2 cosas - 1- práctico (Esto) → Ejercicios en papel

2. Laboratorio. (Miercoles y Viernes) → Ejercicios en computadora.

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Práctico 0.

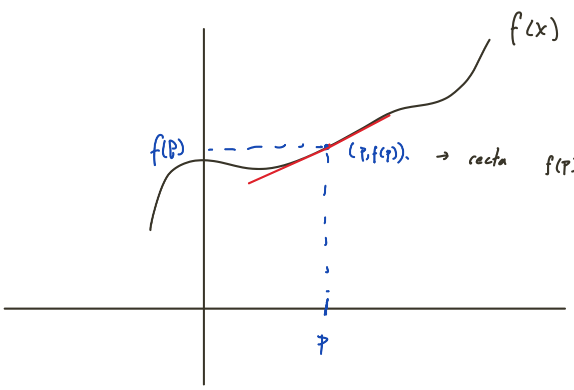
1- Octave.

2- Repaso.

Polinomio de Taylor.

$f: \mathbb{R} \to \mathbb{R}$. y buscamos una aproximación polinomial de grado n.

$f(x)$



→ recta $f(p) + f'(p)(x-p) = r(x)$.

$$f(x) = f(p) + f'(p)(x-p) + \frac{f''(p)}{2}(x-p)^2 + \frac{f'''(p)}{3!}(x-p)^3 + \cdots + \frac{f^{(n)}(p)}{n!}(x-p)^n + r_n(x)$$

Taylor.

Taylor de $f(x) = e^x$ $x_0 = 0$. $n \in \mathbb{N}$.

$n=0 \to f(0) = e^0 = 1$

$n=1 \to f(0), f'(0)$. $f'(0) = (e^x)'(0) = e^0 = 1$.

$T_1 f_0(x) = f(0) + f'(0)(x-0) = 1 + 1(x) = 1 + x$.

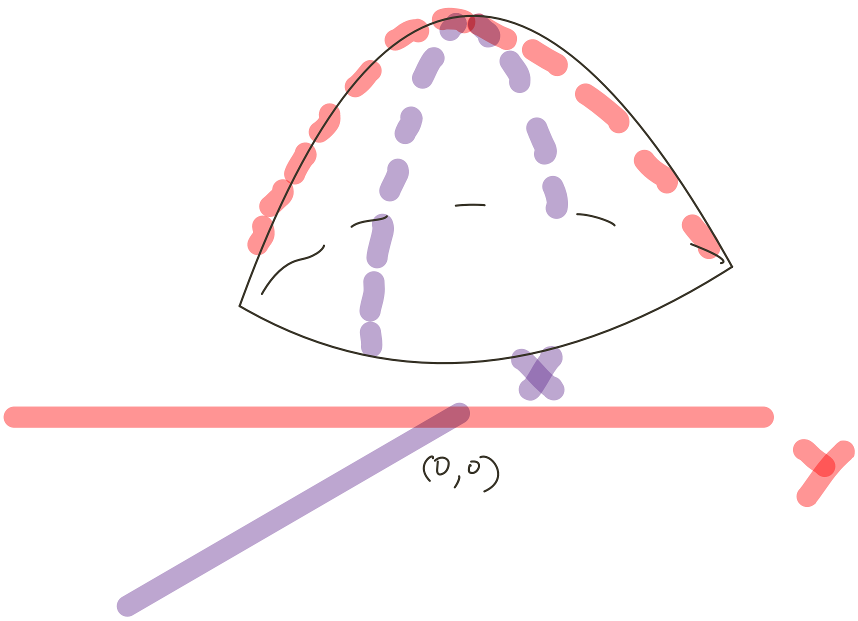
$f''(x) = (e^x)'' = ((e^x)')' = (e^x)' = e^x$ → y vale en general. $f^{(n)}(x) = e^x$.

$$f^{(n)}(x)\Big|_{x=0} = e^{x}\Big|_{x=0} = e^{0} = 1.$$

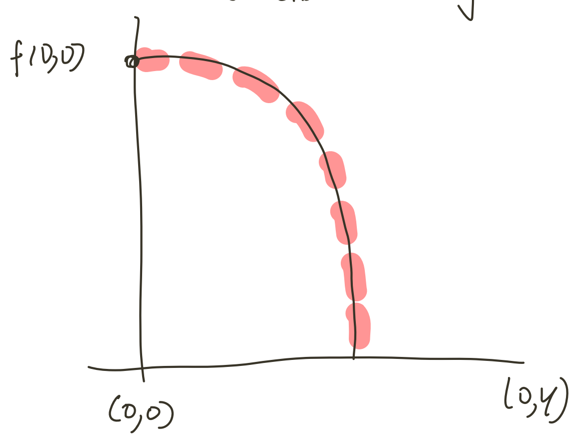
$$T_n f_0 (x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = \sum_{i=0}^{n} \frac{x^i}{i!}$$

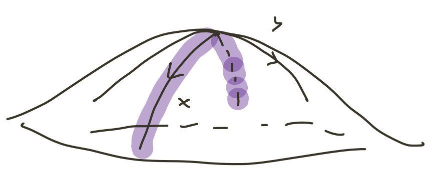
$$\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x.$$

Taylor en varias variables.



sección del dibujo.





