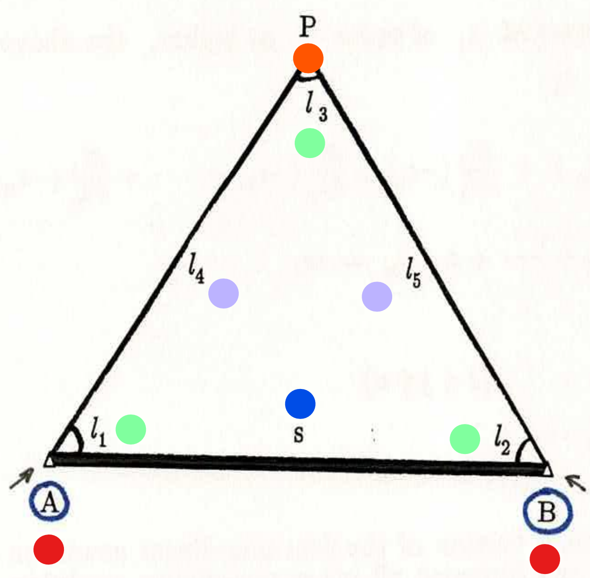
Ejemplo de Datum Defect o Rank Defect de una Red Geodésica

Fig. 2.4 illustrates a single point positioning network where A and B are known points at distance s = 100 meters. Three angles (l_1, l_2, l_3) and two distances (l_4, l_5) have been measured to:

$$l_1 = 60^0 \ 00' \ 05''$$
$$l_2 = 60^0 \ 00' \ 03''$$
$$l_3 = 59^0 \ 59' \ 58''$$
$$l_4 = 100.008 \text{ m}$$
$$l_5 = 99.997 \text{ m}$$

Fig. 2.4



For this network, we have : $m_0 = 4$, $d = 0$, $n_p = 1$, $n_d = 2$, $n_p \cdot n_d = 2$, $e = 0$, $m = 2$, $n = 5$, and finally $t = n - m = 3$. The first condition equation, of totally $t = 3$ independent condition equations, is obtained from the triangular misclosure :

$$\tilde{l}_1 + \tilde{l}_2 + \tilde{l}_3 - 180^0 = 0$$

or: $$\epsilon_1 + \epsilon_2 + \epsilon_3 = w_1 = l_1 + l_2 + l_3 - 180^0 = +6''$$

The 2nd and 3rd condition equations come from the sine theorem for plane triangles :

$$\frac{\tilde{l}_5}{\sin\tilde{l}_1} = \frac{\tilde{l}_4}{\sin\tilde{l}_2} = \frac{s}{\sin\tilde{l}_3}$$

or: $f_1(\tilde{l}_1, \tilde{l}_2, \tilde{l}_3, \tilde{l}_4, \tilde{l}_5) = s\sin\tilde{l}_1 - \sin\tilde{l}_3\,\tilde{l}_5 = 0$

$$f_2(\tilde{l}_1, \tilde{l}_2, \tilde{l}_3, \tilde{l}_4, \tilde{l}_5) = s\sin\tilde{l}_2 - \sin\tilde{l}_3\,\tilde{l}_4 = 0$$

The last two condition equations are non–linear, but can be linearized using (2.2.6), (2.2.7) and (2.2.8):

$$w_1 = f_1(l_1, l_2, l_3, l_4, l_5) = s\sin l_1 - \sin l_3\, l_5 = +4.295 \text{ mm}$$
$$w_2 = f_2(l_1, l_2, l_3, l_4, l_5) = s\sin l_2 - \sin l_3\, l_4 = -5.716 \text{ mm}$$

$$\frac{\partial f_1}{\partial l_1} = s\cos l_1 = +49997.9007 \text{ mm} \qquad \frac{\partial f_2}{\partial l_1} = 0$$

$$\frac{\partial f_1}{\partial l_2} = 0 \qquad \frac{\partial f_2}{\partial l_2} = s\cos l_2 = +49998.7404$$

$$\frac{\partial f_1}{\partial l_3} = -l_5\cos l_3 = -49999.3397 \text{ mm} \qquad \frac{\partial f_2}{\partial l_3} = -l_4\cos l_3 = -50004.8398$$

$$\frac{\partial f_1}{\partial l_4} = 0 \qquad \frac{\partial f_2}{\partial l_4} = -\sin l_3 = -0.866\,020\,556$$

$$\frac{\partial f_1}{\partial l_5} = -\sin l_3 = -0.866\,020\,556 \qquad \frac{\partial f_2}{\partial l_5} = 0$$

The two linearized condition equations then follows:

$$+49997.9007\,\epsilon_1 - 49999.3397\,\epsilon_3 - 0.866020556\,\epsilon_5 = +4.295 \text{ mm}$$
$$+49998.7404\,\epsilon_2 - 50004.8398\,\epsilon_3 - 0.866020556\,\epsilon_4 = -5.716 \text{ mm}$$

