GIS-E3010
Least-Squares Methods in Geoscience
Lecture 9/2018

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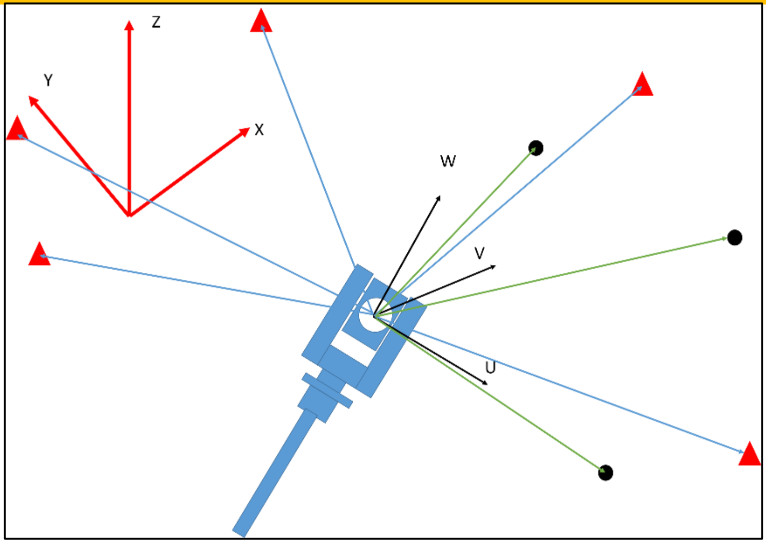
Local area 3D terrestrial network adjustment

About GPS-baseline processing

Context, motivation



- Local area networks
- Local reference frame or connection global
- For construction work
- For monitoring
- Special purpose networks
- Industrial measurements
- Networks form a control for laser scanning or photogrammetry
- Small area networks
    - Terrestrial (tachymeter) 3D networks are nowadays mostly used for precision measurements, like monitoring measurements in small area. (Maximum 1 square kilometer)
    - Distances between points only 5m-200m
    - The uncertainty of refraction does not dominate in vertical angles (like in the case of larger networks with vertices several kilometers) and they are as usefull in adjustment as the horizontal angles and distances

Local area network in global frame

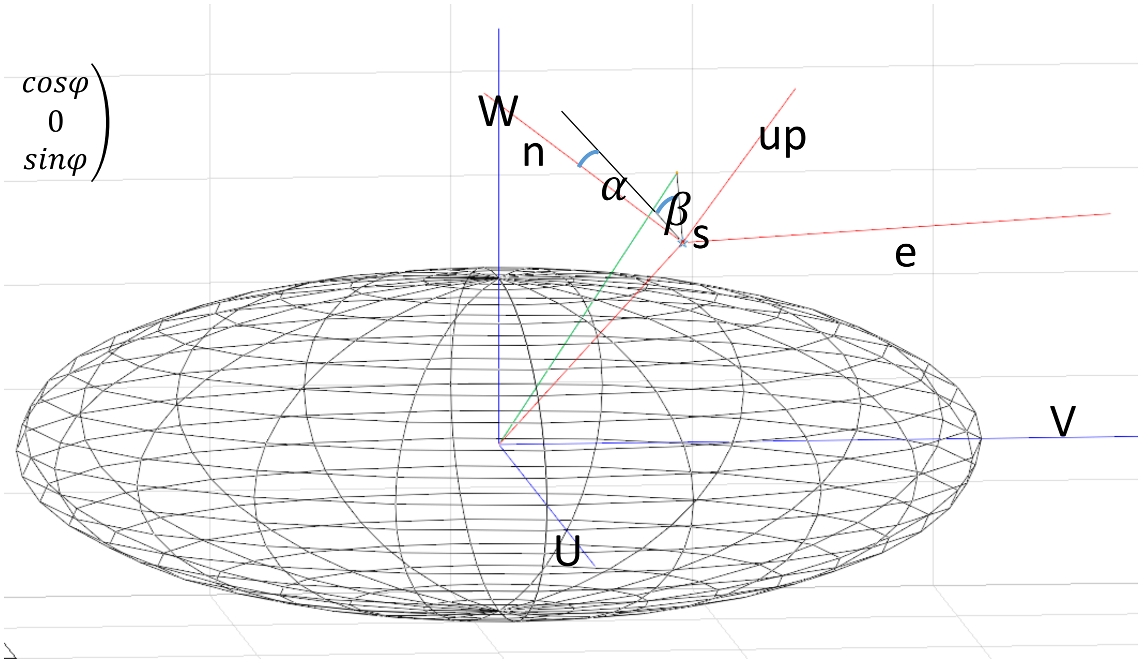
$$\alpha = \tan^{-1}\left(\frac{e}{n}\right)$$

$$\beta = \sin^{-1}\left(\frac{up}{s}\right)$$

$$s = \sqrt{\mathrm{d}u^2 + \mathrm{d}v^2 + \mathrm{d}w^2} = \sqrt{n^2 + e^2 + up^2}$$

$$\mathrm{R}(\varphi, \lambda) = \begin{pmatrix} -sin\varphi cos\lambda & -sin\varphi sin\lambda & cos\varphi \\ -sin\lambda & cos\lambda & 0 \\ cos\varphi cos\lambda & cos\varphi sin\lambda & sin\varphi \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{d}n \\ \mathrm{d}e \\ \mathrm{d}up \end{pmatrix} = \mathrm{R}(\varphi, \lambda) \begin{pmatrix} \mathrm{d}u \\ \mathrm{d}v \\ \mathrm{d}w \end{pmatrix}$$



Model

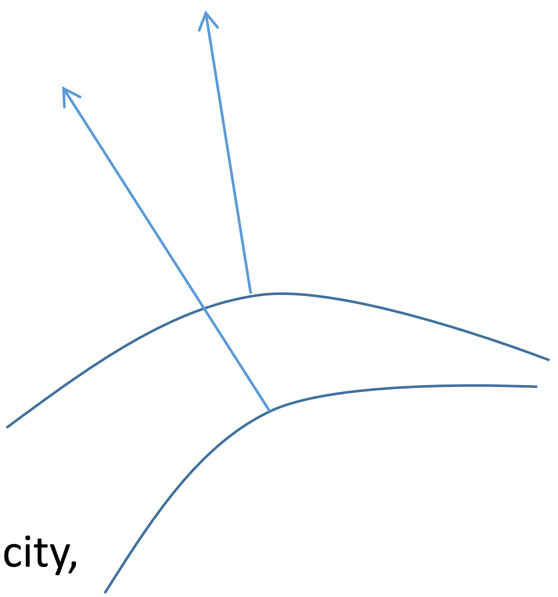
$$\alpha_0 = \alpha - t_0$$

$$\alpha = tan^{-1}\left(\frac{-sin(\lambda)\cdot \mathrm{d}u + cos(\lambda)\cdot \mathrm{d}v}{-sin(\varphi)\cdot cos(\lambda)\cdot \mathrm{d}u - sin(\varphi)\cdot sin(\lambda)\cdot \mathrm{d}v + cos(p)\cdot \mathrm{d}w}\right)$$

$$\beta = sin^{-1}\left(\frac{cos(\varphi)\cdot cos\cdot \mathrm{d}u + cos(\varphi)\cdot sin(\lambda)\cdot \mathrm{d}v + sin(\varphi)\cdot \mathrm{d}w}{|(\mathrm{d}u \quad \mathrm{d}v \quad \mathrm{d}w)|}\right)$$

$$s = |(\mathrm{d}u \quad \mathrm{d}v \quad \mathrm{d}w)|$$

Deflection of vertical correction is necessary because the normal of the **reference ellipsoid** and the normal of the geoid are not same. We assume that we have oriented to gravity our istruments and targets.



