minimally constrained adjustment,* the data need to satisfy the appropriate geometric closures and are not influenced by control errors. After the adjustment, a χ^2 test[1] can be used to check the a priori value of the reference variance against its a posteriori estimate. However, this test is not a good indicator of the presence of a blunder since it is sensitive to poor relative weighting. Thus, the a posteriori residuals should also be checked for the presence of large discrepancies. If no large discrepancies are present, the observational weights should be altered and the adjustment rerun. Since this test is sensitive to weights, the procedures described in Chapters 7 through 10 should be used for building the stochastic model of the adjustment.

Besides the sizes of the residuals, the *signs of the residuals* may also indicate a problem in the data. From normal probability theory, residuals are expected to be small and randomly distributed. A small section of a larger network is shown in Figure 21.3. Notice that the distance residuals between stations *A* and *B* are all positive. This is not expected from normally distributed data. Thus, it is possible that either a blunder or a systematic error is present in some or all of the survey. If both *A* and *B* are control stations, part of the problem could stem from control coordinate discrepancies. This possibility can be isolated by doing a minimally constrained adjustment.

Although residual sizes can suggest observational errors, they do not necessarily identify the observations that contain blunders. This is due to the fact that least squares generally spreads a large observational error or blunder out radially from its source. However, this condition is not unique to least squares adjustments since any arbitrary adjustment method, including the compass rule for traverse adjustment, will also spread a single observational error throughout the entire observational set.

[1] Statistical testing was discussed in Chapter 4.

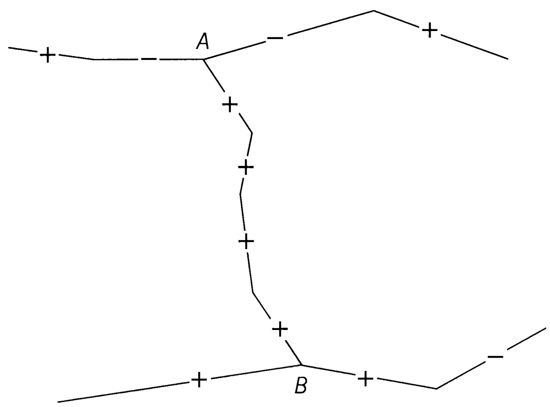

Figure 21.3 Distribution of residuals by sign.

Although an abnormally large residual may suggest the presence of a blunder in an observation, this is not always true. One reason for this could be poor relative weighting in the observations. For example, suppose that angle *GAH* in Figure 21.4 has a small blunder but has been given a relatively high weight. In this case the largest residual may well appear in a length between stations *G* and *H*, *B* and *H*, *C* and *F*, and most noticeably between *D* and *E*, due to their distances from station *A*. This is because the angular blunder will cause the network to spread or compress. When this happens, the signs of the distance residuals between *G* and *H*, *B* and *H*, *C* and *F*, and *D* and *E* may all be the same and thus indicate the problem. Again this situation can

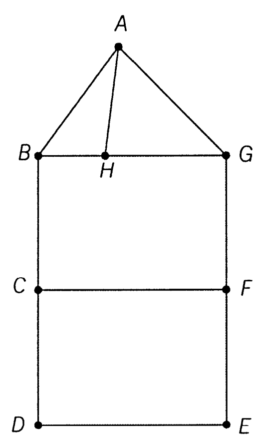

Figure 21.4 Survey network.

be minimized by using proper methods to determine observational variances so that they truly reflect the estimated errors in the observations.

21.4 DEVELOPMENT OF THE COVARIANCE MATRIX FOR THE RESIDUALS

In Chapter 5 it was shown how a sample data set could be tested at any confidence level to isolate observational residuals that were too large. The concept of statistical blunder detection in surveying was introduced in the mid-1960s and utilizes the cofactor matrix for the residuals. To develop this matrix, the adjustment of a linear problem can be expressed in matrix form as

$$L + V = AX + C \tag{21.1}$$

where C is a constants vector, A the coefficient matrix, X the estimated parameter matrix, L the observation matrix, and V the residual vector. Equation (21.1) can be rewritten in terms of V as

$$V = AX - T \tag{21.2}$$

where $T = L - C$, which has a covariance matrix of $W^{-1} = S^2Q_{ll}$. The solution of Equation (21.2) results in the expression

$$X = (A^{\mathrm{T}}WA)^{-1}\, A^{\mathrm{T}}WT \tag{21.3}$$

Letting ε represent a vector of true errors for the observations, Equation (21.1) can be written as

$$L - \varepsilon = A\overline{X} + C \tag{21.4}$$

where $\overline{X}$ is the true value for the unknown parameter X and thus

$$T = L - C = A\overline{X} + \varepsilon \tag{21.5}$$

Substituting Equations (21.3) and (21.5) into Equation (21.2) yields

$$V = A(A^{\mathrm{T}}WA)^{-1}A^{\mathrm{T}}W(AX + \varepsilon) - (A\overline{X} + \varepsilon) \tag{21.6}$$

Expanding Equation (21.6) results in

$$V = A(A^{\mathrm{T}}WA)^{-1}A^{\mathrm{T}}W\varepsilon - \varepsilon + A(A^{\mathrm{T}}WA)^{-1}A^{\mathrm{T}}WA\overline{X} - A\overline{X} \tag{21.7}$$

Since $(A^{\mathrm{T}}WA)^{-1} = A^{-1}W^{-1}A^{-\mathrm{T}}$, Equation (21.7) can be simplified to

$$V = A(A^{T}WA)^{-1}A^{T}W\varepsilon - \varepsilon + A\overline{X} - A\overline{X} \tag{21.8}$$

Factoring $W\varepsilon$ from Equation (21.8) yields

$$V = -[W^{-1} - A(A^{T}WA)^{-1}A^{T}]W\varepsilon \tag{21.9}$$

Recognizing $(A^{T}WA)^{-1} = Q_{xx}$ and defining $Q_{vv} = W^{-1} - AQ_{xx}A^{T}$, Equation (21.9) can be rewritten as

$$V = -Q_{vv}W\varepsilon \tag{21.10}$$

where $Q_{vv} = W^{-1} - AQ_{xx}A^{T} = W^{-1} - Q_{ll}$.

