Uncertainty assessment in geodetic network adjustment by combining GUM and MC

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Abstract. This article extends the classical concept of geodetic network adjustment by introducing a new two-step analysis method, starting with a quality assessment of the original input quantities.

In the first step the original readings and possible influence factors for pre-processing the raw data are analysed using an uncertainty modelling according to GUM (Guidelines to the Expression of Uncertainty in Measurements). This approach is well established in metrology, but rarely adapted within Geodesy.

As second step Monte-Carlo-Simulations (MC-simulations) are carried out. To perform these simulations, possible realisations of the raw readings and the influencing factors are generated, using assigned probability distributions for all variables and the established concept of pseudo-random number generators. Final result is a point cloud which represents the uncertainty of the estimated coordinates; a confidence region can be assigned to these points clouds, as well.

This concept may replace the common variance-covariance propagation and the use of the covariance matrix Σ_{xx} of the parameter vector x for an estimation of the achieved precision. It allows a new way for quality assessment of precision in accordance with the GUM concept for uncertainty modelling.

As practical example the local tie network in "Metsähovi Fundamental Station", Finland is used, where classical geodetic observations are combined with GNSS data.

Keywords. GUM analysis, geodetic network adjustment, quality assessment, Monte-Carlo Simulations, local tie

1 Uncertainty according to GUM

1.1 Concept of GUM (Guidelines to the expression of uncertainty in measurements)

Because no measurement is exact, a measured value, i.e. the result of a real measurement, is only complete with a quantity representing the associated uncertainty. In statistics here the dispersion of the variable is introduced, classical numerical quantities are variance or standard deviation.

The traditional statistical concept to carry out repeated independent (!) observations and to compute the dispersion of the resulting measuring quantity does not cover the complexity of the measuring processes, which we find today. Due to the advent and rapid developments of electronic sensors and - in general - low knowledge on internal measuring processes and pre-set computational steps it is not sufficient to analyse repeated observations to get an adequate measure for the dispersion of measurements! The measurement action itself often is identical with pushing a button and therefore these "observations" do not contain any information on the real variability of the complete measuring process. Aside, often the variability of environmental conditions is not taken into account, i.e. important influencing parameters are not considered.

Being aware of these deficiencies, on initiative of the BIHM (BIPM 1987) a group of international experts of metrology started to develop a new concept to better assess the complete uncertainty of measurements. As result the "Guide to the Expression of Uncertainty in Measurements", abbreviated by GUM, was developed, which nowadays is the international standard (DIN V ENV 13005, ISO 1995) in metrology and beyond. The GUM allows to compute uncertainty quantities for each measuring sensor or system; uncertainty is a non-negative parameters characterizing the dispersion of a value attributed to a measuring quantity and by this nowadays uncertainty is considered to be the adequate precision parameter.

Of course, this GUM concept is studied and discussed within geodetic literature, see e.g. Hennes/Heister (2007), Hennes (2013), Kutterer/Schön (2004) and Niemeier (2008). But these discussions are mainly limited to uncertainty assessment for measurements, only. Due to the knowledge of the authors the here emphasised network approach is not considered yet.

As consequence GUM introduces a unified method for the evaluation of these uncertainty quantities (Sommer/Siebert 2004, Weise /Wöger 1999). The uncertainty according to GUM has a probabilistic basis, but tries to include all knowledge on factors influencing the quantity of interest. Within GUM, two types of influence factors are considered:

Type A: Random dispersion of measurements

- Common statistical approaches, mainly Gaussian distribution.
- Values inferred from repeated measured values (internal accuracy?).

Type B: External influences, systematic effects, approximations, etc.

- What are influencing factors?
- Quantification: Variability within specified interval [a, b]
- Probability distribution: Assume type of distribution, mostly uniform, rectangular or Gaussian

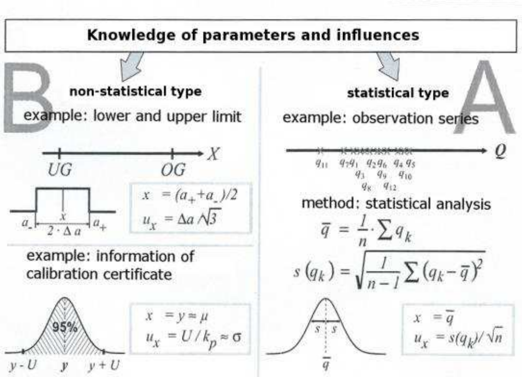


Figure 1 GUM concepts for modelling input quantities of Type A and Type B (after Sommer/Siebert 2004)

This concept allows it to consider the classical random effects (Type A), which might influence the measuring results, i.e. the established approach.

