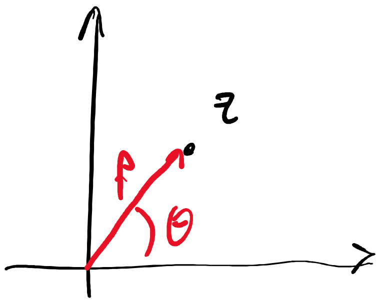
Clase 1 (6/8/24)

Wednesday, July 31, 2024 5:40 PM

$z \in \mathbb{C}, \quad z = \rho e^{i\theta}$




$A \in \mathbb{R}^{n\times n}$ invertible $\Rightarrow \exists\, Q \in \mathbb{R}^{n\times n}$ ortogonal

$(QQ^T = Id)$

$\exists\, R \in \mathbb{R}^{n\times n}$ triangular superior.

$$\left( R = \begin{pmatrix} * & & * \\ 0 & * & \\ 0 & 0 & * \end{pmatrix} \right)$$

$$A = QR$$

$A = \begin{pmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{pmatrix}$ $a_i \in \mathbb{R}^n$ son las columnas de $A$.

$x \in \mathbb{R}^n, \quad x = (x_1, \dots, x_n)$

$$Ax = \begin{pmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= a_1 x_1 + a_2 x_2 + \dots a_n x_n \in \mathbb{R}^n$$

$$Q = \begin{pmatrix} | & & | \\ q_1 & \cdots & q_n \\ | & & | \end{pmatrix} \quad \text{donde} \quad \{q_1, \dots, q_n\} \xrightarrow{\text{bon}} \mathbb{R}^n$$

$$R = \begin{pmatrix} r_{11} & \cdots & \cdots & r_{1n} \\ 0 & r_{22} & & \vdots \\ \vdots & 0 & \ddots & \\ 0 & 0 & & r_{nn} \end{pmatrix}$$

$$\begin{pmatrix} | & & | \\ a_1 & \cdots & a_n \\ | & & | \end{pmatrix} = \underbrace{\begin{pmatrix} | & & | \\ q_1 & \cdots & q_n \\ | & & | \end{pmatrix} \begin{pmatrix} r_{11} & \cdots & \cdots & r_{1n} \\ 0 & r_{22} & & \vdots \\ \vdots & 0 & \ddots & \\ 0 & 0 & & r_{nn} \end{pmatrix}}$$

$$= \left( \begin{array}{c|c|c|c} | & | & & \\ r_{11} q_1 & r_{22} q_2 + r_{12} q_1 & \cdots & \sum_{i=1}^{n} r_{in} q_i \\ | & | & & \end{array} \right)$$

$$\Rightarrow a_k = \underset{= \langle q_1, a_k \rangle}{r_{1k}} q_1 + r_{2k} q_2 + \dots + r_{kk} q_k$$

$$v \in \mathbb{R}^n \Rightarrow v = \overbrace{\langle v, q_1\rangle}^{=\langle q_1, a_k\rangle} q_1 + \dots + \langle v, q_n\rangle q_n$$

$$\Rightarrow a_k = \langle q_1, a_k\rangle q_1 + \langle q_2, a_k\rangle q_2 + \dots + \langle q_k, a_k\rangle q_k$$

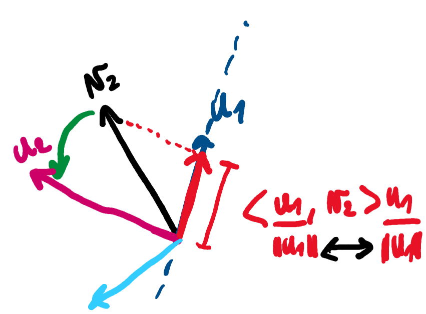
$$r_{ij} = \begin{cases} \langle q_i, a_j\rangle & i \leq j \\ 0 & i > j \end{cases}$$

## Método de Gram-Schmidt.

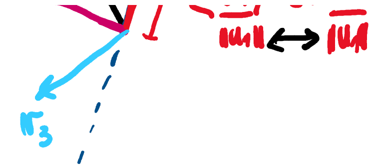
Dadas $\{v_1, \dots, v_k\} \subset \mathbb{R}^n$ vectores LI devuelve $\{q_1, \dots, q_k\}$ conjunto ortonormal que genera lo mismo que $\{v_1, \dots, v_k\}$.