Corrections to observations before adjustment:

- Deflection of vertical (to horizontal and vertical angles)
- Refraction (to vertical angle)
- The first velocity correction (to distances)
  - In larger networks more corrections are needed (2. velocity, curvature)
- Centering elements and their covariance matrix must be converted to global system (NEU to UVW and UVW to φ,λ,h conversions with covariances)

3D model for terrestrial network in global coordinate system, partial derivates

$$g11 = \frac{\partial\alpha}{\partial u_1} = -\frac{\partial\alpha}{\partial u_2} = \frac{(-\sin(\varphi)\cdot\cos(\lambda)\cdot\sin(\alpha)+\sin(\lambda)\cdot\cos(\alpha))}{(s\cdot\cos(\beta))};$$

$$g12 = \frac{\partial\alpha}{\partial v} = -\frac{\partial\alpha}{\partial v_2} = \frac{(-\sin(\varphi)\cdot\sin(\lambda)\cdot\sin(\alpha)-\cos(\lambda)\cdot\cos(\alpha))}{(s\cdot\cos(\beta))};$$

$$g13 = \frac{\partial\alpha}{\partial w_1} = -\frac{\partial\alpha}{\partial w_2} = \frac{(\cos(\varphi)*\sin(\alpha))}{(s\cdot\cos(\beta))};$$

$$g14 = \frac{\partial\alpha_0}{\partial t_0} - 1;$$

$$g21 = \frac{\partial\beta}{\partial u_1} = -\frac{\partial\beta}{\partial u_2} = \frac{(-s\cdot\cos(\varphi)\cdot\cos(\lambda)+\sin(\beta)\cdot \mathrm{d}u)}{(s^2\cdot\cos(\beta))};$$

$$g22 = \frac{\partial\beta}{\partial v} = -\frac{\partial\beta}{\partial v_2} = \frac{(-s\cdot\cos(\varphi)\cdot\sin(\lambda)+\sin(\beta)\cdot \mathrm{d}v)}{(s^2\cdot\cos(\beta))};$$

$$g23 = \frac{\partial\beta}{\partial w_1} = -\frac{\partial\beta}{\partial w_2} = \frac{(-s\cdot\sin(\varphi)+\sin(\beta)\cdot \mathrm{d}w)}{(s^2\cdot\cos(\beta))};$$

$$g31 = \frac{\partial s}{\partial u_1} = -\frac{\partial s}{\partial u_2} = -\frac{\mathrm{d}u}{|(\mathrm{d}u \quad \mathrm{d}v \quad \mathrm{d}w)|};$$

$$g32 = \frac{\partial s}{\partial v_1} = -\frac{\partial s}{\partial v_2} = -\frac{du}{|(du \quad dv \quad dw)|};$$

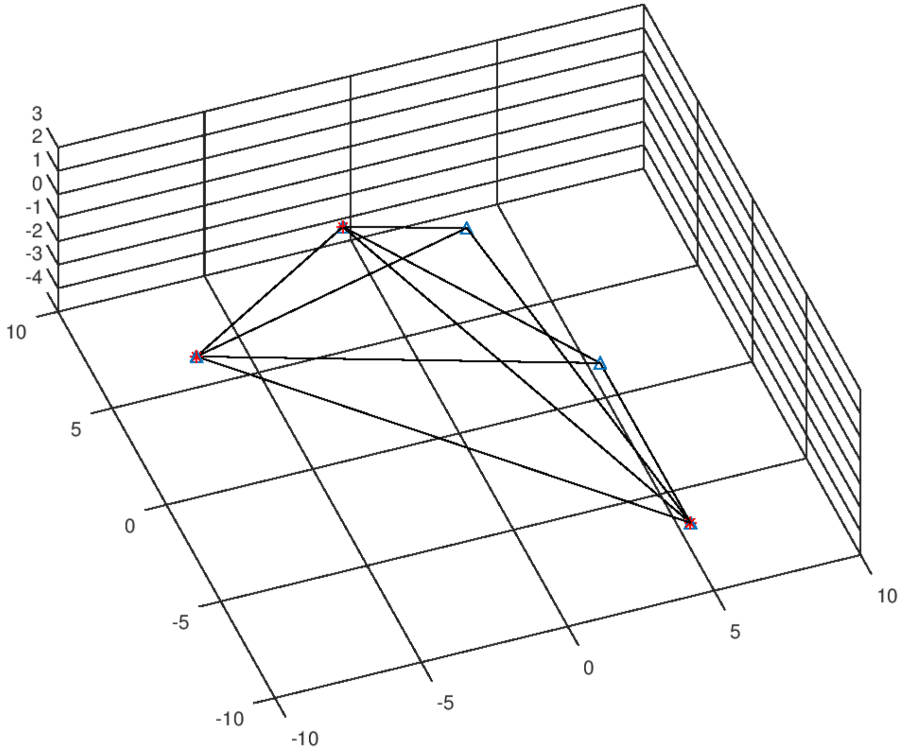
$$g33 = \frac{\partial s}{\partial w_1} = -\frac{\partial s}{\partial w_2} = -\frac{du}{|(du \quad dv \quad dw)|};$$

It is quite easy to add new observation types: Height differences, coordinate differences…

$$A_i = \begin{pmatrix} g11 & g12 & g13 & -g11 & -g12 & -g13 & g14 \\ g21 & g22 & g23 & -g21 & -g22 & -g23 & 0 \\ g31 & g32 & g33 & -g31 & -g32 & -g33 & 0 \end{pmatrix}$$

$$B_i = \begin{pmatrix} g11 & g12 & g13 & -g11 & -g12 & -g13 & -1 & 0 & 0 \\ g21 & g22 & g23 & -g21 & -g22 & -g23 & 0 & -1 & 0 \\ g31 & g32 & g33 & -g31 & -g32 & -g33 & 0 & 0 & -1 \end{pmatrix}$$

Example network



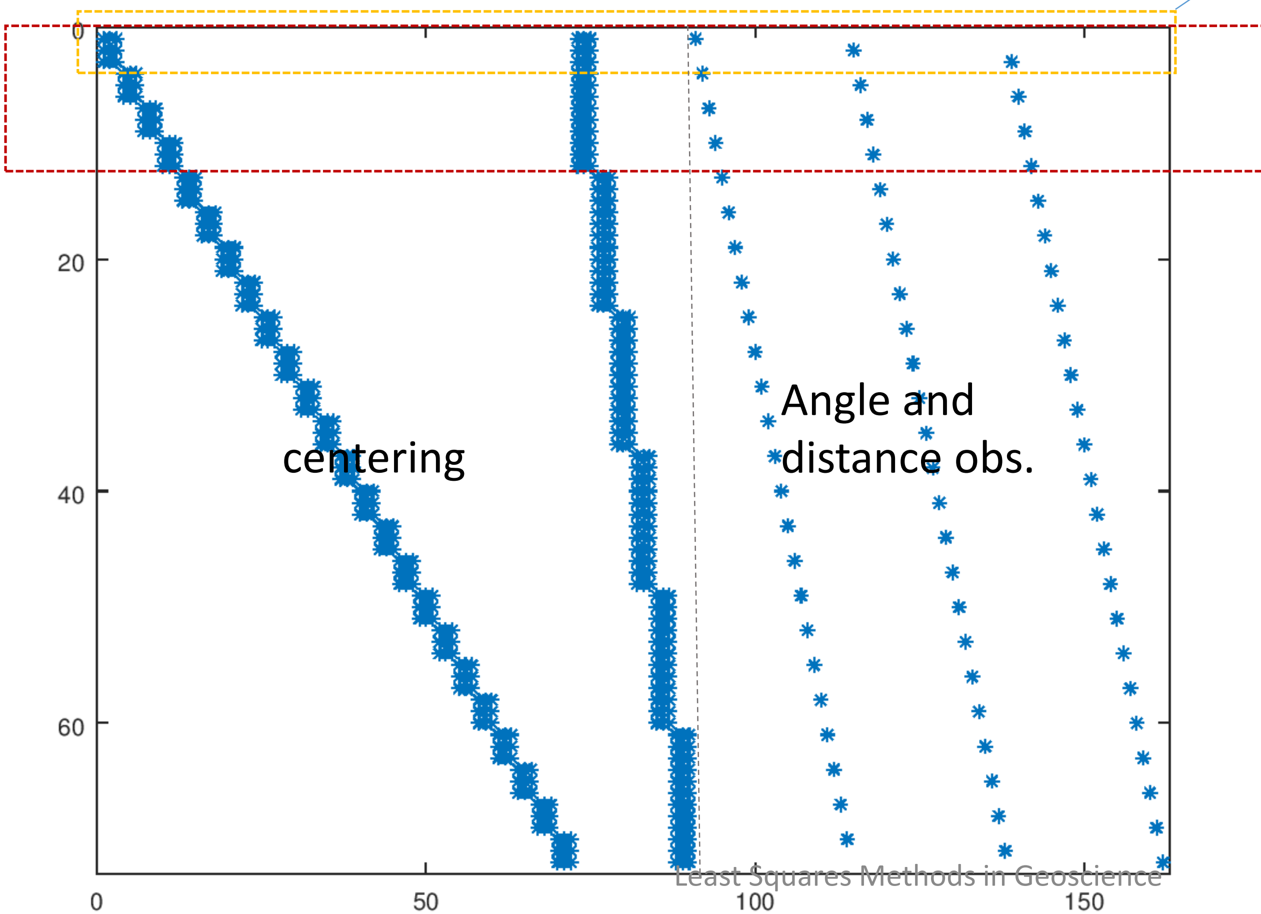
Red: station points, two setups at each station points
Blue: prism points