But additionally it allows to include all relevant additional influences (Type B), e.g. external effect (e.g. environment and observer) and remaining systematic errors (e.g. due to measuring procedure, calibration errors or instrumental effects). Even approximations in a computational formula have to be and can be considered here.

1.2 Approach to perform a GUM analysis

From the view point of classical error analysis the introduction of GUM corresponds to a radical "*paradigm change"*, which leads to several new subtasks, which have to be solved to be able to perform an uncertainty analysis according to GUM:

1.2.1 Modeling the (pre)-processing chain

As describes in the GUM documents, e.g. (DIN 1999, § 4.1), it is necessary to model the complete process to derive the „final“ result Y out of the raw readings. Here all and really all input quantities X_1, X_2, X_3… have to be included, which influence the value of the raw readings (starting quantities). As this model contains an algorithm, how the raw readings are changed whenever one input quantity is changed, this model is named *"carrier of information"* for GUM analysis. Mathematically this model can be described as - often nonlinear - function

$$Y = f(X_1, X_2, X_3, \dots, X_n) \quad (1.1)$$

To establish this function in a complete way is one of the most difficult subtasks for deriving the uncertainty of measurements. A very profound understanding of the sensor, the measuring task, the data processing and the possible influences are mandatory. Here it has to be taken into account, that the purpose is to provide information about the quantity of interest - the *measurand*. No definitive theory can be applied, just some recommendations are available (see e.g. Kessel 2001, Sommer und Siebert 2004).

For distance measurements, one of the basic geodetic techniques, the computational model according to Eq. (1.1) is depicted in Figure 2. Here not the complete mathematical formulas are

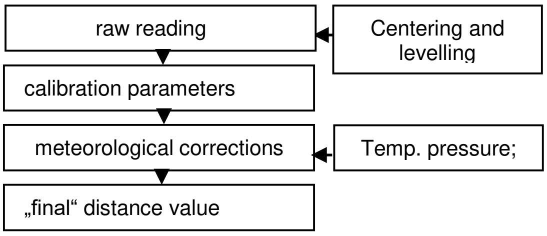


Figure 2 Pre-processing chain for the derivation of a "final" value for distance measurements

listed, just a description of the specific computational steps are given.

The "final" distance value, i.e. the numerical result after all these pre-processing steps, is used as input quantity for a network adjustment. Now it is necessary to assign a dispersion or an uncertainty value to this quantity. Within the classical approach of network adjustment (e.g. Niemeier 2008) just rough estimates for the dispersion, given as variances and covariances, will be introduced. Only the GUM approach allows to include uncertainties of all input variables and influencing factors which are used during the complete pre-processing stages. This will be explained in the following sections.

1.2.2 Assessment of a statistical distribution to the pre-processing variables

For all original readings and influencing quantities in Eq. (1.1) now statistical distributions with expectations and dispersion values have to be assigned:

For Type A quantities the common probability functions with *Gaussian* or *Normal* distribution (defined by expectation μ and variance σ^2) are used, following the classical statistical concepts, see Fig. 1. Here all the classical methods for estimation of variances can be applied, i.e. the analysis of real measurements from own or other experiences, data sheet information, etc.

For influences of Type B, which can be summed up as external influences, remaining systematic effects and insufficient approximations, according e.g. to Weise and Wöger (1999) and Sivia (1996) it seems to be justified to introduce *probability functions* as well. But the individual assignment of an adequate statistical distribution is much more complex as here as statistical concepts aside the classical *normal* distribution, *uniform* and *triangle* distributions are allowed, as well, see Figure 3. and Table 1.

For each distribution the expected mean value and its dispersion have to be introduced, i.e. these quantities have to be pre-selected as starting values for a GUM analysis! The task for the engineer is to estimate e.g. the variability of the effective temperature during the measuring period, to assess a quantity to the centering quality, to evaluate the validity of the calibration parameters, etc.

A simple example for this first step a GUM analysis is given in Table 1 for geodetic distance measurements. In the extended numerical example in section 4 the assessment of statistical distributions to all influencing parameters is performed for a simple 3D-local geodetic network with total station, levelling and GNSS observations. These still simplified estimated probability functions are used in the same section for a Monte-Carlo (MC) simulation of the local tie network of the geodetic fundamental station Metsähovi, Finland.