The Q_{vv} matrix is both singular and *idempotent*. Being singular, it has no inverse. When a matrix is idempotent, the following properties exist for the matrix: (a) The square of the matrix is equal to the original matrix (i.e., $Q_{vv}Q_{vv} = Q_{vv}$), (b) every diagonal element is between zero and 1, and (c) the sum of the diagonal elements, known as the *trace of the matrix,* equals the degrees of freedom in the adjustment. The latter property is expressed mathematically as

$$q_{11} + q_{22} + \cdots + q_{mm} = \text{degrees of freedom} \tag{21.11}$$

(d) The sum of the square of the elements in any single row or column equals the diagonal element. That is,

$$q_{ii} = q_{i1}^2 + q_{i2}^2 + \cdots + q_{im}^2 = q_{1i}^2 + q_{2i}^2 + \cdots + q_{mi}^2 \tag{21.12}$$

Now consider the case when all observations have zero errors except for a particular observation l_i that contains a blunder of size Δl_i. A vector of the true errors is expressed as

$$\Delta\varepsilon = \Delta l_i\,\varepsilon_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \Delta l_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \Delta l_i \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{21.13}$$

If the original observations are uncorrelated, the specific correction for Δv_i can be expressed as

$$\Delta v_i = -q_{ii}\,w_{ii}\,\Delta l_i = -r_i\,\Delta l_i \tag{21.14}$$

where q_{ii} is the ith diagonal of the Q_{vv} matrix, w_{ii} the ith diagonal term of the weight matrix, W, and $r_i = q_{ii}\, w_{ii}$ is the observational *redundancy number*.

When the system has a unique solution, r_i will equal zero, and if the observation is fully constrained, r_i would equal 1. The redundancy numbers provide insight into the geometric strength of the adjustment. An adjustment that in general has low redundancy numbers will have observations that lack sufficient checks to isolate blunders, and thus the chance for undetected blunders to exist in the observations is high. Conversely, a high overall redundancy number enables a high level of internal checking of the observations and thus there is a lower chance of accepting observations that contain blunders. The quotient of r/m, where r is the total number of redundant observations in the system and m is the number of observations, is called the *relative redundancy* of the adjustment.

21.5 DETECTION OF OUTLIERS IN OBSERVATIONS

Equation (21.10) defines the covariance matrix for the vector of residuals, v_i. From this the *standardized residual* is computed using the appropriate diagonal element of the Q_{vv} matrix as

$$\bar{v}_i = \frac{v_i}{\sqrt{q_{ii}}} \tag{21.15}$$

where $\bar{v}_i$ is the standardized residual, v_i the computed residual, and q_{ii} the diagonal element of the Q_{vv} matrix. Using the Q_{vv} matrix, the standard deviation in the residual is $S_0\sqrt{q_{ii}}$. Thus, if the denominator of Equation (21.15) is multiplied by S_0, a t statistic is defined. If the residual is significantly different from zero, the observation used to derive the statistic is considered to be a blunder. The test statistic for this hypothesis test is

$$t_i = \frac{v_i}{S_0\sqrt{q_{ii}}} = \frac{v_i}{S_v} = \frac{\bar{v}_i}{S_0} \tag{21.16}$$

Baarda (1968) computed rejection criteria for various significance levels (see Table 21.1) determining the α and β levels for Type I and Type II errors. The interpretation of these criteria is shown in Figure 21.5. When a blunder is present in the data set, the t distribution is shifted, and a statistical test for this shift may be performed. As with any other statistical test, two types of errors can occur. A Type I error occurs when data are rejected that do not contain blunders, and a Type II error occurs when a blunder is not detected in a data set where one is actually present. The rejection criteria are represented by the vertical line in Figure 21.5 and their corresponding significance

TABLE 21.1 Rejection Criteria with Corresponding Significance Levels

α	1 − α	β	1 − β	Rejection Criterion
0.05	0.95	0.80	0.20	2.8
0.001	0.999	0.80	0.20	4.1
0.001	0.999	0.999	0.001	6.6

levels are shown in Table 21.1. In practice, authors[2] have reported that 3.29 also works as a criterion for rejection of blunders.

Thus, the approach is to use a rejection level given by a *t* distribution with $r - 1$ degrees of freedom. The observation with the largest absolute value of t_i as given by Equation (21.17) is rejected when it is greater than the rejection level. That is, the observation is rejected when

$$\frac{|v_i|}{S_0\sqrt{q_{ii}}} > \text{rejection level} \tag{21.17}$$

Since the existence of any blunder in the data set will affect the remaining observations and since Equation (21.18) depends on S_0, whose value was computed from data containing blunders, all observations that are detected as blunders should not be removed in a single pass. Instead, only the largest or largest independent group of observations should be deleted. Furthermore, since Equation (21.18) depends on S_0, it is possible to rewrite the equation so that it can be computed during the final iteration of a nonlinear adjustment. In this case the appropriate equation is

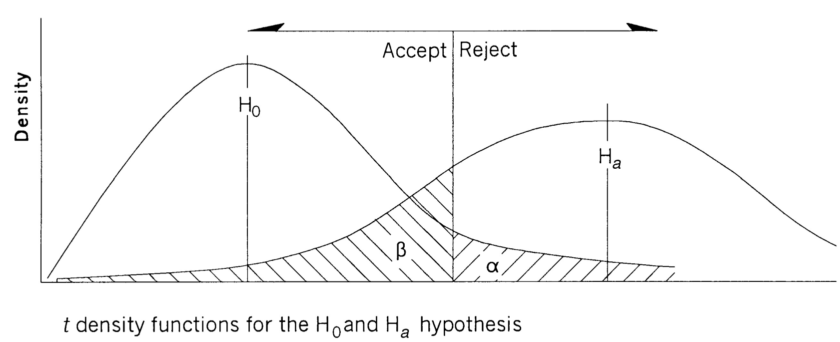

Figure 21.5 Effects of a blunder on the *t* distribution.

