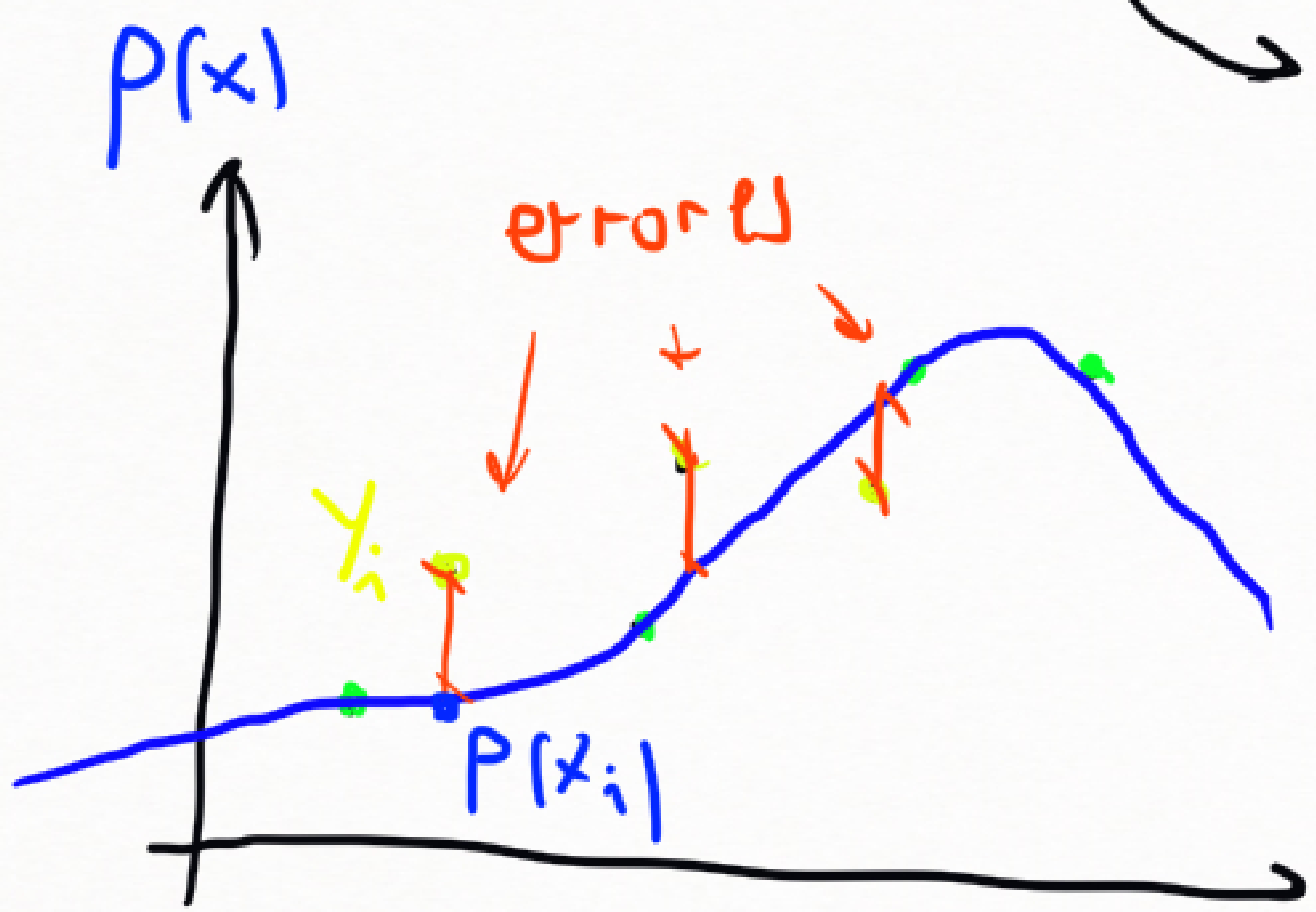
$\{(x_i, y_i) \mid i = 1..n\}$

"Entrenamiento"

"Verificación"



x_i error $|P(x_i) - y_i|$

Interpolación polinomial

Error de interpolación

Interpolación a trozos.