Table 1: Type A and Type B influence factors, their probability distribution and variability range for typical geodetic observations

Influence factors		**Distribution**	**Examples**
Type A	Total station		
	- directions	normal	σ_h= 0,2 mgon
	- vertical angles	normal	σ_v= 0,3 mgon
	- slope distances	normal	σ_d= 0,6 mm+1ppm
	Levelling		
	- height differences	normal	$\sigma_{\Delta h}$= 0,6 mm/√ km
	GNSS		
	- baselines $\Delta x, \Delta y, \Delta z$	normal	σ_Δ = 2 mm
Type B	Pillar und centering		
	- centering direction	uniform	[0..360°]
	- centering offset	triangle	[0..0,1 mm]
	- Target centre definition	uniform	σ_t = 0,1mm
	Instrument and target height	uniform	[0..0,2 mm]
	Calibration parameters		
	- additional constant	normal	σ_A = 0,5mm
	- scale factor	normal	σ_s = 0,2 ppm
	Atmospheric parameters		
	- temperature	uniform	[0.1 K]
	- air pressure	uniform	[0.1 mbar]
	- humidity	uniform	[0..5%]

For the here used basic geodetic observations in Table 1 the main influencing factors with corresponding probability distributions and domain of variability are given. Missing are additional computational influences, according to the required reduction to a reference height, possible personal effects, due to observer resp. team, and further environmental influences, as bad weather, strong insolation, etc.

The selection of the distribution type and the variability range are a preliminary selection, only. At least for GNSS a much more detailed analysis of all influencing factors has to be performed, which is an actual task for scientists from the Finish Geodetic Institute.

1.3 Derivation of uncertainty quantities

1.3.1 Combining the individual influences

Within the classical GUM approach the combination of the different influences is done by a straight forward application of the well-known law of variance propagation. In Figure 3 on the left hand side the different types of probability distributions are depicted, i.e. normal, uniform and triangle distribution, where to each type of distribution several pre-processing variables can be assigned. On the right hand side the principle for combining the different uncertainties u_{xi} of Type A and Type B are visualized, using the classical concept of variance propagation for uncorrelated quantities.

The uncertainty for the function y, see Eq. (1.1), is numerically given in Eq. (1.2), where instead of uncertainties u_{xi} explicitly the uncertainties of Type A (u_{Ai}) and Type B (u_{Bi}) are given to show that here both types of influence

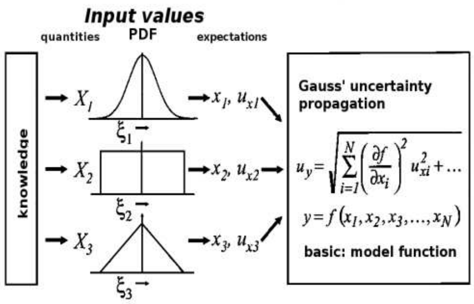


Figure 3 Concept of standard GUM approach to combine Type A and B influence parameters, using Gauss principle of variance propagation (Sommer/Siebert 2004)

factors are combined, applying the law of variance propagation:

$$u_y = \sqrt{u_{A1}^2 + \cdots + u_{An}^2 + u_{B1}^2 + \cdots + u_{Bm}^2} \quad (1.2)$$

Within GUM two extensions are foreseen, which corresponds – to a certain extend – to classical statistical approaches and can be interpreted as contribution to classical thinking:

- One can introduce the *extended uncertainty*, defined as

$$U = ku_C$$

U is considered to content the majority of all values, which are probable as measuring results. Often k= 2 is used, what would correspond to a 95%-confidence interval, if we would compare it to normal distributed quantities.

- To account for not considered unknown influence factors, an additional uncertainty u_u^2 can be introduced, what leads to the *complete uncertainty*

$$u_{II} = \sqrt{u^2 + u_u^2}$$

and finally the extended complete uncertainty

$$U_{II} = k \cdot u_{II}$$

Criticism:

Both extensions can be discussed critically, but this is outside the scope of this paper.

The application of statistical distributions to Type B factors and the application of the variance propagation to get an uncertainty estimate u_y are main contradictions to the GUM approach (Kutterer/Schön 2004). Of course, the assignment of a statistical distribution to a great variety of influence factors is a sensible step, and of course, the use of the law of variance propagation is a crude method, but it allows the community to continue their subsequent computations within established statistical concepts.

1.3.2 More complex and/or combined systems

The basic GUM concepts are restricted to one output quantity, i.e. an uncertainty assessment to a specific *measurand*, what can be the volume of a vessel, the potential difference between the terminals of a battery, or a mass concentration. Of course, this output quantity, for which information is required, can be related to several input quantities.