Example network

from	to	α_{0obs}	β_{obs}	s_{obs}	σ_α	β_β	σ_s	h_{sp}	h_{tp}
1	2	303.4648	91.12726	12.66408	0.0003	0.0005	0.00032	1.4887	1.4619
1	10	333.6974	78.90239	0	0.0003	0.0005	100.0003	1.4887	0
1	0	369.1179	89.32862	12.84218	0.0003	0.0005	0.00032	1.4887	0
1	3	378.2358	99.35329	16.91204	0.0003	0.0005	0.00032	1.4887	1.4719
1	2	41.00568	91.34972	12.65804	0.0003	0.0005	0.00032	1.489	1.4168
1	10	71.23692	78.8885	0	0.0003	0.0005	100.0003	1.489	0
1	0	106.6563	89.32219	12.84244	0.0003	0.0005	0.00032	1.489	0
1	3	115.7755	99.33971	16.91205	0.0003	0.0005	0.00032	1.489	1.4756
2	3	23.72309	105.8412	16.78997	0.0003	0.0005	0.00032	1.4144	1.4749
2	0	32.58689	97.79492	12.43505	0.0003	0.0005	0.00032	1.4144	0
2	10	42.9914	81.64232	0	0.0003	0.0005	100.0003	1.4144	0
2	1	100.3017	108.622	12.65716	0.0003	0.0005	0.00032	1.4144	1.4936
2	3	147.661	105.7652	16.78828	0.0003	0.0005	0.00032	1.4201	1.5026
2	0	156.5248	97.82854	12.43496	0.0003	0.0005	0.00032	1.4201	0
2	10	166.9293	81.72781	0	0.0003	0.0005	100.0003	1.4201	0
2	1	224.2395	108.6503	12.65797	0.0003	0.0005	0.00032	1.4201	1.4943
3	1	242.0176	100.8088	16.91266	0.0003	0.0005	0.00032	1.5031	1.4756
3	0	266.9403	75.23898	5.11965	0.0003	0.0005	0.00032	1.5031	0
3	10	280.789	82.16708	0	0.0003	0.0005	100.0003	1.5031	0
3	2	290.666	94.37759	16.7844	0.0003	0.0005	0.00032	1.5031	1.3847
3	1	375.5538	100.8701	16.91306	0.0003	0.0005	0.00032	1.5178	1.4766
3	0	0.480085	75.42832	5.11365	0.0003	0.0005	0.00032	1.5178	0
3	10	14.32567	82.25481	0	0.0003	0.0005	100.0003	1.5178	0
3	2	24.2044	94.45305	16.7829	0.0003	0.0005	0.00032	1.5178	1.3814

B-matrix

$$B_i = \begin{pmatrix} g11 & g12 & g13 & -g11 & -g12 & -g13 & -1 & 0 & 0 \\ g21 & g22 & g23 & -g21 & -g22 & -g23 & 0 & -1 & 0 \\ g31 & g32 & g33 & -g31 & -g32 & -g33 & 0 & 0 & -1 \end{pmatrix}$$



Observations to one target

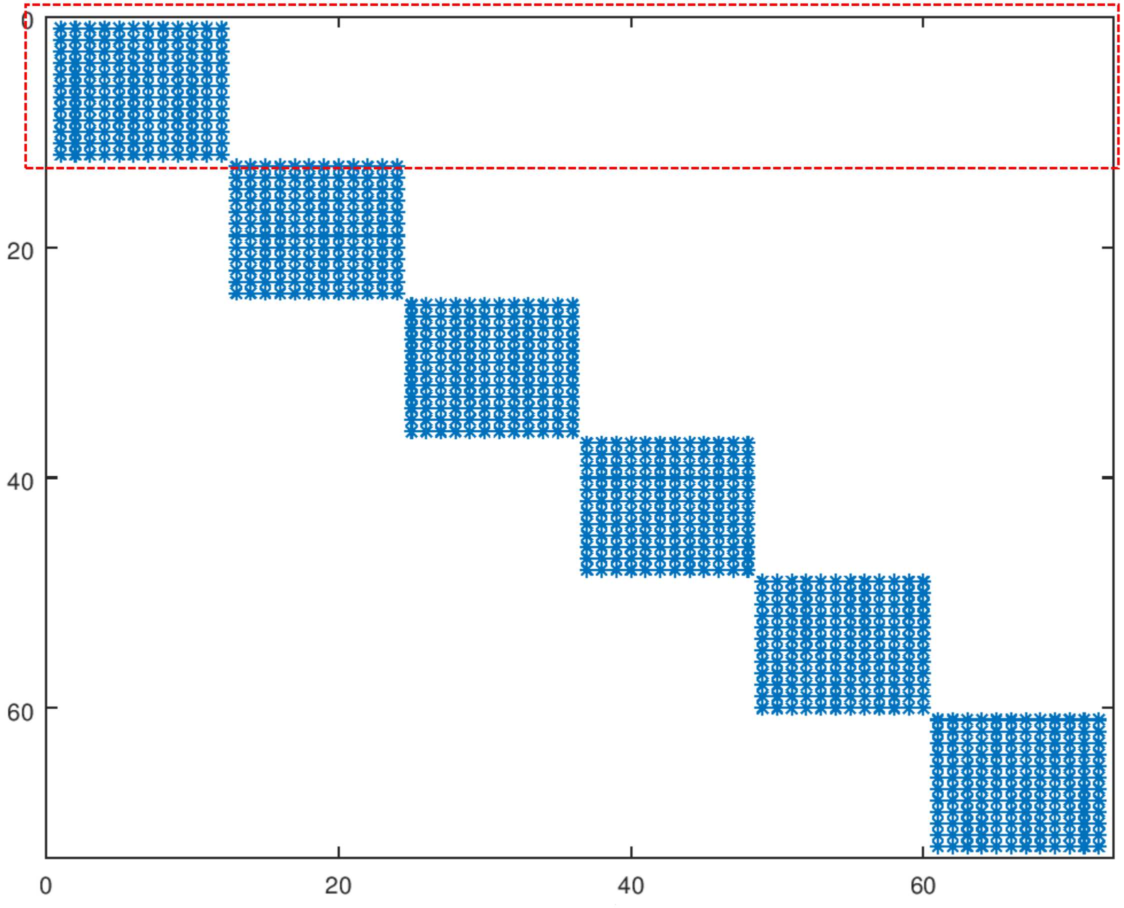
Observation from one station point

Number of rows equals to number of all angle and distance measurement in network

Number of columns equals to number of all centerings elements plus angle and distance measurement in network