[2]References relating to the use of 3.29 as the rejection criterion are made in Amer (1979) and Harvey (1994).

$$\bar{v}_i = \frac{|v_i|}{\sqrt{q_{ii}}} > S_0 \times \text{rejection level} \tag{21.18}$$

A summary of procedures for this manner of blunder detection is as follows:

Step 1: Locate all standardized residuals that meet the rejection criteria of Equation (21.17) or (21.18).

Step 2: Remove the largest detected blunder or unrelated blunder groups.

Step 3: Rerun the adjustment.

Step 4: Continue steps 1 through 3 until all detected blunders are removed.

Step 5: If more than one observation is removed in steps 1 through 4, reenter the observations in the adjustment *one at a time*. Check the observation after each adjustment to see if it is again detected as a blunder. If it is, remove it from the adjustment or have that observation reobserved.

Again it should be noted that this form of blunder detection is sensitive to improper relative weighting in observations. Thus, it is important to use weights that are reflective of the observational errors. Proper methods of computing estimated errors in observations, and weighting, were discussed in Chapters 7 through 10.

21.6 TECHNIQUES USED IN ADJUSTING CONTROL

As discussed in Chapter 20, some control is necessary in each adjustment. However, since control itself is not perfect, this raises the question of how control should be managed. If control stations that contain errors are heavily weighted, the adjustment will improperly associate the control errors with the observations. This effect can be removed by using only the minimum amount of control required to fix the project. Table 21.2 lists the type of survey versus the minimum amount of control. Thus, in a horizontal adjustment, if the coordinates of only one station and the direction of only one line are held

TABLE 21.2 Requirements for a Minimally Constrained Adjustment

Survey Type	Minimum Amount of Control
Differential leveling	1 benchmark
Horizontal survey	1 point with known *xy* coordinates
	1 course with known azimuth
GPS survey	1 point with known geodetic coordinates

fixed, the observations will not be constricted by the control. That is, the observations will need to satisfy the internal geometric constraints of the network only. If more than minimum control is used, these additional constraints will be factored into the adjustment.

Statistical blunder detection can help identify weak control or the presence of systematic errors in observations. Using a minimally constrained adjustment, the data set is screened for blunders. After becoming confident that the blunders are removed from the data set, a fully constrained adjustment is performed. Following the fully constrained adjustment, an *F* test is used to compare the ratio of the minimally and fully constrained reference variances. The ratio should be 1.[3] If the two reference variances are found to be statistically different, two possible causes might exist. The first is that there are errors in the control that must be isolated and removed. The second is that the observations contain systematic error. Since systematic errors are not compensating in nature, they will appear as blunders in the fully constrained adjustment. If systematic errors are suspected, they should be identified and removed from the original data set and the entire adjustment procedure redone. If no systematic errors are identified,[4] different combinations of control stations should be used in the constrained adjustments until the problem is isolated. By following this type of systematic approach, a single control station that has questionable coordinates can be isolated.

With this stated, it should be realized that the ideal amount of control in each survey type is greater than the minimum. In fact, for all three survey types, a minimum of three controls is always preferable. For example, in a differential leveling survey with only two benchmarks, it would be impossible to isolate the problem simply by removing one benchmark from the adjustment. However, if three benchmarks are used, a separate adjustment containing only two of the benchmarks can be run until the offending benchmark is isolated.

Extreme caution should always be used when dealing with control stations. Although it is possible that a control station was disturbed or that the original published coordinates contained errors, that is seldom the case. A prudent surveyor should check for physical evidence of disturbance and talk with other surveyors before deciding to discard control. If the station was set by a local, state, or federal agency, the surveyor should contact the proper

[3] The ratio of the reference variances from the minimally and fully constrained adjustments should be 1, since both reference variances should be statistically equal. That is, $\sigma^2_{\text{minimally constrained}} = \sigma^2_{\text{fully constrained}}$.

[4] When adjusting data that cover a large region (e.g., spherical excess, reduction to the ellipsoid) it is essential that geodetic corrections to the data be considered and applied where necessary. These corrections are systematic in nature and can cause errors when fitting to more than minimal control.

authorities and report any suspected problems. People in the agency familiar with the control may help explain any apparent problem. For example, it is possible that the control used in the survey was established by two previously nonconnecting surveys. In this case, the relative accuracy of the stations was never checked when they were established. Another problem with control common in surveys is the connection of two control points from different datums. As an example, suppose that a first-order control station and a high-accuracy reference network (HARN) station are used as control in a survey. These two stations come from different national adjustments and are thus in different datums. They will probably not agree with each other in an adjustment.

21.7 DATA SET WITH BLUNDERS

Example 21.1 The network shown in Figure 21.6 was established to provide control for mapping in the area of stations 1 through 6. It began from two National Geodetic Survey second-order class II (1:20,000 precision) control stations, 2000 and 2001. The data for the job were gathered by five field crews in a class environment. The procedures discussed in Chapter 7 were used to estimate the observational errors. The problem is to check for blunders in the data set using a rejection level of $3.29S_0$.