For many real problems is it not possible to measure the quantities of interest directly but they have to be calculated from a combination of several measurements, following an - often non-linear - complex algorithm. If it comes to geodetic networks with several points, the objective is to determine the 1D, 2D or 3D coordinates of all network points. Here redundant and sometimes physically different observations (horizontal and vertical directions, slope distances, 3D coordinate differences, height differences, etc) are taken to estimate the "best" results for the final coordinates of the network points and to improve the reliability of the system. To solve this overdetermined adjustment problem mainly the principle of least squares is applied (e.g. Niemeier 2008). The main aspects for the funtional and stochastic model are given in section 1.

To the knowlegde of the authors, the here discussed complex network adjustment process is not included in or covered by a GUM analysis, yet. In a classical geodetic adjustment only variance and covariance estimates for the final input quantities can be taken into account, see section 2.2. In theory these estimates take into account GUM Type A influences, only. But we are aware that often the quality of the pre-processing steps and the environmental conditions, i.e. information of GUM Type B effects, are included - better to say "added" - in an intuitive way into the variance estimates, at least by experienced observers. But these additional influencing effects cannot be considered in a rigorous way within the classical concept; this is possible only by the here proposed new approach.

3 Monte - Carlo (MC) Approach

3.1 Basic idea of MC simulations

Monte Carlo (MC) methods characterise a class of computational algorithms to solve complex numerical problems by repeated random experiments (Kroese, Taimre, & Botev, 2011), i.e. "real" experiments are replicated or simulated within a computer. Monte Carlo simulations use repeated random sampling for input quantities to obtain the variability of numerical results. Typically the simulation runs many times (1000 – 100,000 or more) in order to obtain a realistic or probabilistic distribution of the quantities of interest.

Nowadays with efficient computers and good random number generators large samples are easy to generate. The variability of input quantities is computed by deterministic, pseudorandom sequences what makes it easy to evaluate and re-run simulations. MC simulations allow to model phenomena with well-defined, significant uncertainties for all input variables.

Here this MC approach will be applied to perform extended uncertainty modelling within geodetic data processing, especially network adjustments according to the GM model. It will be shown that this methodology allows it to combine MC simulations with a GUM analysis of the influencing factors in a rigorous way.

The algorithm used here follows a particular pattern:

- Define the functional relation between all input data and quantities of interest.
- Define expectation and uncertainty domain for input data.
- Generate input data randomly from a probability distribution over this domain.
- Perform a deterministic computation with these inputs and get the quantities of interest.
- Aggregate and analyze the computed quantities of interest.

MC approaches are used since a long time in Geodesy. Dupraz and Niemeier (1979) applied this method to estimate the precision of a simple geodetic network with just 1000 samples. They could demonstrate the correlation between neighboring stations and by this the usefulness of a singular value decomposition for geodetic networks. Koch (2002) and Koch (2007) studied the applicability of MC simulations for regulation parameters and for outlier detection.

3.2 Combination of MC simulations with GUM

In the approach here the combination of the different influencing factors is done by Monte-Carlo (MC) simulations, an approach, which overcomes at least the before mentioned criticism to use the classical variance propagation concept. The use MC-simulations in connection with GUM is propagated by the BIPM expert group since 2004, see e.g. Bich (2008).

The here proposed algorithm for an extended uncertainty assessment for geodetic network by rigorous combination of MC simulations with concepts of GUM is depicted in Figure 4. Starting point is the analysis of the complete pre-processing chain for each observation L_i, i.e. an analysis of all influencing factors of Type A and Type B, as given in Eq. (1.1), the “carrier of information” for GUM analyses. For all these influencing factors the most probable numerical value of this quantity, often a mean value or a real observation, a probability distribution function (according to Figure 3 an equal, triangle or normal distribution) and a variability domain (expressed by a variance) have to be selected.

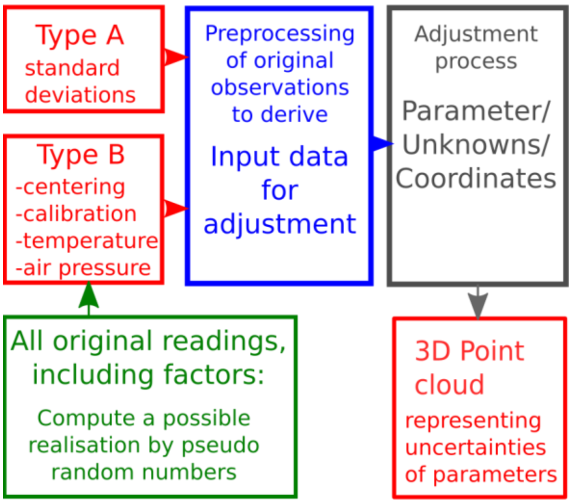


Figure 4 GUM-conform input data generation for Monte-Carlo (MC)-simulation of a network adjustment

With an established *pseudo random number generators* (prn-generator) then in each MC-run one realization for each influence factor is computed and these values are combined within the pre-processing chain (see Eq. 1.1) to get one simulated input quantity L_i for the adjustment approach. If these first steps according to GUM are performed for all input data of an adjustment, e.g. the distances, directions, height differences or coordinate differences, for each run the well-known adjustment approach can start to estimate the adjustment parameters, what are in most applications the station coordinates.