$$u_1 = v_1$$

$$u_2 = v_2 - \frac{\langle u_1, v_2\rangle}{\|u_1\|^2} u_1$$



$$u_2 = v_2 - \frac{\langle u_1, v_2\rangle}{\|u_1\|^2} u_1$$




$$u_3 = v_3 - \frac{\langle u_2, v_3\rangle}{\|u_2\|^2} u_2 - \frac{\langle u_1, v_3\rangle}{\|u_1\|^2} u_1$$

$$\vdots$$

$$u_\ell = v_\ell - \sum_{i=1}^{\ell-1} \frac{\langle u_i, v_\ell\rangle}{\|u_i\|^2} u_i \leftarrow$$

$$\boxed{q_j := \frac{u_j}{\|u_j\|}}$$

$$\frac{\langle u_i, v_\ell\rangle}{\|u_i\| \cdot \|u_i\|} u_i$$

$$= \langle q_i, v_\ell\rangle q_i$$

$$u_\ell = v_\ell - \sum_{i=1}^{\ell-1} \langle q_i, v_\ell\rangle q_i$$

10) $A = \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \leftarrow$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ v_1 & v_2 & v_3 \end{array}$$

$$u_1 = v_1 = (2,2,0)$$

$$u_2 = v_2 - \frac{\langle u_1, v_2 \rangle}{\|u_1\|^2} u_1$$

$$= (3,-1,0) - \frac{\langle (2,2,0),(3,-1,0) \rangle}{8} (2,2,0)$$

$$= (3,-1,0) - \tfrac{1}{2}(2,2,0) = \underline{(2,-2,0)}$$

$$q_3 = (0,0,1)$$

$$(x,y,z) \perp (2,2,0), (2,-2,0)$$

$$\left.\begin{array}{l} 2x + 2y = 0 \\ 2x - 2y = 0 \end{array}\right\} x = y = 0$$

$$z \in \mathbb{R}$$

$$q_1 = u_1 / \|u_1\|$$

$$= \left(2/\sqrt{8}, 2/\sqrt{8}, 0\right)$$

$$= \left(1/\sqrt{2}, 1/\sqrt{2}, 0\right)$$

$$q_2 = u_2 / \|u_2\| = \left(2/\sqrt{8}, -2/\sqrt{8}, 0\right) = \left(1/\sqrt{2}, -1/\sqrt{2}, 0\right)$$

$$q_2 = \frac{u_2}{\|u_2\|} = \left(\frac{2}{\sqrt{8}}, -\frac{2}{\sqrt{8}}, 0\right) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix} = \begin{pmatrix} 2\sqrt{2} & \sqrt{2} & -\sqrt{2} \\ 0 & 2\sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$r_{ij} = \begin{cases} \langle q_i, v_j \rangle & i \leq j \\ 0 & i > j \end{cases}$$

$$r_{11} = \langle q_1, v_1 \rangle = \left\langle \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), (2, 2, 0) \right\rangle = 2\sqrt{2}$$

$$r_{12} = \left\langle \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), (3, -1, 0) \right\rangle = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$QR = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2\sqrt{2} & \sqrt{2} & -\sqrt{2} \\ 0 & 2\sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = A$$

$$\begin{array}{l} Ax = b \iff Rx = Q^T b \\ \parallel \\ QRx \end{array}$$

## Taylor de varias variables.

(k derivadas continuas)

$f : U \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$ de clase $C^k$

$(x_0, y_0) \in U$.

Definimos, dado $(x_0, y_0) \in U$.

(En una variable $P_{f,x_0}(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$)

$$P_{f,m,(x_0,y_0)}(x,y) = \sum_{\substack{j,k=0 \\ j+k \leq m}}^{m} \frac{\partial^{j+k}}{\partial x^j \partial y^k} f(x_0,y_0) \frac{(x-x_0)^j}{j!} \frac{(y-y_0)^k}{k!}$$

$\Rightarrow$ Si $r_m(x,y) = f(x,y) - P_{f,m,(x_0,y_0)}(x,y)$

$$\frac{r_m(x_0+h_1, y_0+h_2)}{\underbrace{\|h\|^m}_{h=(h_1,h_2)}} \xrightarrow{\|h\| \to 0} 0$$

Otra forma de escribirlo:

$$" = \sum_{|\alpha| \leq m} \frac{\partial^{|\alpha|}}{\partial x^\alpha} f(x_0)(x-x_0)^\alpha$$

$$\bigcup_{|\alpha| \leq m} \quad \partial x^{\alpha}$$

- $\alpha = (n_1, \dots, n_m) \in \mathbb{N}^m, \ |\alpha| := n_1 + \dots + n_m$
- $x^{\alpha} := x_1^{n_1} \cdot x_2^{n_2} \cdots x_m^{n_m}$
- $\dfrac{\partial^{|\alpha|}}{\partial x^{\alpha}} := \dfrac{\partial^{|\alpha|}}{\partial x_1^{n_1} \partial x_2^{n_2} \cdots \partial x_m^{n_m}}$