Notice that in the last two equations above, ϵ_1, ϵ_2 and ϵ_3 are in radian while ϵ_4 and ϵ_5 are in mm just as w_4 and w_5. If we want ϵ_1, ϵ_2 and ϵ_3 to be in arc second ("), we then have to divide their corresponding coefficients by the conversion constant $\rho'' \approx 206\,265''$. After this conversion, the whole condition equation system will become:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ +0.24239644 & 0 & -0.24240341 & 0 & -0.86602056 \\ 0 & +0.24240050 & -0.24243008 & -0.86602056 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_1\,'' \\ \epsilon_2\,'' \\ \epsilon_3\,'' \\ \epsilon_4\,^{mm} \\ \epsilon_5\,^{mm} \end{bmatrix}$$

$$= \begin{bmatrix} +6.000\,'' \\ +4.295\,^{mm} \\ -5.716\,^{mm} \end{bmatrix}, \qquad \Longrightarrow \qquad \underset{3\times 5}{B}\ \underset{5\times 1}{\epsilon} = \underset{3\times 1}{W}$$

A*v = f   CONDITION ADJUSTMENT

→ In this example, the network to be adjusted contains two different types of observations (i.e. angles and distances) and thus empirical weighting strategy outlined in §1.1.3 is not directly applicable. Instead, we use the definition (1.1.5b) to calculate weights of all observations. Assume that all three angles have a priori standard error of ±6" and that both two distances have a priori standard error of ±3 mm. For a unit weight standard error chosen as $\sigma_0 = \pm 3$ mm, we can obtain the weight matrix of ϵ :

$$\underset{5\times5}{P} = \begin{bmatrix} 0.25\,(\text{mm}/'')^2 & & & & \\ & 0.25\,(\text{mm}/'')^2 & & & \\ & & 0.25\,(\text{mm}/'')^2 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

where the last two weights are unitless, as the chosen σ_0 has unit mm.

With different types of observations, the relative weighting between different types becomes a difficult but important problem in geodetic adjustment. In Chapter §4, we will describe some methods which use the available observation data to obtain better weighting results for the observation data.

Using matrices B, P and W obtained above, the condition adjustment can be carried out in the usual way. The normal equation matrix can be found:

$$BP^{-1}B^T = \begin{bmatrix} 12.00000000 & -0.00002791 & -0.00011828 \\ -0.00002791 & 1.22005339 & 0.23506351 \\ -0.00011828 & 0.23506351 & 1.22011299 \end{bmatrix}$$

The estimated residuals are given by (2.1.10):

$$\hat{\epsilon} = P^{-1}B^T(BP^{-1}B^T)^{-1}W = \begin{bmatrix} +6.45\,'' \\ -3.40\,'' \\ +2.95\,'' \\ +4.82\,\text{mm} \\ -3.98\,\text{mm} \end{bmatrix}$$

The adjusted observations are given by (2.1.11):

$$\hat{L} = L - \hat{\epsilon} = \begin{bmatrix} 59^0\ 59'\ 58.55'' \\ 60^0\ 00'\ 06.40'' \\ 59^0\ 59'\ 55.05'' \\ 100.0032\ \text{m} \\ 100.0010\ \text{m} \end{bmatrix}$$

The a posteriori estimate of the variance factor $\sigma_0{}^2$ is obtained from (2.1.13):

$$\rightarrow\quad \hat{\sigma}_0{}^2 = \frac{\hat{\epsilon}^T P \hat{\epsilon}}{t} = \frac{1}{t}\sum_{i=1}^{n}\left\{p_i\,\hat{\epsilon}_i\,\hat{\epsilon}_i\right\} \approx 18.1885\ \text{mm}^2 \quad \text{or:} \quad \hat{\sigma}_0 \approx \pm 4.26\ \text{mm}$$

The cofactor matrix of the adjusted observations is obtained from (2.1.32):

$$Q_{\hat{L}\hat{L}} = P^{-1} - P^{-1}B^T(BP^{-1}B^T)^{-1}BP^{-1}$$

$$= \begin{bmatrix} +1.8664 & -1.1792 & -0.6872 & -0.1377 & +0.7148 \\ -1.1792 & +1.8664 & -0.6872 & +0.7148 & -0.1377 \\ -0.6872 & -0.6872 & +1.3744 & -0.5771 & -0.5771 \\ -0.1377 & +0.7148 & -0.5771 & +0.3616 & +0.1230 \\ +0.7148 & -0.1377 & -0.5771 & +0.1230 & +0.3616 \end{bmatrix}$$

From $\hat{\sigma}_0$ and $Q_{\hat{L}\hat{L}}$ computed above, one can estimate the standard errors of the observations after adjustment:

$$\sigma_{\hat{l}_1} = \hat{\sigma}_0 \sqrt{1.8664} = \pm 5.83''$$

$$\sigma_{\hat{l}_2} = \hat{\sigma}_0 \sqrt{1.8664} = \pm 5.8''$$

$$\sigma_{\hat{l}_3} = \hat{\sigma}_0 \sqrt{1.3744} = \pm 5.0''$$

$$\sigma_{\hat{l}_4} = \hat{\sigma}_0 \sqrt{0.3616} = \pm 2.6''$$

$$\sigma_{\hat{l}_5} = \hat{\sigma}_0 \sqrt{0.3616} = \pm 2.6''$$