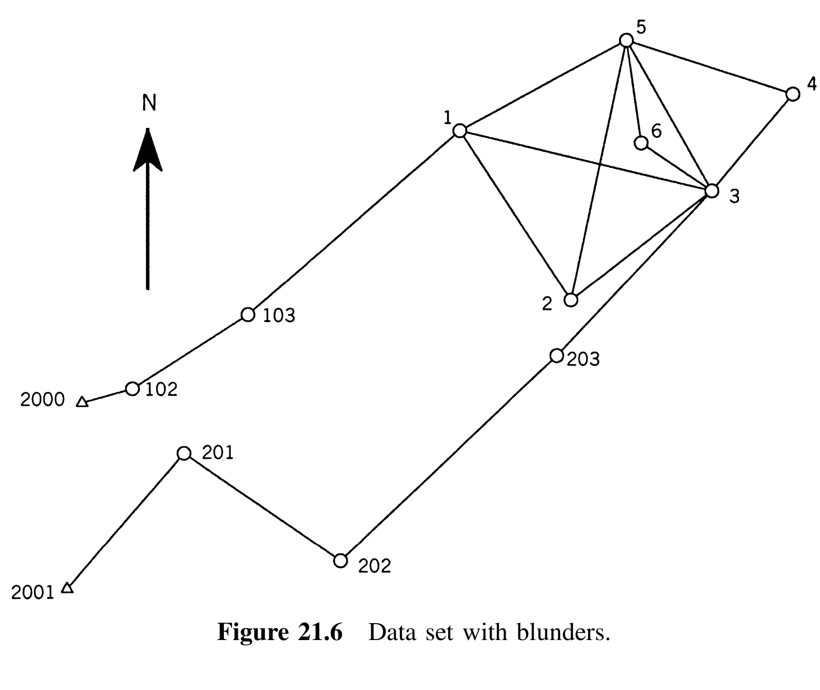

Figure 21.6 Data set with blunders.

Control stations

Station	Northing (ft)	Easting (ft)
2000	419,710.09	2,476,334.60
2001	419,266.82	2,476,297.98

Angle observations

Backsight	Occupied	Foresight	Angle	*S* (″)
102	2000	2001	109°10′54.0″	25.5
2000	102	103	162°58′16.0″	28.9
102	103	1	172°01′43.0″	11.8
2000	2001	201	36°04′26.2″	7.4
2001	201	202	263°54′18.7″	9.7
201	202	203	101°49′55.0″	8.1
202	203	3	176°49′10.0″	8.4
203	3	2	8°59′56.0″	6.5
2	1	3	316°48′00.5″	6.3
3	5	4	324°17′44.0″	8.1
6	5	3	338°36′38.5″	10.7
1	5	3	268°49′32.5″	9.8
2	5	3	318°20′54.5″	7.0
2	3	1	51°07′11.0″	7.2
2	3	5	98°09′36.5″	10.3
2	3	6	71°42′51.5″	15.1
2	3	4	167°32′28.0″	14.5

Distance observations

From	To	Distance (ft)	*S* (ft)
2001	201	425.90	0.022
201	202	453.10	0.022
202	203	709.78	0.022
203	3	537.18	0.022
5	3	410.46	0.022
5	4	397.89	0.022
5	6	246.61	0.022
5	1	450.67	0.022
5	2	629.58	0.022
3	2	422.70	0.022
3	1	615.74	0.022
3	5	410.44	0.022
3	6	201.98	0.022
3	4	298.10	0.022
1	2	480.71	0.022
1	3	615.74	0.022
2000	102	125.24	0.022
102	103	327.37	0.022
103	1	665.79	0.022

Initial approximations were computed for the stations as follows:

Station	Northing (ft)	Easting (ft)
1	420,353.62	2,477,233.88
2	419,951.98	2,477,497.99
3	420,210.17	2,477,832.67
4	420,438.88	2,478,023.86
5	420,567.44	2,477,630.64
6	420,323.31	2,477,665.36
102	419,743.39	2,476,454.17
103	419,919.69	2,476,728.88
201	419,589.24	2,476,576.25
202	419,331.29	2,476,948.76
203	419,819.56	2,477,463.90

SOLUTION Do the a priori check of the computed observations versus their *K*-matrix values. In this check, only one angle is detected as having a difference great enough to suspect that it contains a blunder. This is angle 3–5–4, which was measured as 324°17′44.0″ but was computed as 317°35′31.2″. Since this difference should not create a problem with convergence during the adjustment, the angle remained in the data set and the adjustment was attempted. The results of the first trial adjustment are shown below. The software used the rejection criteria procedure based on Equation (21.18) for its blunder detection. A rejection level of $3.29S_0$ is used for comparison against the standardized residuals. The column headed Std. Res. represents the standardized residual of the observation as defined by Equation (21.15) and the column headed Red. Num. represents the redundancy number of the observation as defined by Equation (21.14).

**** Adjusted Distance Observations ****

No.	From	To	Distance	Residual	Std. Res.	Red. Num.
1	1	3	616.234	0.494	26.148	0.7458
2	1	2	480.943	0.233	12.926	0.6871
3	1	3	616.234	0.494	26.148	0.7458
4	**3**	**4**	**267.044**	**−31.056**	**−1821.579**	**0.6169**
5	3	6	203.746	1.766	107.428	0.5748
6	3	5	413.726	3.286	171.934	0.7719
7	3	2	422.765	0.065	3.500	0.7312
8	5	2	630.949	1.369	75.909	0.6791
9	5	1	449.398	−1.272	−79.651	0.5377
10	5	6	247.822	1.212	75.418	0.5488
11	5	4	407.125	9.235	631.032	0.4529

12	5	3	413.726	3.266	170.888	0.7719
13	102	103	327.250	−0.120	−17.338	0.1018
14	103	1	665.702	−0.088	−12.395	0.1050
15	201	202	453.362	0.262	91.903	0.0172
16	202	203	709.856	0.076	10.737	0.1048
17	203	3	537.241	0.061	8.775	0.1026
18	2000	102	125.056	−0.184	−28.821	0.0868
19	2001	201	425.949	0.049	7.074	0.1008