Cofactor matrix of observationsQ

$$Q_{obs} = B\Sigma_{cx,cy,cz,\alpha,\beta,s}B^T$$



Observations from one station point are correlating

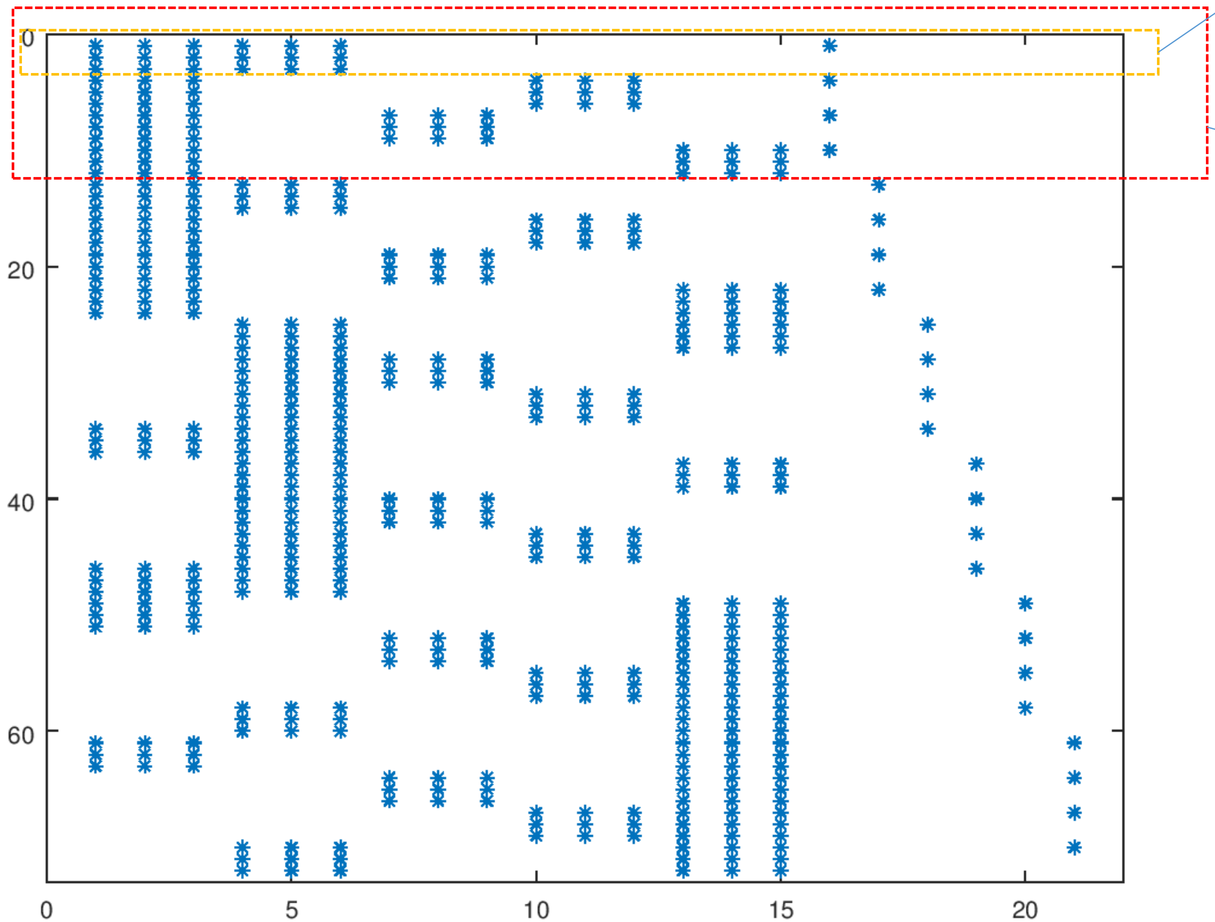
$$\boldsymbol{P = Q_{obs}{}^{-1}}$$

$$\sigma_{0apri}{}^2 = 1$$

Number of rows and columns equal to number of angle and distance observations in network

A-matrix

$$A_i = \begin{pmatrix} g11 & g12 & g13 & -g11 & -g12 & -g13 & g14 \\ g21 & g22 & g23 & -g21 & -g22 & -g23 & 0 \\ g31 & g32 & g33 & -g31 & -g32 & -g33 & 0 \end{pmatrix}$$



Observatons to one target points

Observations from one station point

Number of rows equals to Number of angle and distance observations (equations in network)

Number of columns equals to number of coordinates and orientation unknowns in network

y-vector

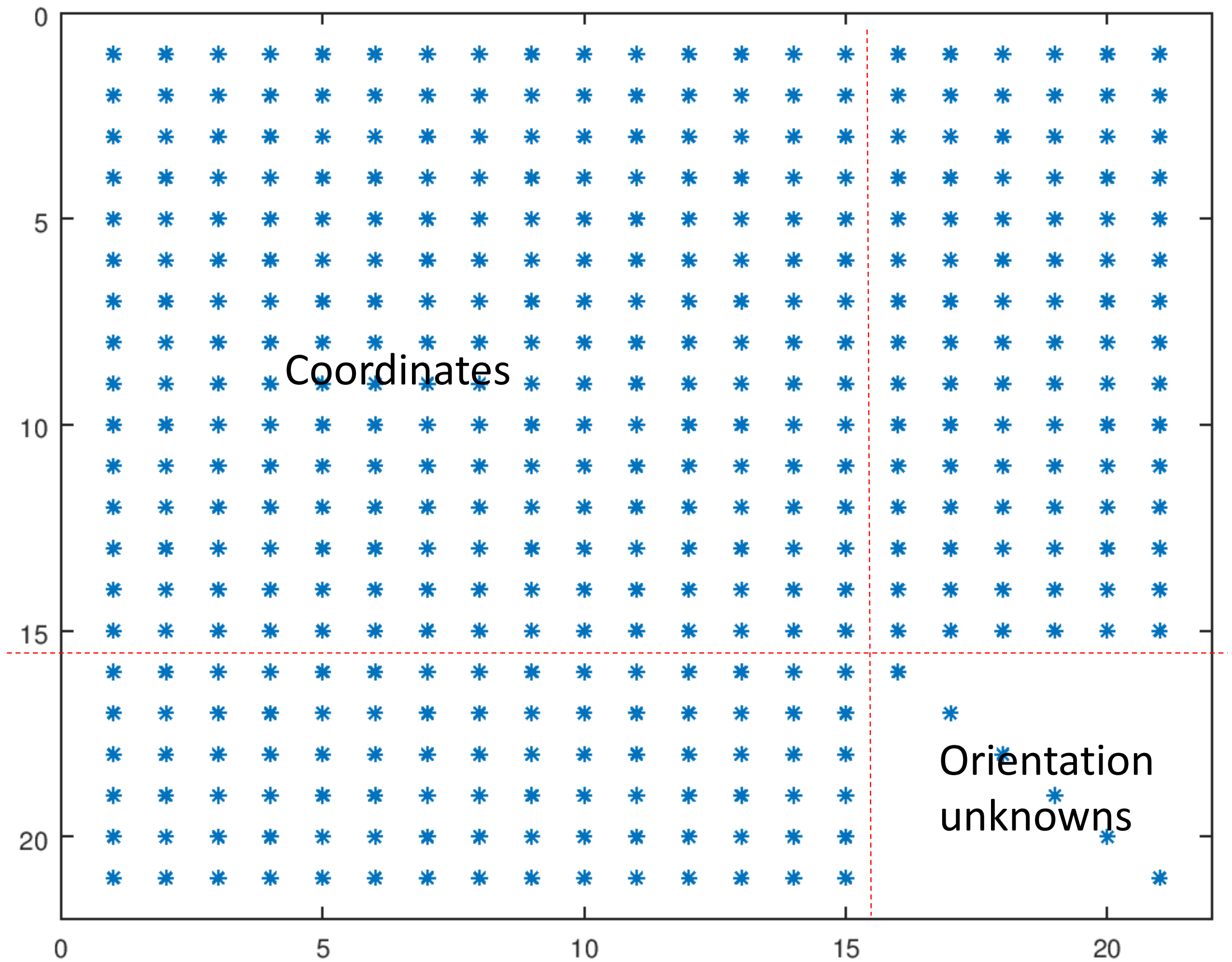
$$y_{\alpha_{0ij}} = \alpha_{0_{obs}} - \alpha_0(u_i + c_{ui}, v_i + c, w_i + c_{wi}, u_j + c_{uj}, v_j, w_j + c_{wj}, t_{0i})$$

$$y_{\beta_{0ij}} = \beta_{obs} - \beta(u_i+c_{ui}, v_i + c, w_i + c_{wi}, u_j + c_{uj}, v_j, w_j + c_{wj})$$

$$y_{s_{0ij}} = s_{obs} - s\ (u_i + c_{ui}, v_i + c, w_i + c_{wi}, u_j + c_{uj}, v_j, w_j + c_{wj})$$

Observed minus calculated for all observations. Size of y-vector is number of angle and distance measurement in network times one. The centering elements (in global system) are added to approximative coordinates

N-matrix without constraints



We have as many rows and colums as there are unknown parameters, here number of coordinates plus number of orientation unknowns

3D model for network with tilted polar instruments

- Instruments are not levelled, they can be in arbitary attitude.
- R_i is rotation from object coordinate system to instrument coordinate system
- Ra is rotation around the primary axis.
- Rz is rotation around the secondary axis.
- *dR1, dR2,dR3* are 3x3 matrices with partials of three rotation angles (Partial derivates are taken element by element of *Ri*.)
- k is distance and $Ra \ast Rz$ include the angle observations, *E0* and *E* are eccentric vector of the instrument in instrument system and p is unit vector of aiming in zero angle position
- The observations of one station are correlated
- Suitable for industrial measurements