The preliminary step to get input quantities according to GUM does not influence the adjustment process itself, or – better to say – it is open for different coordinate systems and datum definitions, even for different adjustment concepts with arbitrary target functions.

As result of each adjustment run one sample set of estimated parameters are computed. Within a MC simulation these computational steps are repeated m times (m between 1000 and 1 Million), resulting in m coordinate estimates for the position of each station.

Final step is a visualization of these results,

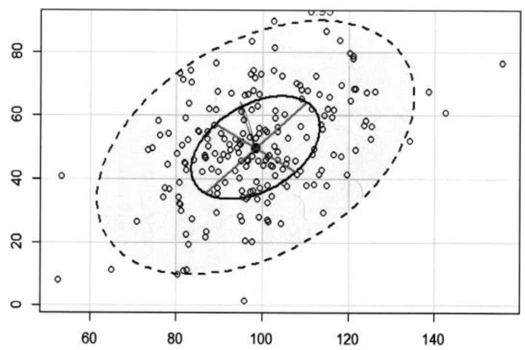


Figure 5 2D-point-cloud, representing the different solutions for coordinate-estimates in a Monte-Carlo-Simulation. The ellipses are numerically derived regions for 68% and 95% confidence levels.

which forms a *point cloud*, as depicted in Figure 5 for just one station in a 2D network. The variability of the estimated coordinates after MC simulation is a realistic picture of the uncertainty of these coordinate estimates according to GUM-analysis of all influencing factors and the adjustment process. It is easy to compute confidence regions numerically within these points clouds, covering 68% or 95% of the total variability, as shown by the ellipses in Figure 5. Therefore by this MC approach confidence regions can be derived easily and subsequent analyses can be performed.

One has to note that the probability distribution of this new uncertainty quantity has to be discussed. Basis are several influencing factors with different and in principle arbitrary probability distributions. At first glance one could apply the well-known central limit theorem (Cramer 1943), which state that as result of the combination of a large number of input quantities the result has a normal distribution. This would result in subsequent analyses, e.g. for deformation studies, without any problems. But here more detailed studies have to be performed.

4 Application to Local Tie Network "Metsähovi" in Finland

4.1 What is a Local Tie?

A "Local Tie Vector" can be defined an the 3D-coordinate difference between instantaneous phase centres of different space based geodetic techniques, defined in the coordinate system of the space techniques. (e.g. Abbonondanza and Sarti 2012). As intermediate step often the 3D-coordinate difference between a specific space technique and physically defined reference stations are determined. The uncertainty requirement for local tie vectors is extremely high, within the global observing systems one to wants to achieve this information with an uncertainty of 1 mm.

Nowadays different space geodetic techniques are available, as example these are:

- VLBI (Very Long Baseline Interferometry)
- GNSS (Global Navigation Satellite Systems)
- SLR (Satellite Laser Ranging)
- DORIS (Doppler Satellite Orbitography)

All have advantages and deficiencies. The different techniques complement each other and the combination of all is desirable. The combination requires a common reference system and the knowledge of the relative position between the reference points of each contributing instrument, the above mentioned local tie vector.

In a resolution (Draft, Version 2014-01-22) the participants of the IERS Workshop on Local Surveying and Co-Locations requested a unique lithospheric reference points whose validity as such is established through pillar networks or the like. The local tie vector then is defined as the vector between the reference points of the space techniques and the lithospheric reference point.

As common reference frame the International Terrestrial Reference Frame (ITRF) coordinate system is chosen. The International GPS Service (IGS) is a global system of satellite tracking stations, which helps to improve and extend the ITRF. The IGS Reference Frame is a realisation of the ITRF. With the IGS generated precise satellite ephemerides processed GNSS baselines refer to the ITRF too (Rebischung, et al., 2012).

The coordinates of the points in a network realize one possible solution. To determine the local tie vector only the orientation of the network is important not the absolute position of the reference points because only the coordinate differences are required for the local tie vector.