$$Tf_{(P_1,P_2)}(x,y) = f(P_1,P_2) + f_x(P_1,P_2)(x-P_1) + f_y(P_1,P_2)(y-P_2)$$

$$+ f_{xx}(P_1,P_2)\frac{(x-P_1)^2}{2} + f_{yy}(P_1,P_2)\frac{(y-P_2)^2}{2} + \cancel{\tfrac{1}{2} f_{xy}(P_1,P_2)(x-P_1)(y-P_2)}$$

$$\cancel{\tfrac{1}{2} f_{yx}(P_1,P_2)(y-P_2)(x-P_1)}$$ son iguales.

$$+ f_{yx}(P_1,P_2)(x-P_1)(y-P_2)$$

$$f_{xy} = f_{yx}.$$

$f(x,y) = \frac{y}{x}$ $\quad (x_0, y_0) = (1,1) = P \quad n = 2.$

$f(p), \quad f_x(p), \quad f_y(p), \quad f_{xx}(p), \quad f_{xy}(p), \quad f_{yy}(p).$

$f(p) = \frac{1}{1} = 1. \quad f_x(p) = \partial_x\left(\frac{y}{x}\right) = y\,\partial_x\left(\frac{1}{x}\right) = y\,\partial_x\left(x^{-1}\right) = -\frac{y}{x^2}\Big|_p = -1.$

$$f_y(p) = \frac{1}{x}\,\partial_y(y) = \frac{1}{x}\Big|_p = 1.$$

$$f_{xx} = \partial_x(f_x) = \partial_x\left(-\frac{y}{x^2}\right) = -y\,\partial_x\left(\frac{1}{x^2}\right) = -y\,\partial_x\left(x^{-2}\right) = \frac{2y}{x^3}\Big|_p = 2.$$

$$f_{yy} = \partial_y(f_y) = \partial_y\left(\frac{1}{x}\right) = 0.$$

$$f_{xy} = \partial_x\left(\frac{1}{x}\right) = -\frac{1}{x^2}\Big|_p = -1.$$

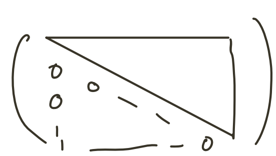
$$Tf_{(1,1)}(x,y) = 1 + (-1)(x-1) + 1\,(y-1) + \frac{\cancel{2}}{\cancel{2}}(x-1)^2 + \cancel{0 \cdot \frac{(y-1)^2}{2}} + (-1)(x-1)(y-1)$$

10) $A = \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad$ hallar $Q, R \;/\; A = QR$, Q ortogonal
R triangular superior.

Q ortogonal? $\quad Q = \begin{pmatrix} | & & | \\ c_1 & \cdots & c_n \\ | & & | \end{pmatrix}$ $\quad \{c_1, \dots, c_n\} \overset{\text{Bon}}{\subset} \mathbb{R}^n.$ $\quad$ Prop $Q^TQ = QQ^T = I.$

$\|c_i\| = 1$

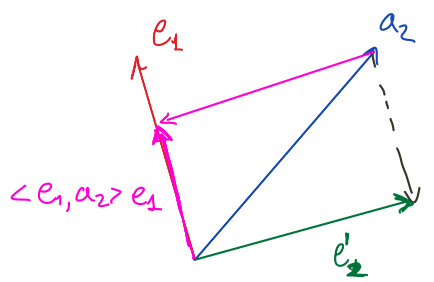
$\langle c_i, c_j \rangle = 0. \quad i \neq j$

Triang. sup. $\quad R =$ 

$$A = \begin{pmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{pmatrix}$$ y quiero bon.

$$e_1 = \frac{a_1}{\|a_1\|} \rightarrow \|e_1\| = 1.$$

$$\left\| \frac{1}{a}(v_1, v_2) \right\| = \sqrt{\frac{v_1^2}{a^2} + \frac{v_2^2}{a^2}} = \sqrt{\frac{1}{a^2}(v_1^2+v_2^2)}$$
$$= \sqrt{\frac{1}{a^2}}\sqrt{(v_1^2+v_2^2)}$$
$$= \frac{1}{a}\sqrt{(v_1^2+v_2^2)}$$
$$= \frac{1}{a}\|v\|.$$



$$e_2' = a_2 - \langle e_1, a_2 \rangle e_1 \rightarrow e_2 = \frac{e_2'}{\|e_2'\|}.$$

$$e_3' = a_3 - \langle a_3, e_2 \rangle e_2 - \langle a_3, e_1 \rangle e_1. \rightarrow e_3 = \frac{e_3'}{\|e_3'\|}.$$

$$Q = \begin{pmatrix} | & | & | \\ e_1 & e_2 & e_3 \\ | & | & | \end{pmatrix} \sim \begin{pmatrix} \|a_1\| & \langle e_1, a_2 \rangle & \langle e_1, a_3 \rangle \\ 0 & \|e_2'\| & \langle e_2, a_3 \rangle \\ 0 & 0 & \|e_3'\| \end{pmatrix}.$$

$$e_1 = \frac{a_1}{\|a_1\|} \rightarrow a_1 = \|a_1\| e_1$$

$$e_2 = \frac{e_2'}{\|e_2'\|} \rightarrow \|e_2'\| e_2 = e_2' = a_2 - \langle e_1, a_2 \rangle e_1 \rightarrow \|e_2'\| e_2 + \langle e_1, a_2 \rangle e_1 = a_2.$$

$A = QR$. Ya armé Q.

$A = QR. \Rightarrow Q^T A = Q^T Q R = I.R = R.$

2 maneras : 1- GS y despejar.

2- hallar Q, hacer $Q^T A = R$.

Ejercicio: hacer el del práctico.