3D model for network with tilted polar instruments

$$Q_{obsi} = Q_{obs} = B\Sigma_{cx,cy,cz,alfa,zen,k}B^T$$

y-vector

$yi = (k * Ra * Rz * p) - Ri * (X - X0) + E0 - E;$

A-matrix

#X0

$A11 = -Ri;$

#X

$A12 = Ri;$

#Ri

$A13 = [dR1 * (X - X0), dR2 * (X - X0), dR3 * (X - X0)];$

#

$Ai = [A11, A12, A13];$

B-matrix

#alfa

$B13 = -k * dRa * Rz * p;$

#zen

$B14 = -k * Ra * dRz * p;$

#k

$B15 = -Ra * Rz * p;$

#E0

$B11 = -eye(3);$

#E

$B12 = eye(3);$

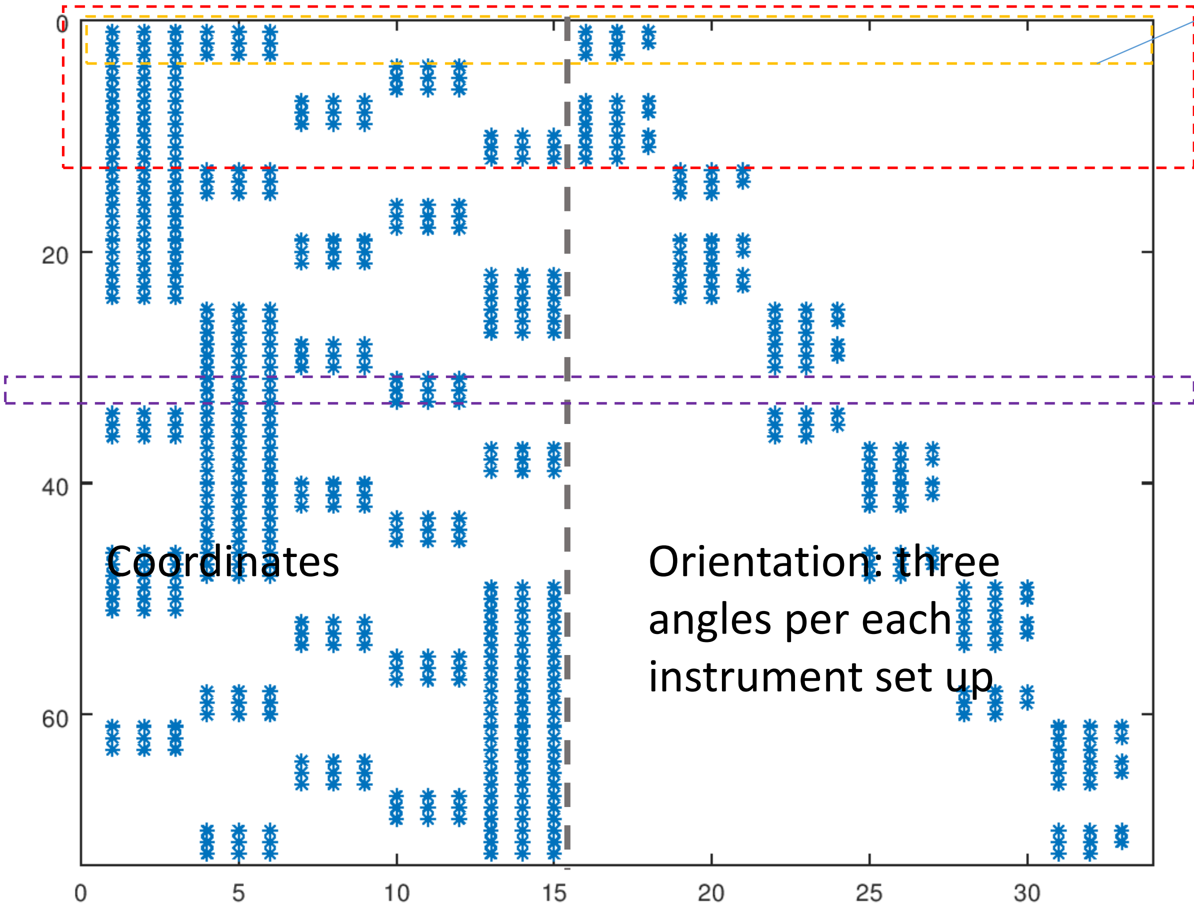
#

$Bi = [B11, B12, B13, B14, B15];$

- Instruments are not levelled, they can be in arbitary attitude.
- R_i is rotation from object coordinate system to instrument coordinate system
- Ra is rotation around the primary axis.
- Rz is rotation around the secondary axis.
- *dR1, dR2,dR3* are 3x3 matrices with partials of three rotation angles (Partial derivates are taken element by element of *Ri*.)
- k is distance and $Ra * Rz$ include the angle observations, *E0* and *E* are eccentric vector of the instrument in instrument system and p is unit vector of aiming in zero angle position
- The observations of one station are correlated

A-matrix (an example)

$$Ai = [A11, A12, A13];$$

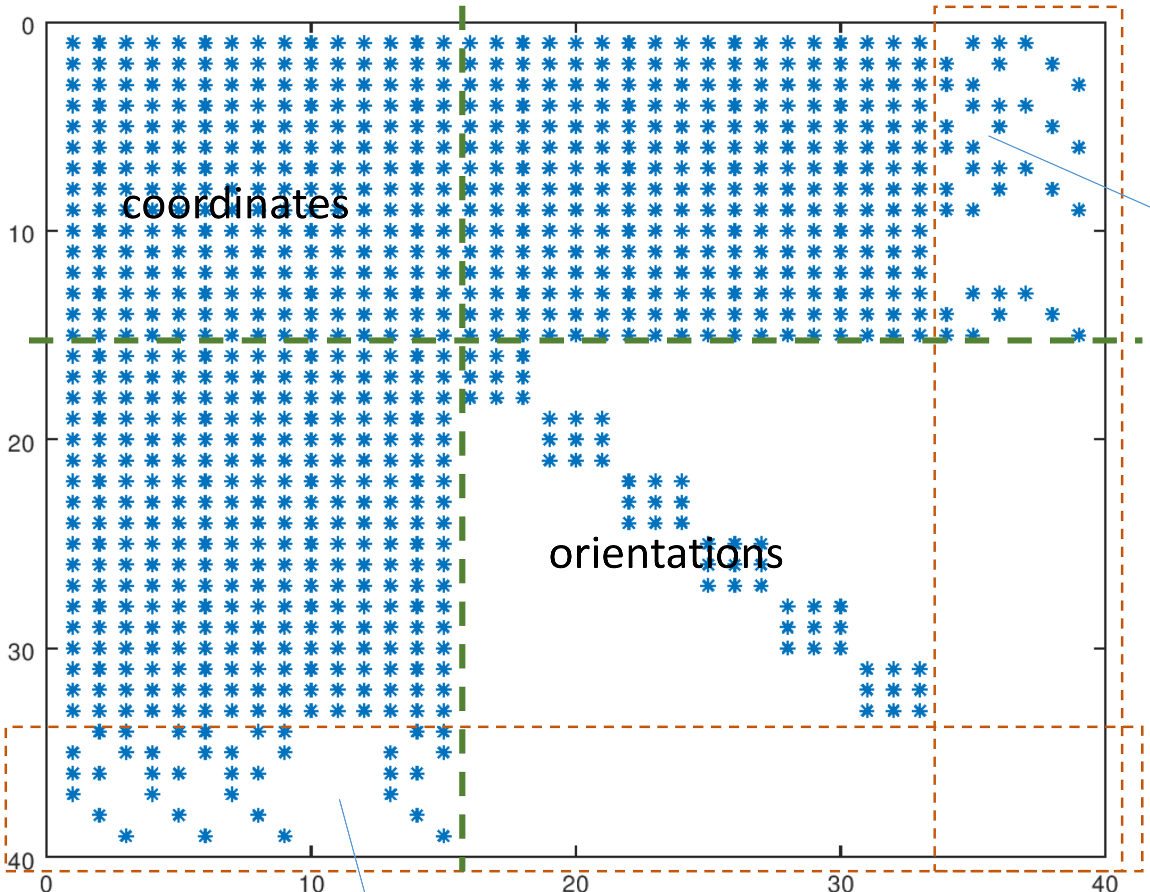


Observations to one target point

Observations from one station point

This target point is not participating the estimation of orientation

Structure of N with constraint equations



Constraints: in this case we have 6 (3 rotations and 3 translations)