**** Adjusted Angle Observations ****

No.	From	Occ	To	Angle	Residual	Std. Res.	Red Num
1	2	1	3	316°49′55.1″	114.6″	28.041	0.4164
2	2	3	4	167°36′00.2″	212.2″	25.577	0.3260
3	2	3	6	71°43′01.5″	10.0″	1.054	0.3990
4	2	3	5	97°55′09.3″	−867.2″	−101.159	0.6876
5	2	3	1	51°06′14.6″	−56.4″	−11.156	0.4985
6	203	3	2	8°59′36.3″	−19.7″	−13.003	0.0550
7	2	5	3	318°25′14.4″	259.9″	44.471	0.6949
8	1	5	3	268°58′49.8″	557.3″	78.590	0.5288
9	6	5	3	338°42′53.4″	374.9″	63.507	0.3058
10	**3**	**5**	**4**	**322°02′24.7″**	**−8119.3″**	**−1781.060**	**0.3197**
11	2000	102	103	162°23′50.9″	−2065.1″	−110.371	0.4194
12	102	103	1	171°57′46.9″	−236.1″	−112.246	0.0317
13	2001	201	202	263°58′31.6″	252.9″	104.430	0.0619
14	201	202	203	101°52′56.4″	181.4″	57.971	0.1493
15	202	203	3	176°50′15.9″	65.9″	23.278	0.1138
16	102	2000	2001	109°40′18.6″	1764.6″	106.331	0.4234
17	2000	2001	201	36°07′56.4″	210.2″	104.450	0.0731

****** Adjustment Statistics ******
Adjustment's Reference Standard Deviation = 487.79
Rejection Level = 1604.82

The proper procedure for removing blunders is to remove the single observation that is greater in magnitude than the rejection level selected for the adjustment and is greater in magnitude than the value of any other standardized residual in the adjustment. This procedure prevents removing observations that are connected to blunders and thus are inherently affected by their presence. By comparing the values of the standardized residuals against the rejection level of the adjustment, it can be seen that both a single distance (3–4) and an angle (3–5–4) are possible blunders since their standardized residuals are greater than the rejection level chosen. However, upon inspection of Figure 21.6, it can be seen that a blunder in distance 3–4 will directly affect angle 3–5–4, and distance 3–4 has the standardized residual that is greatest in magnitude. This explains the previous a priori rejection of this angle observation. That is, distance 3–4 directly affects the size of angle 3–5–4 in the adjustment. Thus, only distance 3–4 should be removed from the observations. After removing this distance from the observations, the adjustment was rerun with the results shown below.

**** Adjusted Distance Observations ****

No.	From	To	Distance	Residual	Std. Res.	Red. Num.
1	1	3	615.693	−0.047	−2.495	0.7457
2	1	2	480.644	−0.066	−3.647	0.6868
3	1	3	615.693	−0.047	−2.495	0.7457
4	2001	201	425.902	0.002	0.265	0.1009
5	3	6	201.963	−0.017	−1.032	0.5765
6	3	5	410.439	−0.001	−0.032	0.7661
7	3	2	422.684	−0.016	−0.858	0.7314
8	5	2	629.557	−0.023	−1.280	0.6784
9	5	1	450.656	−0.014	−0.858	0.5389
10	5	6	246.590	−0.020	−1.241	0.5519
11	5	4	397.885	−0.005	−0.380	0.4313
12	5	3	410.439	−0.021	−1.082	0.7661
13	102	103	327.298	−0.072	−10.380	0.1018
14	103	1	665.751	−0.039	−5.506	0.1049
15	201	202	453.346	0.246	86.073	0.0172
16	202	203	709.807	0.027	3.857	0.1049
17	203	3	537.193	0.013	1.922	0.1027
18	2000	102	125.101	−0.139	−21.759	0.0868

**** Adjusted Angle Observations ****

No.	From	Occ	To	Angle	Residual	Std. Res.	Red Num
1	2	1	3	316°47′54.2″	−6.3″	−1.551	0.4160
2	2	3	4	167°32′31.0″	3.0″	0.380	0.2988
3	2	3	6	71°42′46.0″	−5.5″	−0.576	0.3953
4	2	3	5	98°09′18.6″	−17.9″	−2.088	0.6839
5	2	3	1	51°07′04.1″	−6.9″	−1.360	0.4978
6	203	3	2	8°59′26.7″	−29.3″	−19.340	0.0550
7	2	5	3	318°20′51.4″	−3.1″	−0.532	0.6933
8	1	5	3	268°50′03.4″	30.9″	4.353	0.5282
9	6	5	3	338°36′37.1″	−1.4″	−0.238	0.3049
10	3	5	4	324°17′43.6″	−0.4″	−0.381	0.0160
11	**2000**	**102**	**103**	**162°24′10.2″**	**−2045.8″**	**−109.353**	**0.4193**
12	**102**	**103**	**1**	**171°57′51.2″**	**−231.8″**	**−110.360**	**0.0316**
13	2001	201	202	263°58′20.3″	241.6″	99.714	0.0619

14	201	202	203	101°52′34.7″	159.7″	51.023	0.1494
15	202	203	3	176°49′56.1″	46.1″	16.273	0.1138
16	**102**	**2000**	**2001**	**109°40′17.7″**	**1763.7″**	**106.280**	**0.4233**
17	2000	2001	201	36°07′46.9″	200.7″	99.688	0.0732

```
****** Adjustment Statistics ******
Adjustment's Reference Standard Deviation = 30.62
Rejection Level = 100.73
```

After this adjustment, analysis of standardized residuals indicates that the angles most likely still to contain blunders are observations 11, 12, and 16. Of these, observation 12 displays the highest standardized residual. Looking at Figure 21.6, it is seen that this angle attaches the northern traverse leg to control station 2000. This is a crucial observation in the network if any hopes of redundancy in the orientation of the network are to be maintained. Since this is a flat angle (i.e., nearly 180°), it is possible that the backsight and foresight stations were reported incorrectly, which can be checked by reversing stations 102 and 1. However, without further field checking, it cannot be guaranteed that this occurred. A decision must ultimately be made about whether this angle should be reobserved. However, for now, this observation will be discarded and another trial adjustment made. In this stepwise blunder detection process, it is always wise to remove as few observations as possible. In no case should observations that are blunder-free be deleted. This can and does happen, however, in trial blunder detection adjustments. But through persistent and careful processing, ultimately only those observations that contain blunders can be identified and eliminated. The results of the adjustment after removing the angle 12 are shown below.

```
****************
Adjusted stations
****************
```

