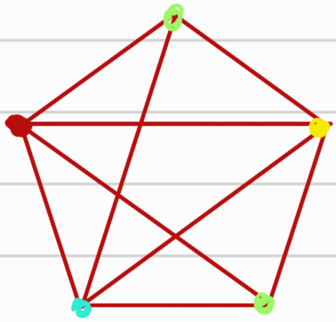
$P_{2,n}$

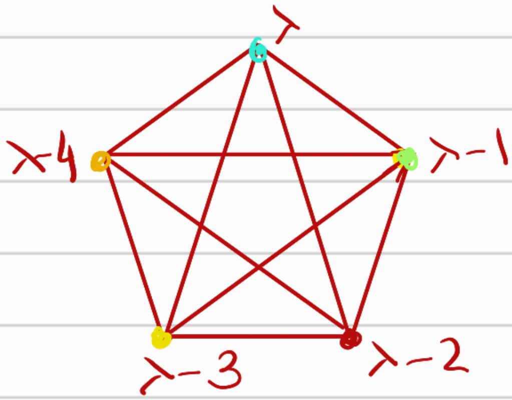
$\chi(P_{2,n})=2$

K_5 menos una arista



$\chi(K_5-e)=4.$

K_n:



$$P_{K_n}(\lambda)=\lambda(\lambda-1)(\lambda-2)\cdots(\lambda-(n-1))$$
$$=\lambda\cdots(\lambda-n+1)$$

$\chi(K_n)=5$

P_n:

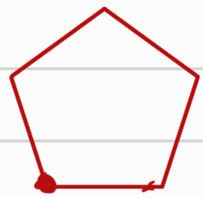
$$P_{P_n}(\lambda)=\lambda(\lambda-1)^{n-1}$$

$\chi(K_n)=2$

C_n : $p(C_n) = p(C_n - \{u,v\}) - p(C_{n_{u=v}})$

$C_n - \{u,v\}$

$C_{n_{u=v}}$



$$P_{C_6}(\lambda) = P_{P_6}(\lambda) - P_{C_5}(\lambda)$$

Cn gnl

$$P_{C_n}(\lambda) = P_{P_n}(\lambda) - P_{C_{n-1}}(\lambda)$$

$$P_{C_3}(\lambda) = \lambda(\lambda-1)(\lambda-2) = (\lambda-1)(\lambda^2-2\lambda) = (\lambda-1)((\lambda-1)^2-1) = (\lambda-1)^3 - (\lambda-1)$$

$$P_{C_4}(\lambda) = \lambda(\lambda-1)(\lambda-1) + \lambda(\lambda-1)(\lambda-2)(\lambda-2)$$
$$= \lambda(\lambda-1)^2 + (\lambda-1)(\lambda(\lambda-2)^2)$$
$$= (\lambda-1)^4 + (\lambda-1)$$

$$P_{C_n}(\lambda) = P_{P_n}(\lambda) - P_{C_{n-1}}(\lambda).$$

$$P_{C_3}(\lambda) = (\lambda-1)^3 - (\lambda-1)$$

$$P_{C_4}(\lambda) = (\lambda-1)^4 + (\lambda-1)$$

$$P_{C_5}(\lambda) = \lambda(\lambda-1)^4 - (\lambda-1)^4 - (\lambda-1)$$
$$= (\lambda-1)^4(\lambda-1) - (\lambda-1)$$
$$= (\lambda-1)^5 - (\lambda-1)$$

$$P_{C_6}(\lambda) = \lambda(\lambda-1)^5 - (\lambda-1)^5 + (\lambda-1)$$
$$= (\lambda-1)^5(\lambda-1) + (\lambda-1)$$
$$= (\lambda-1)^6 + (\lambda-1)$$

¿$P_{C_n}(\lambda) = (\lambda-1)^n + (-1)^n(\lambda-1)$?

Prueba por inducción:

P.B: $P_{C_3}(\lambda) = (\lambda-1)^3 + (-1)^3(\lambda-1)$ ✓

H.I.: $P_{C_k}(\lambda) = (\lambda-1)^k + (-1)^k(\lambda-1)$

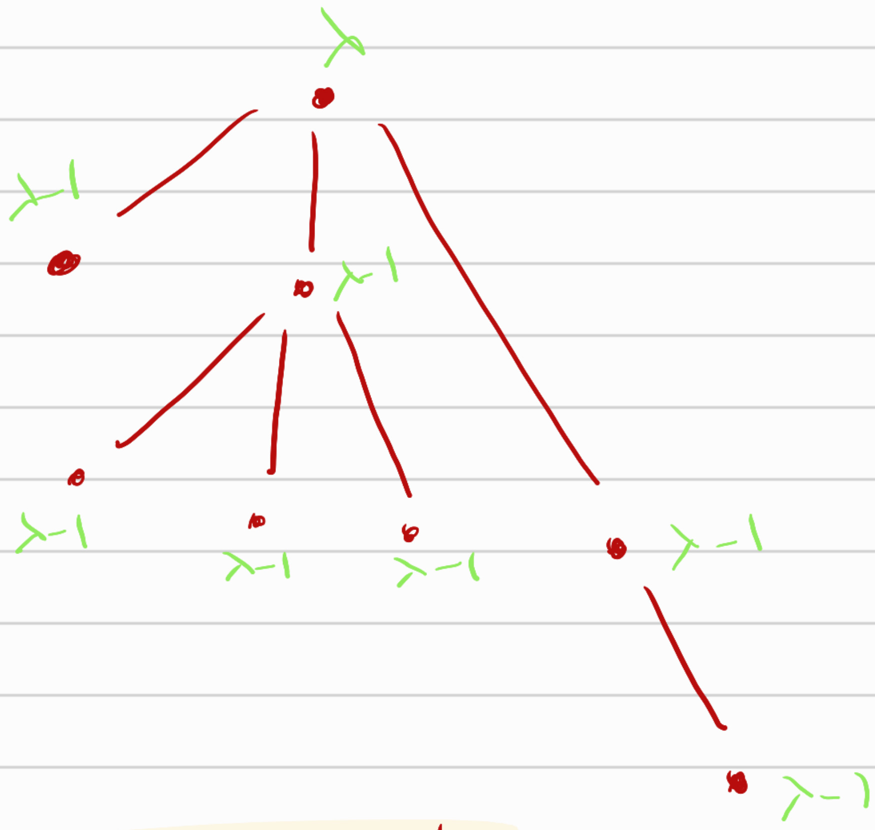
T.I. $P_{C_{k+1}}(\lambda) = (\lambda-1)^{k+1} + (-1)^k(\lambda-1)$?

dem: $P_{C_{k+1}}(\lambda) = \underbrace{P_{P_{n+1}}(\lambda)}_{\lambda(\lambda-1)^n} - \underbrace{P_{C_k}(\lambda)}_{H.I.}$

...

$$X(P_n) = \begin{cases} 3 & n \text{ impar} \\ 2 & n \text{ par} \end{cases}$$

Ejercicio 10. Sea G un árbol con $n \geq 2$ vértices. Hallar el polinomio y número cromático de G.



$P_{T_n}(\lambda) = \lambda(\lambda-1)^{n-1}$

Prueba por inducción en nº vértices.

P.B: 2 vértices $P_{T_2}(\lambda) = \lambda(\lambda-1)$ ✓

P. Inductivo:

H.I.: n vértices: $P_{T_n}(\lambda) = \lambda(\lambda-1)^{n-1}$

T.I. n+1 vértices $P_{T_{n+1}}(\lambda) = \lambda(\lambda-1)^{n}$

dem: Si le sacamos una hoja a T_{n+1} nos queda un árbol con n vértices. Por H.I. tenemos $\lambda(\lambda-1)^{n-1}$ coloraciones para este subarbol y $(\lambda-1)$ formas de colorear la hoja

$\Rightarrow$ Por la Regla del Producto hay $\lambda(\lambda-1)^{n-1} \cdot (\lambda-1) = \lambda(\lambda-1)^{n}$ coloraciones

Ejercicio 12. Sea $G = (V, E)$ un grafo y $\Delta(G) = \max_{v \in V(G)} gr(v)$.

(a) Demostrar que $\chi(G) \leq \Delta(G) + 1$.

(b) Dar un ejemplo de grafo que cumpla la igualdad.

Inducción en $n = |V|$

P.B.: • $n = 1$ $\quad \chi(G) = 1 \quad \Delta(G)+1 = 1$ ✓

H.I. Si G tiene n vértices $\Rightarrow$ $\chi(G) \leq \Delta(G)+1$

T.I. Si G tiene $n+1$ vértices $\Rightarrow$ $\chi(G) \leq \Delta(G)+1$

dem:

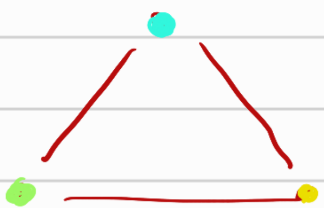
Sea G un grafo con $n+1$ vértices, sea u un vértice cualquiera. $G-u$ es grafo con n vértices.

$\Rightarrow \chi(G-u) \leq \Delta(G-u)+1$

$\leq \Delta(G)+1$

Finalmente, u es ady a $gr(u) \leq \Delta(G)$ vértices $\Rightarrow$ si tengo $\Delta(G)+1$ colores $\Rightarrow$ puedo colorear u.

Ej:



$\Delta(G) = 2$

$\chi(G) = 3$

Ej: $\Delta(K_n) = n-1$

$\chi(K_n) = n$