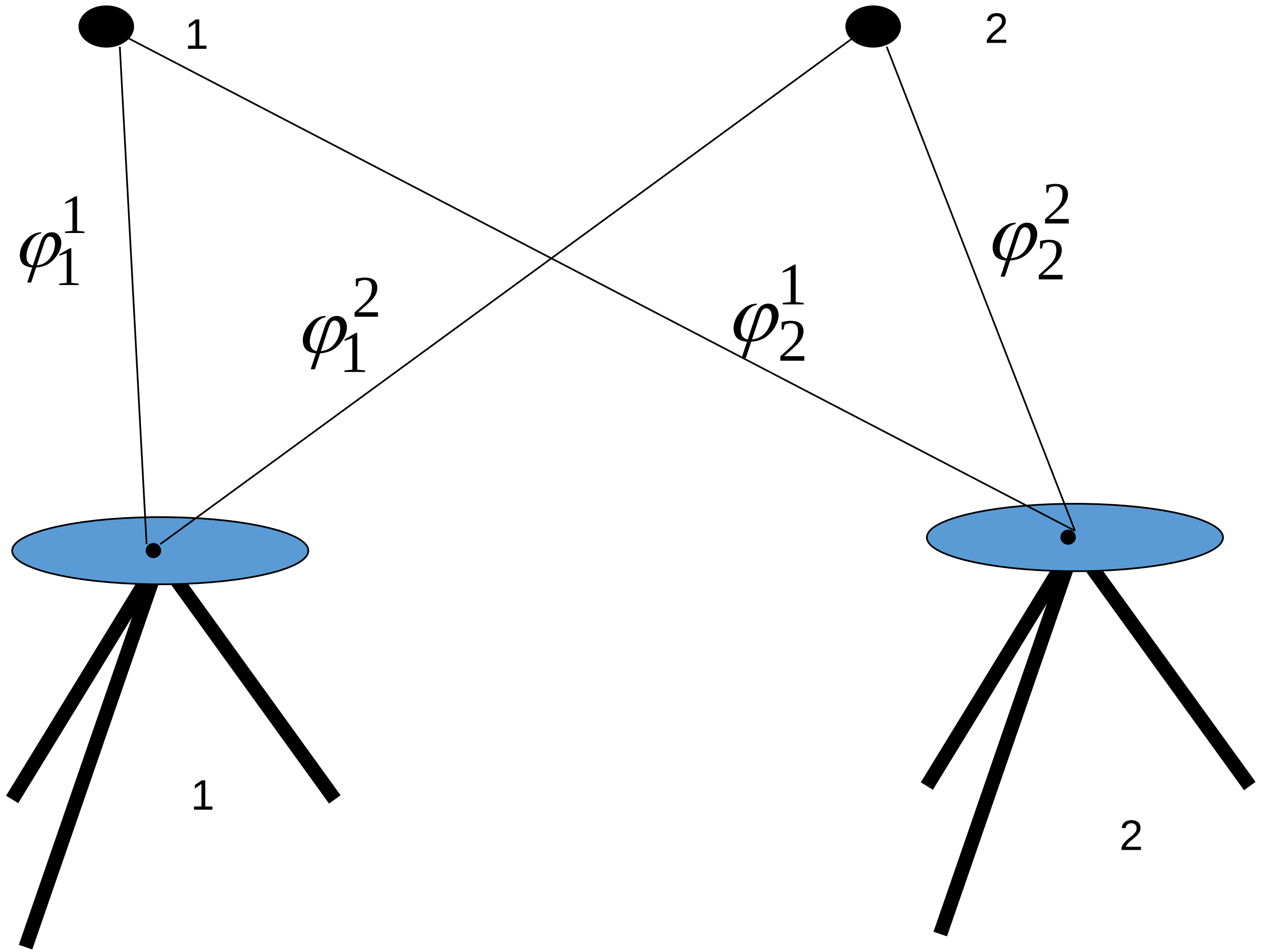
One point in this network is not a datum point

Algorithm

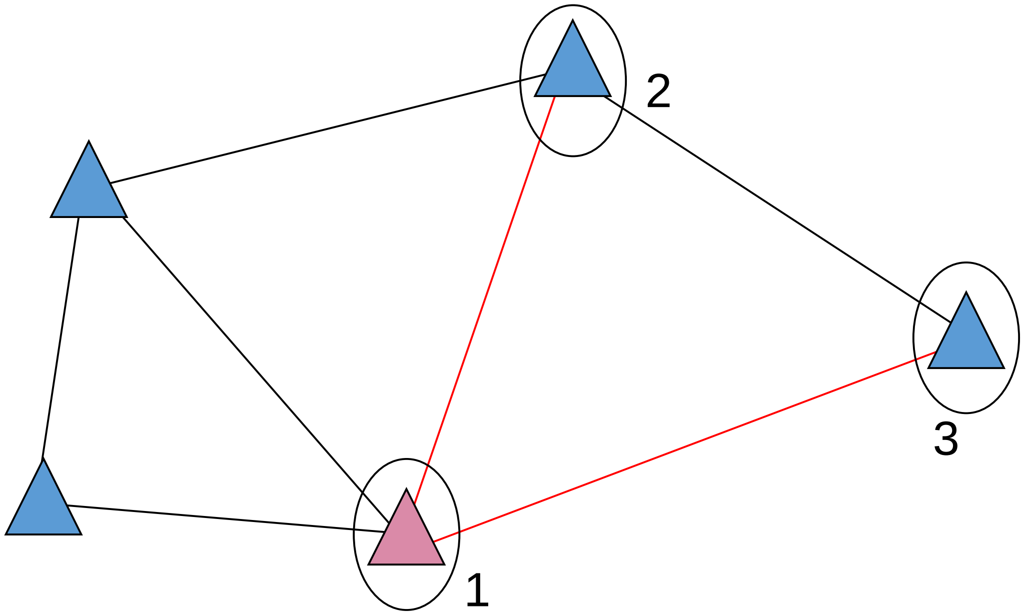
1. Read initial coordinates
2. Read datum points
3. Read observations, centering and precision of observations and centering elements
4. Form A, B, Qobs, P, y
5. Calculate normal matrix $N = A^T PA$ and normal equation vector $t = A^T Py$
6. Add datum information (3 translations and 1 rotation) to normal equations. Constraints or fixed points.
7. Solve for the corrections to initial values and add them to initial values
8. Iterate (back to 3) with new approximative values until corrections practically zeros

9. Precision, reliability, residuals, outliers

Double difference observations



Observations, unknowns, model constants



- <u>Observations:</u> double differences of phase observations
- <u>Constants ?:</u> Coordinates of satellites from pre calculated orbits
- <u>Unknown parameters:</u> Coordinates of the points

Double differences are linear combinations of phase observation

- We can form T(S-1)(R-1) linearily indipendent double differences per frequency
  - S is number of satellites, R is number of receivers, T number of epochs
- Here double differences are formed for three receivers one epoch and four satellites (one frequency)

$$\Sigma_{Dt} = J \Sigma_{\phi} J^T$$

$$J = \begin{pmatrix} 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

$$D_{12}^{12} = (\varphi_2^2 - \varphi_1^2) - (\varphi_2^1 - \varphi_1^1)$$
$$D_{13}^{12} = (\varphi_3^2 - \varphi_1^2) - (\varphi_3^1 - \varphi_1^1)$$
$$D_{12}^{13} = (\varphi_2^3 - \varphi_1^3) - (\varphi_2^1 - \varphi_1^1)$$
$$D_{13}^{13} = (\varphi_3^3 - \varphi_1^3) - (\varphi_3^1 - \varphi_1^1)$$
$$D_{12}^{14} = (\varphi_2^4 - \varphi_1^4) - (\varphi_2^1 - \varphi_1^1)$$
$$D_{13}^{14} = (\varphi_3^4 - \varphi_1^4) - (\varphi_3^1 - \varphi_1^1)$$

$$P_t = \Sigma_{Dt}^{-1}$$ Weight matrix for epoch t

Functional model

$$\varphi_r^s(t) = -\frac{f}{c} s_r^s - f(\delta^s - \delta_r) - \frac{f}{c}(-d_{ion} + d_{trop}) + N_r^s$$

Phase observation

$$\phi_r^s(t) = s_r^s + c(\delta^s - \delta_r) - d_{ion} + d_{trop} + \lambda N$$

Phase in metric form

$$\begin{aligned} D_{km}^{pq} &= (\phi_m^q - \phi_k^q) - (\phi_m^p - \phi_k^p) \\ &= s_{km}^{pq} + \lambda N_{km}^{pq} \\ f_{D_{km}^{pq}} &= (s_m^q - s_k^q) - (s_m^p - s_k^p) + \lambda N_{km}^{pq} - D_{km}^{pq} = 0 \end{aligned}$$