			Standard error ellipses computed				
Station	X	Y	Sx	Sy	Su	Sv	t
1	2,477,233.72	420,353.59	0.071	0.069	0.092	0.036	133.47°
2	2,477,497.89	419,951.98	0.050	0.083	0.090	0.037	156.01°
3	2,477,832.55	420,210.21	0.062	0.107	0.119	0.034	152.80°
4	2,477,991.64	420,400.58	0.077	0.121	0.138	0.039	149.71°
5	2,477,630.43	420,567.45	0.088	0.093	0.123	0.036	136.74°
6	2,477,665.22	420,323.32	0.071	0.096	0.114	0.036	145.44°
102	2,476,455.89	419,741.38	0.024	0.018	0.024	0.017	80.86°
103	2,476,735.05	419,912.42	0.051	0.070	0.081	0.031	147.25°
201	2,476,576.23	419,589.23	0.020	0.022	0.024	0.017	37.73°
202	2,476,948.74	419,331.29	0.029	0.041	0.042	0.029	14.24°
203	2,477,463.84	419,819.58	0.040	0.077	0.081	0.032	160.84°

```
*******************************
Adjusted Distance Observations
*******************************
```

Station Occupied	Station Sighted	Distance	V	Std.Res.	Red.#
2001	201	425.88	−0.023	−3.25	0.102
201	202	453.09	−0.005	−3.25	0.006
202	203	709.76	−0.023	−3.25	0.104
203	3	537.16	−0.023	−3.25	0.103
5	3	410.45	−0.011	−0.60	0.767
5	4	397.89	−0.003	−0.19	0.436
5	6	246.60	−0.014	−0.83	0.556
5	1	450.68	0.013	0.80	0.542
5	2	629.58	0.003	0.15	0.678
3	2	422.70	0.003	0.16	0.736
3	1	615.75	0.008	0.40	0.745
3	5	410.45	0.009	0.44	0.767
3	6	201.97	−0.013	−0.78	0.580
1	2	480.71	−0.003	−0.19	0.688
1	3	615.75	0.008	0.40	0.745
2000	102	125.26	0.020	3.25	0.082
102	103	327.39	0.023	3.25	0.101
103	1	665.81	0.023	3.25	0.104

```
***************************
Adjusted Angle Observations
***************************
```

Station Backsighted	Station Occupied	Station Foresighted	Angle	V	Std.Res.	Red.#
102	2000	2001	109°11′11.1″	17.06″	3.25	0.042
2000	102	103	162°58′05.1″	−10.95″	−3.25	0.014
2000	2001	201	36°04′23.8″	−2.45″	−3.25	0.010
2001	201	202	263°54′15.7″	−2.97″	−3.25	0.009
201	202	203	101°49′46.3″	−8.72″	−3.25	0.110
202	203	3	176°49′01.0″	−8.98″	−3.25	0.109
203	3	2	8°59′51.1″	−4.91″	−3.25	0.054
2	1	3	316°48′02.8″	2.29″	0.57	0.410
3	5	4	324°17′43.8″	−0.19″	−0.19	0.016
6	5	3	338°36′37.0″	−1.51″	−0.26	0.302
1	5	3	268°49′43.7″	11.20″	1.57	0.528
2	5	3	318°20′51.1″	−3.44″	−0.59	0.691
2	3	1	51°07′14.4″	3.45″	0.68	0.497
2	3	5	98°09′22.0″	−14.55″	−1.71	0.680
2	3	6	71°42′48.5″	−2.97″	−0.31	0.392
2	3	4	167°32′29.5″	1.48″	0.19	0.294

```
***************************************
        Adjustment Statistics
***************************************
              Iterations = 4
            Redundancies = 12
      Reference Variance = 1.316
            Reference So = ±1.1
Possible blunder in observations with Std.Res. > 4
              Convergence!
```

From analysis of the results, all observations containing blunders appear to have been removed. However, it should also be noted that several remaining distance and angle observations have very low redundancy numbers. This identifies them as unchecked observations, which is also an undesirable situation. Thus, good judgment dictates reobservation of the measurements deleted. This weakness can also be seen in the size of the standard error ellipses for the stations shown in Figure 21.7.

Note, especially, rotation of the error ellipses. That is, the uncertainty is primarily in a direction perpendicular to the line to stations 1 and 102. This condition is predictable since the angle 102–103–1 has been removed from the data set. Furthermore, the crew on the northern leg never observed an

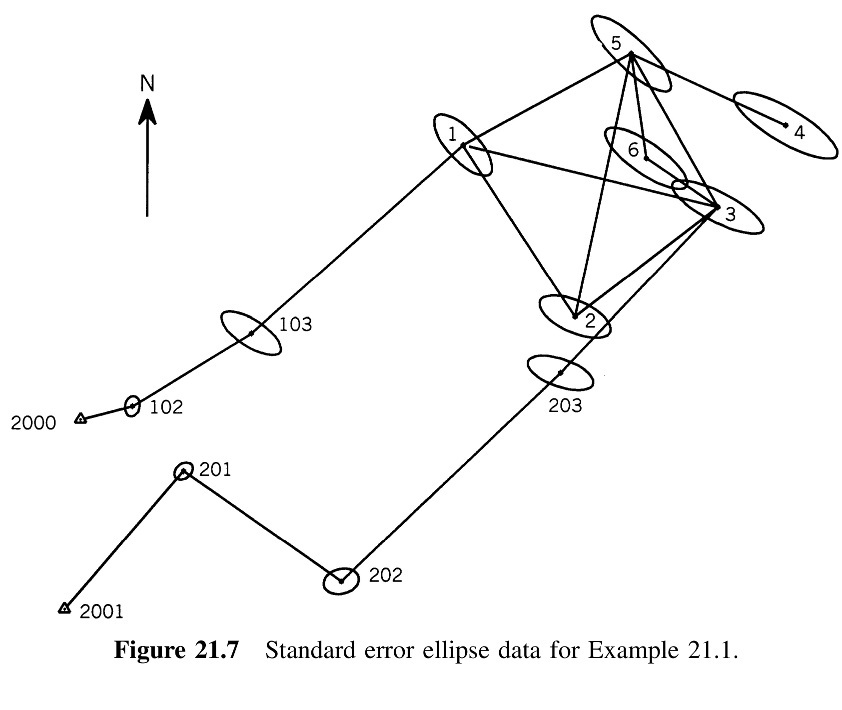

Figure 21.7 Standard error ellipse data for Example 21.1.

angle at station 1 that would tie into station 103, and thus the position of station 103 was found by the intersection of two distances that nearly form a straight line. This results in a larger error in the direction perpendicular to the lines at this station.