A lot a problems exist, dealing with the definition and the realisation of a local tie vector. Just to give an idea, some of these problems are mentioned here:

- Centre of antennas are influenced by gravitational, temperature (Nothnagel 2009) and wind effects. E.g. tilting of the antenna implies mechanical loading on the structure.
- Reference points can't be measured directly, as e.g. for VLBI the reference point is defined as the intersection of the two axes.
- Terrestrial measurements do not refer directly to ITRF, but to the local gravity field.
- Attaching GNSS antennas e.g. to a large VLBI antenna (Ning et al., 2014) will cause near field effects to the GNSS observations, which vary during the operation of the VLBI system.

4.2 Test site "Metsähovi"

The Metsähovi Fundamental Station is a key infrastructure of the Finnish Geospatial research Institut (FGI). Metsähovi is the basic station for the national FIN2000 reference system, and a part of the national permanent GNSS network FinnRef. Metsähovi is a stable part of global network of geodetic core stations which are used in maintaining global terrestrial and celestial reference frames, for computation of satellite orbits, and for several geophysical studies (www.fgi.fi/fgi/node/517, last access: 11.02.2015).

As depicted in Figure 6, the structure of this fundamental station is rather complex, the VLBI antenna is located inside a radome. This results in a two-step network, where the connection between the outside network and the inside network is the most critical part, but this will not be discussed here in detail.

The local tie network consists of 31 points, the local tie vector is defined between the GPS points 11 and 31 and the point 180 in the radome. This is not the reference point of the VLBI antenna, but is located in the neighbourhood. In a 3D so-called free adjustment the following measurements are used: 149 total station measurements, 48 GNSS baselines and levelled 43 height differences.

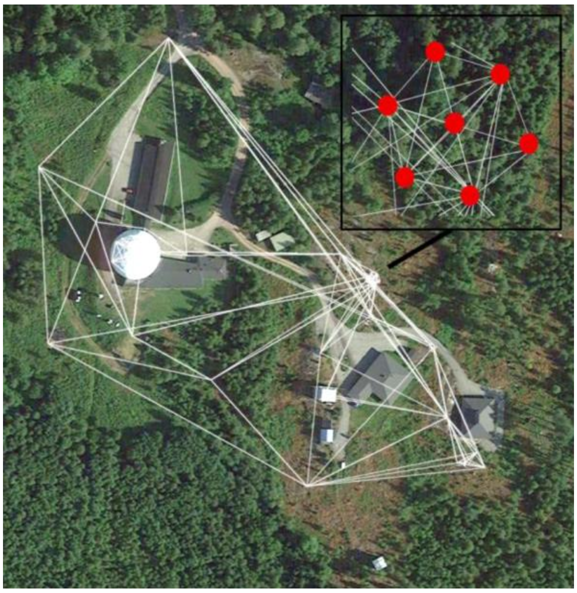


Figure 6 Metsähovi fundamental station, Finland, with local tie network

4.3 Derivation of Input Variables According to GUM Analysis

To get a realistic idea on achievable uncertainty of the local tie vector a computational model with combined GNSS/terrestrial measurements, as is realised for Metsähovi, is simulated according to the ahead given GUM analysis with subsequent Monte Carlo method.

The GNSS uncertainty model is hardly to simulate because the influence of multipath and near field effects are widely unknown, but these are important influence factors for the uncertainty assessment. As mentioned before, colleagues from FGI are working on this problem.

In this first approach, the classical terrestrial local tie network is simulated. Each simulation consists of 1000 runs of the adjustment. In each run a new set of observations and subsequently of station coordinates are generated. A forced centring is assumed so the coordinates may differ only in each simulation according to the pillar centring variations.

The local tie vector is defined as coordinate differences between the GPS points 11 and 31 and the point 180 in the radome, as mentioned before. The uncertainty of this local tie vector can be calculated in each simulation run from the covariance matrix of the adjusted stations.

The following influence factors were considered with their corresponding variability estimates according to the GUM concept:

Type A: Classical approach, Standard deviations

1. Total station observations

Standard deviation of modern total station (for example Leica TS30) is 0.15 mgon for manual angle measurements and 0.3 mgon for measurements with automatic target recognition. With two sets of angles, a precision of 0.2 mgon is assumed.