Double difference observation

$$s_r^s = \sqrt{(X^s - X_r)^2 + (Y^s - Y_r)^2 (Z^s - Z_r)^2}$$

Distance between satellite and receiver antenna

Design matrix and y-vector for epoch t

$$A_{D_{ep}} = \begin{pmatrix} \frac{\partial f_{D_{12}^{12}}}{\partial X_2} & \frac{\partial f_{D_{12}^{12}}}{\partial Y_2} & \frac{\partial f_{D_{12}^{12}}}{\partial Z_2} & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial f_{D_{13}^{12}}}{\partial X_3} & \frac{\partial f_{D_{13}^{12}}}{\partial Y_3} & \frac{\partial f_{D_{13}^{12}}}{\partial Z_3} & 0 & \lambda & 0 & 0 & 0 & 0 \\ \frac{\partial f_{D_{12}^{13}}}{\partial X_2} & \frac{\partial f_{D_{12}^{13}}}{\partial Y_2} & \frac{\partial f_{D_{12}^{13}}}{\partial Z_2} & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial f_{D_{13}^{13}}}{\partial X_3} & \frac{\partial f_{D_{13}^{13}}}{\partial Y_3} & \frac{\partial f_{D_{13}^{13}}}{\partial Z_3} & 0 & 0 & 0 & \lambda & 0 & 0 \\ \frac{\partial f_{D_{12}^{14}}}{\partial X_2} & \frac{\partial f_{D_{12}^{14}}}{\partial Y_2} & \frac{\partial f_{D_{12}^{14}}}{\partial Z_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \frac{\partial f_{D_{13}^{14}}}{\partial X_3} & \frac{\partial f_{D_{13}^{14}}}{\partial Y_3} & \frac{\partial f_{D_{13}^{14}}}{\partial Z_3} & 0 & 0 & 0 & 0 & 0 & \lambda \end{pmatrix}$$

$$y_t = -\begin{pmatrix} f_{D_{12}^{12}} \\ f_{D_{13}^{12}} \\ f_{D_{12}^{13}} \\ f_{D_{13}^{13}} \\ f_{D_{12}^{14}} \\ f_{D_{13}^{14}} \end{pmatrix}$$

Session solution

$$\begin{pmatrix} X_2 - X_{2_0} \\ Y_2 - Y_{2_0} \\ Z_2 - Z_{2_0} \\ X_3 - X_{3_0} \\ Y_3 - Y_{3_0} \\ Z_3 - Z_{3_0} \\ N_{12}^{12} - N_{12\,0}^{12} \\ N_{13}^{12} - N_{13\,0}^{12} \\ N_{12}^{13} - N_{12\,0}^{13} \\ N_{13}^{13} - N_{13\,0}^{13} \\ N_{12}^{14} - N_{12\,0}^{14} \\ N_{13}^{14} - N_{13\,0}^{14} \end{pmatrix} = \left(A^T P A\right)^{-1} A^T P y$$

Corrections to initial values

Ambiguities are still floating points

Iteration needed (non-linear model)

Floating point ambiguites to fixed integer ambiguites

- Ambiguites are tried to fix to integer values
- There might be more than one possible set of integers. The best set gives minimum variance (For short vectors up to 30km ambiguites should be found depending on the session length
- For long vectors it is not always possible to fix ambiguites
- **New adjustment with fixed ambiguites**

Covariance matrix of the session solution:

$$\Sigma_{\Delta X \Delta Y \Delta Z} = \sigma_0^2 (A^T P A)^{-1}$$

$$v^T P v = \min$$

Fixed-solution gives the coordinates (coordinate differences) and their covariances to GPS-vector network adjustment.

Variance propagation

Variances of phase observations

$J \Sigma J^T$

Covariance matrix of double differences

LSQ:

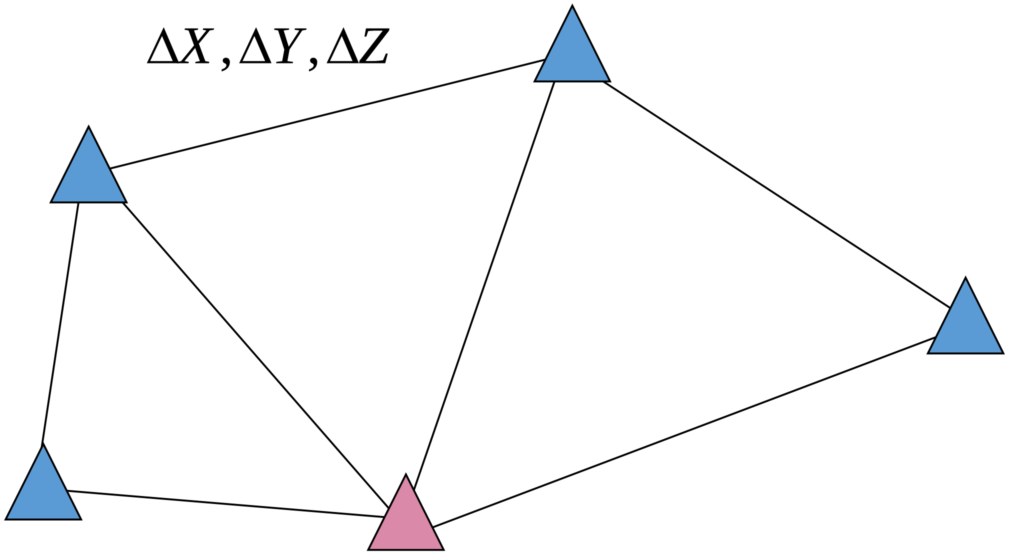
$$\Sigma_{XYZ} = \sigma_0^2 (A^T P A)^{-1}$$

Covariance matrix of vectors

Network adjustment

Covariance matrix from network adjustment

GNSS-vector network



<u>Unknown</u>: Coordinates of the points

<u>Observations:</u> Coordinate differences from vector processing $\Delta X, \Delta Y, \Delta Z$

<u>Weighting</u>: inverse of covariance matrix of vector components

Malli:

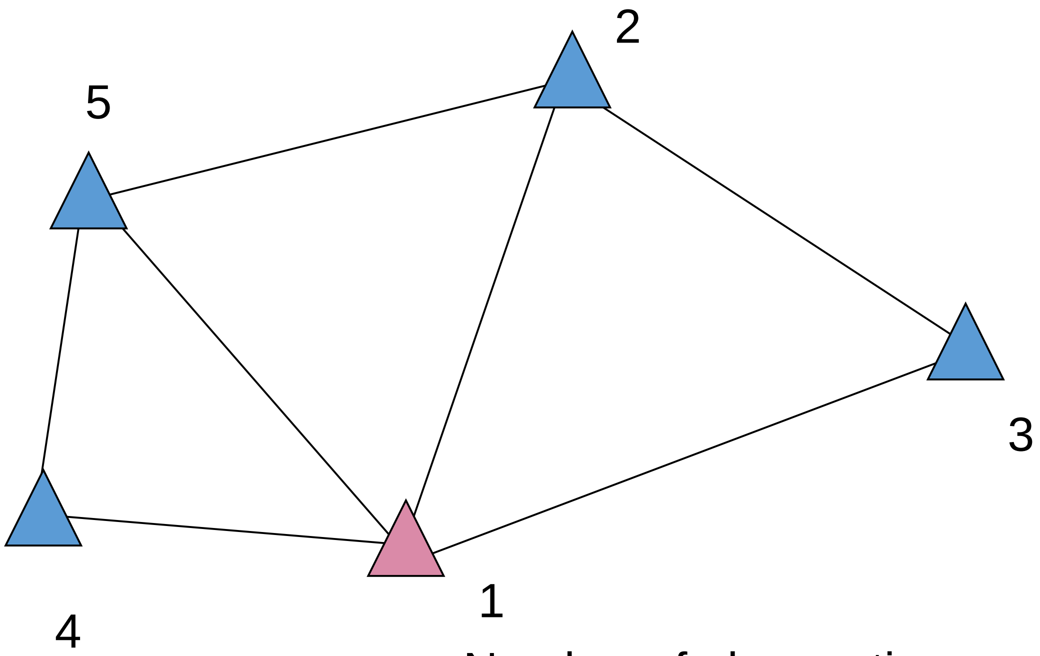
$$\left.\begin{aligned} X_j - X_i - \Delta X_{ij} &= 0 \\ Y_j - Y_i - \Delta Y_{ij} &= 0 \\ Z_j - Z_i - \Delta Z_{ij} &= 0 \end{aligned}\right\}$$

Number of vectors in adjustment

- In one session we get $\frac{R(R-1)}{2}$ vectors to networkadjustment but
- Only R-1 are linearly indipendent
- The rest of vectors $\frac{R(R-1)}{2} - (R-1) = \frac{R}{2} - 1$ are so called trivial vectors
- The network should be measured (sessions should be planned) so that none of the vectors in network is trivial (see JHS184)

- We still take all possible vectors (also trivial ones), to network adjustment, because usual commercial vector processing softwares are not able to solve for covariances between vectors. (Scientific softwares can)
- If we choose vectors, we will lose information
- When we take all vectors we get false redundance and perhaps over optimistic variances.