This example demonstrates the process used to statistically detect and remove observational blunders. Whether the observations should be remeasured depends on the intended use of the survey. Obviously, additional observations would strengthen the network and probably reduce the size of the error ellipses.

Observations between stations 102 and 201 also contribute to the overall strength in the network. However, because a building obstructs that line, these observations could not be obtained. This is a common problem in network design. That is, it is sometimes physically impossible to gather observations that would contribute to the total network strength. Thus, a compromise must be made between the *ideal* network and what is physically obtainable. Balancing these aspects requires careful planning before the observations are collected. Of course, line obstructions that occur due to terrain, vegetation, or buildings, can now be overcome by using GPS.

21.8 SOME FURTHER CONSIDERATIONS

Equation (21.14) shows the relationship between blunders and their effects on residuals as $\Delta v_i = -r_i\ \Delta l_i$. From this relationship note that the effect of the blunder, Δl_i, on the residual, Δv_i, is directly proportional to the redundancy number, r_i. Therefore:

1. If r_i is large ($\approx$ 1), the blunder greatly affects the residual and should be easy to find.
2. If r_i is small ($\approx$ 0), the blunder has little affect on the residual and will be difficult to find.
3. If $r_i = 0$, the blunder is undetectable and the parameters will be incorrect since the error has not been detected.

Since redundancy numbers can range from 0 to 1, it is possible to compute the minimum detectable error for a single blunder. For example, suppose that a value of 4.0 is used to isolate observational blunders. Then, if the reference variance of the adjustment is 6, all observations that have standardized residuals greater than 24.0 (4.0×6) are possible blunders. However, from Equation (21.14), it can be seen that for an observation with a redundancy number of 0.2 ($r_i = 0.2$) and a standardized residual of $\Delta v_i = 24.0$, the minimum detectable error is 24.0/0.2, or 120! Thus, a blunder, Δl_i, in this observation as large as five times the desired level can go undetected due to its low redundancy number. This situation can be extended to observations that have no observational checks; that is, r_i is 0. In this case, Equation (21.14) shows that it is impossible to detect any blunder, Δl_i, in the observation since $\Delta v_i/\Delta r_i$ is indeterminate.

With this taken into consideration, it has been shown that a marginally detectable blunder in an individual observation is

$$\Delta l_i = S \sqrt{\frac{\lambda_0}{q_{ii} w_{ii}^2}} \tag{21.19}$$

where λ_0 is the mean of the noncentral normal distribution shown in Figure 21.5, known as the *noncentrality parameter*. This parameter is the translation of the normal distribution that leads to rejection of the null hypothesis, whose values can be taken from nomograms developed by Baarda (1968). The sizes of the values obtained from Equation (21.19) provide a clear insight into weak areas of the network.

21.8.1 Internal Reliability

Internal reliability is found by examining how well observations check geometrically with each other. As mentioned previously, if a station is determined uniquely, q_{ii} will be zero in Equation (21.19), and the computed value of Δl_i is infinity. This indicates the lack of measurement self-checking. Since Equation (21.19) is independent of the actual observations, it provides a method of detecting weak areas in networks. To minimize the sizes of the undetected blunders in a network, the redundancy numbers of the individual observations should approach their maximum value of 1. Furthermore, for uniform network strength, the individual redundancy numbers, r_i, should be close to the global relative redundancy of r/m, where r is the number of redundant observations and m is the number of observations in the network. Weak areas in the network are located by finding regions where the redundancy numbers become small in comparison to relative redundancy.

21.8.2 External Reliability

An undetected blunder of size Δl_i has a direct effect on the adjusted parameters. External reliability is the effect of the undetected blunders on these parameters. As Δl_i (a blunder) increases, so will its effect on ΔX. The value of ΔX is given by

$$\Delta X = (A^{\mathrm{T}}WA)^{-1}A^{\mathrm{T}}W\,\Delta\varepsilon \tag{21.20}$$

Again, this equation is datum independent. From Equation (21.20) it can be seen that to minimize the value of ΔX_i, the size of redundancy numbers must be increased. Baarda suggested using average coordinate values in determining the effect of an undetected blunder with the following equation

$$\lambda = \Delta X^{T} Q_{xx} \Delta X \tag{21.21}$$

where λ represents the noncentrality parameter.

The noncentrality parameter should remain as small as possible to minimize the effects of undetected blunders on the coordinates. Note that as the redundancy numbers on the observations become small, the effects of undetected blunders become large. Thus, the effect of the coordinates of a station from a undetected blunder is greater when the redundancy number is low. In fact, an observation with a high redundancy number is likely to be detected as a blunder.

A traverse sideshot can be used to explain this phenomenon. Since the angle and distance to the station are unchecked in a sideshot, the coordinates of the station will change according to the size of the observational blunders. The observations creating the sideshot will have redundancy numbers of zero since they are geometrically unchecked. This situation is neither good nor acceptable in a well-designed observational system. In network design, one should always check the redundancy numbers of the anticipated observations and strive to achieve uniformly high values for all observations. Redundancy numbers above 0.5 are generally sufficient to provide well-checked observations.

21.9 SURVEY DESIGN

In Chapters 8 and 19, the topic of observational system design was discussed. Redundancy numbers can now be added to this discussion. A well-designed network will provide sufficient geometric checks to allow observational blunders to be detected and removed. In Section 21.8.1 it was stated that if blunder removal is to occur uniformly throughout the system, the redundancy numbers should be close to the system's global relative redundancy. Furthermore, in Section 21.8.2 it was noted that redundancy numbers should be greater than 0.5. By combining these two additional concepts with the error ellipse sizes and shapes, and stochastic model planning, an overall methodology for designing observational systems can be obtained.