- Standard deviation of directions; normal distribution; σ = 0.2 mgon
- Standard deviation of zenithal angles; normal distribution, σ = 0.2 mgon
- Standard deviation of slope distances; normal distribution, σ = 0.6 mm + 1 ppm

2. GNSS Baselines

Here just a rough estimate is used without considering correlations between baseline components

- Standard deviation of each coordinate component: normal distribution, σ = 3.0 mm

3. Height differences

- Standard deviation of height differences: normal distribution, σ = 2.0 mm/$\sqrt{km}$

Type B: Systematic effects, ext. influences, etc.

1. Pillar and centring variations

- Uniform distribution, range: 0 - 0.1 mm

2. Instrument and target height

- Uniform distribution, range: 0 - 0.1 mm

3. Total station instrument calibration

Schwarz (Schwarz, 2012) describes the possible standard deviations of calibration parameters for total stations:

- Addition constant: normal distribution, σ = 0.2 mm for combination instrument and prism
- Scale unknown: normal distribution, σ = 0.8 ppm

The value for the scale is bases on the problem to determine the temperature along the propagation path of the laser, see atmospheric parameter, too.

4. GNSS Antenna calibration

- Not implemented yet because antenna calibration depends on knowledge of satellite positions (elevation/azimuth), so it is time depended (Campbell, Görres, Siemes, Wirsch, & M., 2004).

5. Atmospheric Parameters

- Temperature: uniform distribution, range 0-0.8 K; (remark: 1 K ≈ 1 ppm)
- Air pressure: uniform distribution, range 0 - 0.5 mbar; (remark: 1 mbar ≈ 0.3 ppm)
- Humidity: uniform distribution, range 0 - 5 %

5. Results

Applying the here proposed concept to the network Metsähovi gave the following results:

In Figure 7 the observation scheme of the

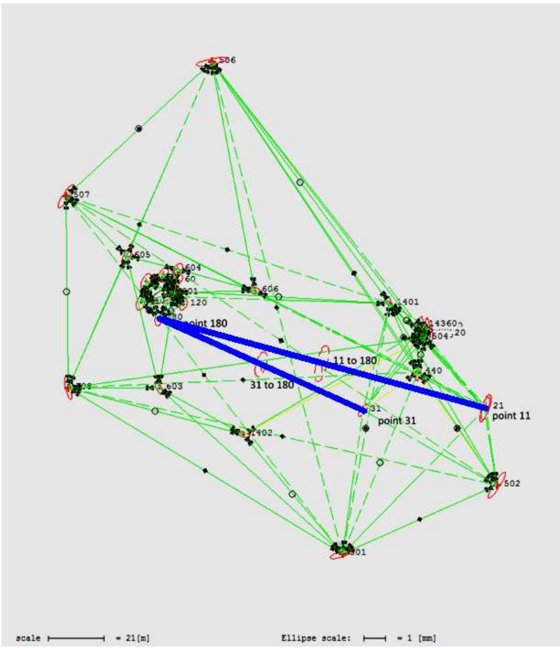


Figure 7 Complete local tie network Metsähovi with error ellipses for all stations and the local tie vectors

complete network and the radome network are depicted, containing of the point connections and the 95%-confidence ellipses, derived out of the MC simulation with 1000 runs.