Non trivial vectors in example network



Observations : $\Delta X, \Delta Y, \Delta Z$

From to	session
1-3	A
1-2	A
2-3	B
2-5	B
1-4	C
1-5	C
4-5	D

Number of observations with trivial vectors: 21

Number of all observations: 36

Simple combination models for GPS network

$$s_t \begin{pmatrix} 1 & \kappa & -\phi \\ -\kappa & 1 & \omega \\ \phi & -\omega & 1 \end{pmatrix}_t \begin{pmatrix} X_{2L} - X_{1L} \\ Y_{2L} - Y_{1L} \\ Z_{2L} - Z_{1L} \end{pmatrix} - \begin{pmatrix} \Delta X_{12GPS} \\ \Delta Y_{12GPS} \\ \Delta Z_{12GPS} \end{pmatrix}_t = 0$$

- Functional model for GPS-vectors in observation epoch: each epoch has own rotation and scale
- Assumptions: between observation sessions rotation and scale difference but no deformation
- For small densification networks it is sufficient to assume no rotation or scale difference between epochs
  - Rotation matrix is unit matrix and scale 1

$$s \begin{pmatrix} 1 & \kappa & -\phi \\ -\kappa & 1 & \omega \\ \phi & -\omega & 1 \end{pmatrix} \begin{pmatrix} X_{2L} - X_{1L} \\ Y_{2L} - Y_{1L} \\ Z_{2L} - Z_{1L} \end{pmatrix} - \begin{pmatrix} \Delta X_{12GPS} \\ \Delta Y_{12GPS} \\ \Delta Z_{12GPS} \end{pmatrix} = 0$$

- Functional model for GPS-vectors in observation epoch: each epoch has same rotation and scale which differ from reference L
- Assumptions: between observation sessions no rotation and scale difference nor deformation, but there is rotation between GPS vectors and the reference system, but no deformation

SINEX-file

sinex

```
-SITE/ECCENTRICITY
*-------------------------------------------------------------------------------
+SOLUTION/EPOCHS
*CODE PT SOLN T _DATA_START_ __DATA_END__ _MEAN_EPOCH_
 1400  A    1 P 16:288:32310 16:289:86395 16:289:16152
 1406  A    1 P 16:288:32585 16:289:86395 16:289:16290
 1412  A    1 P 16:288:35685 16:289:86395 16:289:17840
 1424  A    1 P 16:288:31535 16:289:86395 16:289:15765
 1430  A    1 P 16:288:32035 16:289:86395 16:289:16015
 1436  A    1 P 16:288:31535 16:289:86395 16:289:15765
-SOLUTION/EPOCHS
*-------------------------------------------------------------------------------
+SOLUTION/ESTIMATE
*INDEX TYPE__ CODE PT SOLN _REF_EPOCH__ UNIT S __ESTIMATED VALUE____ _STD_DEV___
     1 STAX   1400  A    1 16:289:15750 m    0 0.289254960202000E+07 .946520E-04
     2 STAY   1400  A    1 16:289:15750 m    0 0.131180745749000E+07 .944962E-04
     3 STAZ   1400  A    1 16:289:15750 m    0 0.551263134307000E+07 .948550E-04
     4 STAX   1406  A    1 16:289:15750 m    2 0.289254744945638E+07 .958337E-04
     5 STAY   1406  A    1 16:289:15750 m    2 0.131180834485352E+07 .951214E-04
     6 STAZ   1406  A    1 16:289:15750 m    2 0.551263226844771E+07 .979107E-04
     7 STAX   1412  A    1 16:289:15750 m    2 0.289254910251813E+07 .958584E-04
     8 STAY   1412  A    1 16:289:15750 m    2 0.131181016583246E+07 .951217E-04
     9 STAZ   1412  A    1 16:289:15750 m    2 0.551263096143799E+07 .979569E-04
    10 STAX   1424  A    1 16:289:15750 m    2 0.289255169261810E+07 .958432E-04
    11 STAY   1424  A    1 16:289:15750 m    2 0.131180648250172E+07 .951265E-04
    12 STAZ   1424  A    1 16:289:15750 m    2 0.551263047556758E+07 .979165E-04
    13 STAX   1430  A    1 16:289:15750 m    2 0.289254997521260E+07 .958178E-04
    14 STAY   1430  A    1 16:289:15750 m    2 0.131180508794785E+07 .951116E-04
    15 STAZ   1430  A    1 16:289:15750 m    2 0.551263171909886E+07 .978960E-04
    16 STAX   1436  A    1 16:289:15750 m    2 0.289254785993023E+07 .958155E-04
    17 STAY   1436  A    1 16:289:15750 m    2 0.131180589355305E+07 .951141E-04
    18 STAZ   1436  A    1 16:289:15750 m    2 0.551263263809010E+07 .978917E-04
-SOLUTION/ESTIMATE
*-------------------------------------------------------------------------------
+SOLUTION/APRIORI
*INDEX TYPE__ CODE PT SOLN _REF_EPOCH__ UNIT S __APRIORI VALUE______ _STD_DEV___
     1 STAX   1400  A    1 16:289:15750 m    0 0.289254960202000E+07 .997027E-04
     2 STAY   1400  A    1 16:289:15750 m    0 0.131180745749000E+07 .995387E-04
     3 STAZ   1400  A    1 16:289:15750 m    0 0.551263134307000E+07 .999166E-04
     4 STAX   1406  A    1 16:289:15750 m    2 0.289254745136000E+07 .316228E+01
     5 STAY   1406  A    1 16:289:15750 m    2 0.131180834595000E+07 .316228E+01
     6 STAZ   1406  A    1 16:289:15750 m    2 0.551263227181000E+07 .316228E+01
     7 STAX   1412  A    1 16:289:15750 m    2 0.289254910503000E+07 .316228E+01
     8 STAY   1412  A    1 16:289:15750 m    2 0.131181016717000E+07 .316228E+01
     9 STAZ   1412  A    1 16:289:15750 m    2 0.551263096649000E+07 .316228E+01
    10 STAX   1424  A    1 16:289:15750 m    2 0.289255169377000E+07 .316228E+01
    11 STAY   1424  A    1 16:289:15750 m    2 0.131180648284000E+07 .316228E+01
```

Combination model

Combination Model: basic equations

$$\left\{\begin{array}{l}\begin{pmatrix} x_s^i \\ y_s^i \\ z_s^i \end{pmatrix} = \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix} + (t_s^i - t_0)\begin{pmatrix} \dot{x}^i \\ \dot{y}^i \\ \dot{z}^i \end{pmatrix} + T_k + D_k \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix} + R_k \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix} \\ \qquad + (t_s^i - t_k)\left[\dot{T}_k + \dot{D}_k \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix} + \dot{R}_k \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix}\right] \\ \begin{pmatrix} \dot{x}_s^i \\ \dot{y}_s^i \\ \dot{z}_s^i \end{pmatrix} = \begin{pmatrix} \dot{x}^i \\ \dot{y}^i \\ \dot{z}^i \end{pmatrix} + \dot{T}_k + \dot{D}_k \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix} + \dot{R}_k \begin{pmatrix} x^i \\ y^i \\ z^i \end{pmatrix}\end{array}\right.$$

$$\left\{\begin{array}{rcl} x_s^p & = & x^p + R2_k \\ y_s^p & = & y^p + R1_k \\ UT_s & = & UT - \frac{1}{f} R3_k \\ \dot{x}_s^p & = & \dot{x}^p \\ \dot{y}_s^p & = & \dot{y}^p \\ LOD_s & = & LOD \end{array}\right.$$

TRF combination

Altamimi, catref-man-Oct_2010.pdf