To begin the design process, the approximate positions for stations to be included in the survey must be determined. These locations can be determined from topographic maps, photo measurements, or previous survey data. The approximate locations of the control stations should be dictated by their desired locations, the surrounding terrain, vegetation, soils, sight-line obstructions, and so on. Field reconnaissance at this phase of the design process is generally worthwhile to verify sight lines and accessibility of stations. Moving a station only a small distance from the original design location may greatly enhance visibility to and from other stations but not change the geometry of the network significantly. By using topographic maps in this process, sight-line ground clearances can be checked by constructing profiles between stations.

When approximate station coordinates are determined, a stochastic model for the observational system can be designed following the procedures discussed in Chapter 7. In this design process, considerations should be given to the abilities of the field personnel, quality of the equipment, and observational procedures. After the design is completed, specifications for field crews can be written based on these design parameters. These specifications should include the type of instrument used, number of turnings for angle observations, accuracy of instrument leveling and centering, misclosure requirements, and many other items.

Once the stochastic model is designed, simulated observations are computed from the station coordinates, and a least squares adjustment of the observations is to be done. Since actual observations have not been made, their values are computed from the station coordinates. The adjustment will converge in a single iteration, with all residuals equaling zero. Thus, the reference variance must be assigned the a priori value of 1 to compute the error ellipse axes and coordinate standard deviations. Having completed the adjustment, the network can be checked for geometrically weak areas, unacceptable error ellipse sizes or shapes, and so on. This inspection may dictate the need for any or all of the following: (1) more observations, (2) different observational procedures, (3) different equipment, (4) more stations, (5) different network geometry, and so on. In any event, a clear picture of results obtainable from the observational system will be provided by the simulated adjustment and additional observations, or different network geometry can be used.

It should be noted that what is expected from the design may not actually occur, for numerous and varied reasons. Thus, systems are generally overdesigned. However, this tendency to overdesign should be tempered with the knowledge that it will raise the costs of the survey. Thus, a balance should be found between the design and costs. Experienced surveyors know what level of accuracy is necessary for each job and design observational systems to achieve the accuracy desired. It would be cost prohibitive and foolish always to design an observational system for maximum accuracy regardless of the survey's intended use. As an example, the final adjustment of the survey in Section 21.7 had sufficient accuracy to be used in a mapping project with a final scale of 1:1200 since the largest error ellipse semimajor axis (0.138 ft) would only plot as 0.0014 in. and is thus less than the width of a line on the final map product.

For convenience, the steps involved in network design are summarized below.

Step 1: Using a topographic map or aerial photos, lay out possible positions for stations.

Step 2: Use the topographic map together with air photos to check sight lines for possible obstructions and ground clearance.

Step 3: Do field reconnaissance, checking sight lines for obstructions not shown on the map or photos, and adjust positions of stations as necessary.

Step 4: Determine approximate coordinates for the stations from the map or photos.

Step 5: Compute values of observations using the coordinates from step 4.

Step 6: Using methods discussed in Chapter 6, compute the standard deviation of each observation based on available equipment and field measuring procedures.

Step 7: Perform a least squares adjustment, to compute observational redundancy numbers, standard deviations of station coordinates, and error ellipses at a specified percent probability.

Step 8: Inspect the solution for weak areas based on redundancy numbers and ellipse shapes. Add or remove observations as necessary, or reevaluate measurement procedures and equipment.

Step 9: Evaluate the costs of the survey, and determine if some other method of measurement (e.g., GPS) may be more cost-effective.

Step 10: Write specifications for field crews.

PROBLEMS

Note: For problems requiring least squares adjustment, if a computer program is not distinctly specified for use in the problem, it is expected that the least squares algorithm will be solved using the program MATRIX, which is included on the CD supplied with the book.

21.1 Discuss the effects of a distance blunder on a traverse closure and explain how it can be identified.

21.2 Discuss the effects of an angle blunder on a traverse closure, and explain how it can be identified.

21.3 Explain why a well-designed network has observational redundancy numbers above 0.5 and approximately equal.

21.4 Create a list of items that should be included in field specifications for a crew in a designed network.

21.5 Summarize the general procedures used in isolating observational blunders.

21.6 How are control problems isolated in an adjustment?

21.7 Discuss possible causes for control problems in an adjustment.

21.8 Why is it recommended that there be at least three control stations in a least squares adjustment?

21.9 Outline the procedures used in survey network design.

21.10 Using the procedures discussed in this chapter, analyze the data in Problem

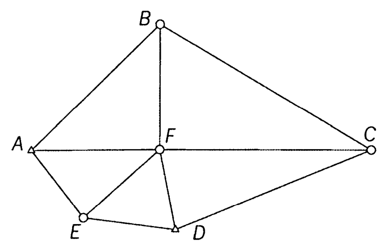

Figure P21.11

21.11 In Figure P21.11 the following data were gathered. Assuming that the control stations have a published precision of 1:20,000, apply the procedures discussed in this chapter to isolate and remove any apparent blunders in the data.

Control stations

Station	Easting	Northing
A	982.083	1000.204
D	2686.270	58.096

Approximate station coordinates

Station	Easting	Northing
B	2507.7	2500.6
C	4999.9	998.6
E	1597.6	200.0
F	2501.0	1009.6

Distance observations

From	To	Distance (m)	*S* (m)	From	To	Distance (m)	*S* (m)
A	*B*	2139.769	0.023	*E*	*F*	1231.086	0.021
A	*F*	1518.945	0.021	*E*	*A*	1009.552	0.020
B	*C*	2909.771	0.025	*D*	*F*	969.386	0.020
B	*F*	1491.007	0.021	*D*	*E*	1097.873	0.021
C	*D*	2497.459	0.023	*C*	*F*	2498.887	0.023