In Figure 8 and 9 the point clouds for the local

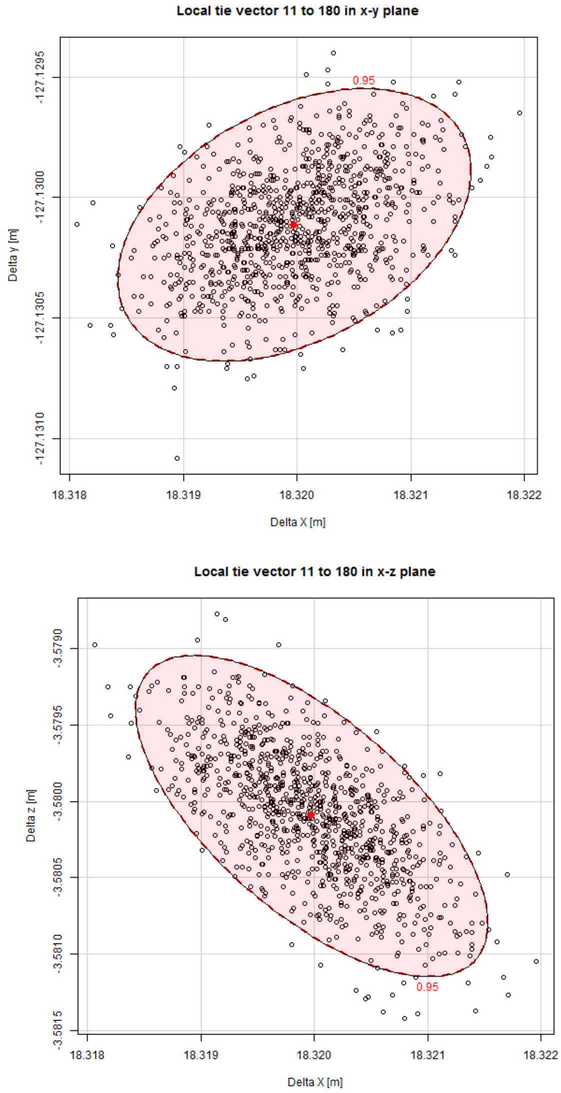


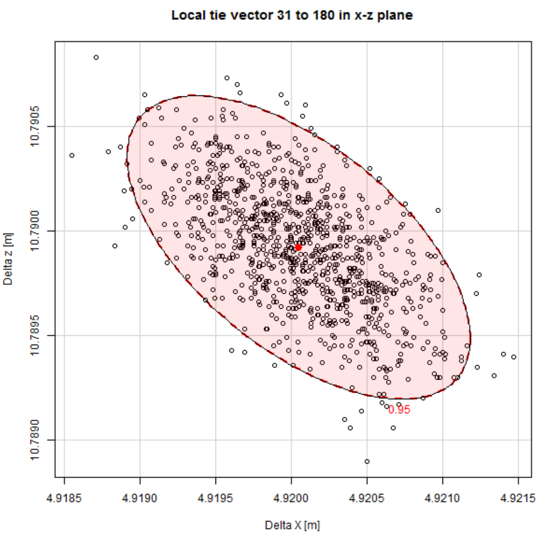
Figure 8 Local tie vector between points 11 and 180. The elliptical contours refer to a confidence level of 95%. The semi major axis is about 1.6 mm (x-y plane) and 2 mm (x-z plane)

tie vector between the stations 11 and 180 resp. 31 and 180 are given. As this is a 3D-geodetic network, the point clouds are presented in the x-y plane and x-z plane.

The semi major axis of the local tie vector is:

- about 0.7 mm between points 11 and 180
- about 0.5 mm between points 31 and 180

These values correspond to the so-called error ellipses, i.e. to a confidence level of about 68 %



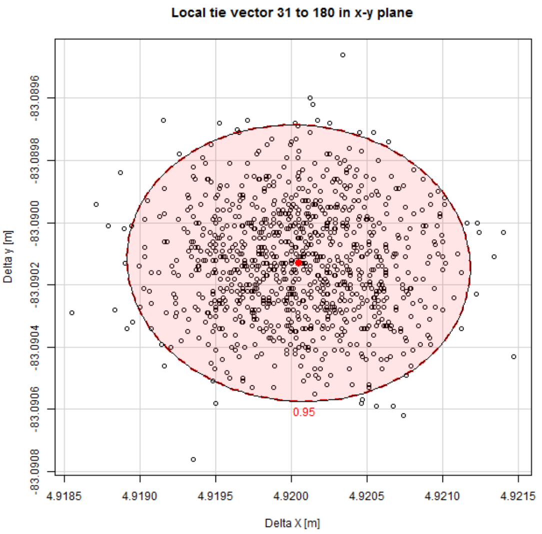


Figure 9 Local tie vector between points 31 and 180. The elliptical contours refer to a confidence level of 95%. The semi major axis is about 1.1 mm (x-y plane) and 1.3 mm (x-z plane)

and can be interpreted to be comparable to standard deviations. Compared to the required uncertainty of 1 mm for local tie vectors, as pre-set by international scientific organisations, this achievement is reached by the existing network. The question is open, whether or not this static information on the local tie vector is valid for the usually applied instantaneous observations, i.e. whether this difference vector covers the variability of the instrument itself to a sufficient level.

The local tie vector was determined between the GPS Reference points 11/31 and the points inside the radome 180. The coordinate differences refer to the global Cartesian coordinate system.

6. Conclusion

In this report first ideas are presented towards an advanced quality assessment for geodetic networks by combining the uncertainty analysis according to GUM with the potential of MC simulations. Starting point is a detailed uncertainty analysis of all influencing factors for the pre-processing chain. In accordance to this the MC simulation starts with this preliminary data and includes the subsequent adjustment computations, as well.

This method is applied to a classical network for determination the local tie vector of the fundamental station Metsähovi, Finland. Here terrestrial measurements and GNSS observations are taken. The complete processing chain is outlined and the numerical results depict the fully sufficient results: It can be shown that the local tie vector can be determined with an uncertainty of 2 mm based on a confidence level of 95%.

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