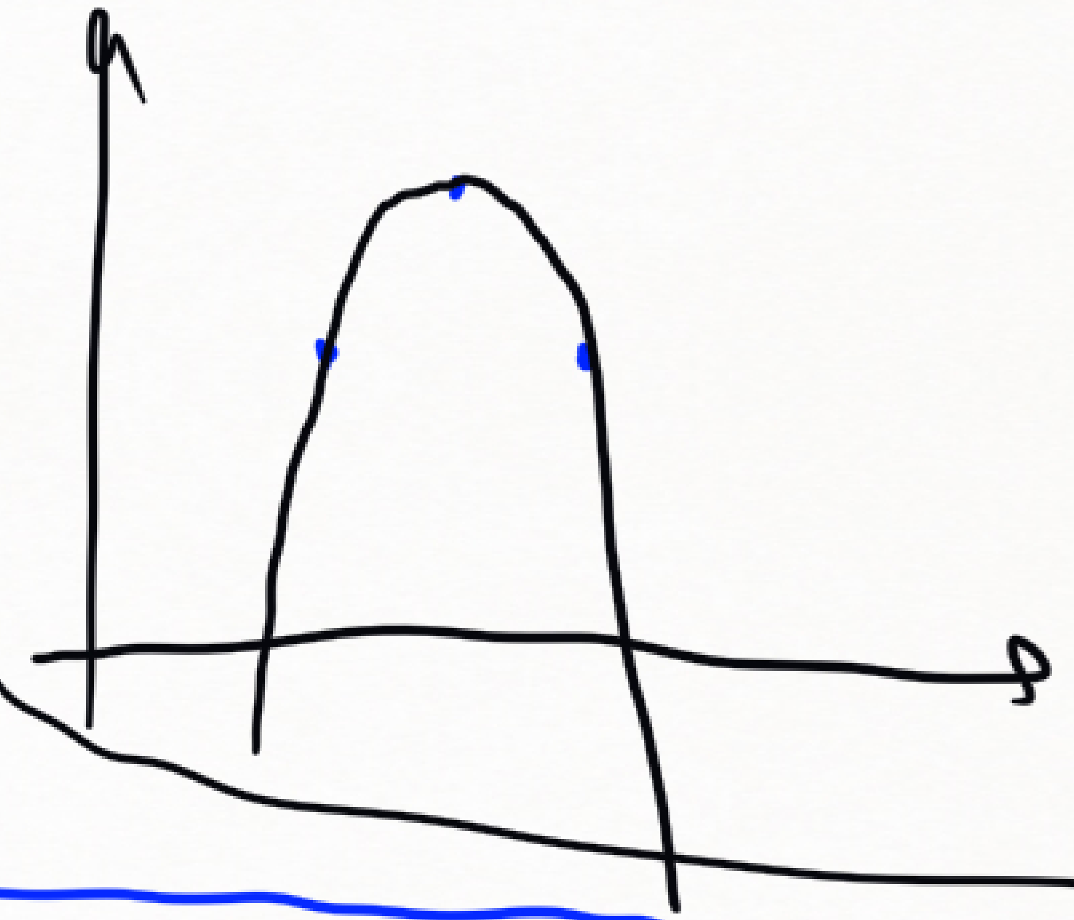
Prop: $n+1$ puntos $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$

$x_i \neq x_j$ si $i \neq j$

$\Rightarrow \exists!\, p(x)$ polinomio de grado $\leq n$ que interpola*

$\forall i = 0..n \quad p(x_i) = y_i$

3 métodos para hallarlo → Vandermonde, Lagrange, Newton.



① $(-1,0) \quad (0,3) \quad (1,2)$

a) $p(x) = \overset{\frown}{a_2} x^2 + \overset{\frown}{a_1} x + \overset{\frown}{a_0}$ incógnitas

$$\begin{aligned}
(-1,0) &\xrightarrow{*} p(-1) = 0 \rightarrow \\
(0,3) &\rightarrow p(0) = 3 \rightarrow \\
(1,2) &\rightarrow p(1) = 2 \rightarrow
\end{aligned}
\left\{\begin{aligned}
a_2 - a_1 + a_0 &= 0 \\
a_0 &= 3 \\
a_2 + a_1 + a_0 &= 2
\end{aligned}\right.$$

sol. $a_2 = -2$, $a_1 = 1$, $a_0 = 3$

$$\boxed{p(x) = -2x^2 + x + 3}$$

b) $(-1,0)$ $(0,3)$ $(1,2)$ (x_0,y_0) (x_1,y_1) (x_2,y_2)

Pol. de base de Lagrange: $L_2^0(x)$, $L_2^1(x)$, $L_2^2(x)$. El pol. interpolante es una combinación lineal de esos polinomios.

$$P(x) = 0\,L_2^0(x) + 3\,L_2^1(x) + 2\,L_2^2(x)$$

Propiedades:

$L_2^0(-1) = 1$	$L_2^1(-1) = 0$	$L_2^2(-1) = 0$
$L_2^0(0) = 0$	$L_2^1(0) = 1$	$L_2^2(0) = 0$
$L_2^0(1) = 0$	$L_2^1(1) = 0$	$L_2^2(1) = 1$

x_0 x_1 x_2

De esas propiedades se deduce la fórmula

$L_2^1(x)$: $L_2^1(x) = c\underbrace{(x+1)}_{L_2^1(-1)=0}\underbrace{(x-1)}_{L_2^1(1)=0}$

Hallamos c con la otra propiedad.

$L_2^1(0) = 1 \iff c\,1\,(-1) = 1$

$\iff c = -1$

$$\boxed{L_2^1(x) = -(x+1)(x-1)}$$

$L_2^2(x) = c(x+1)x$

$L_2^2(1) = 1 \iff c \cdot 2 \cdot 1 = 1 \iff c = \frac{1}{2}$

$$\boxed{L_2^2(x) = \frac{(x+1)x}{2}}$$

$$\boxed{p(x) = -3(x+1)(x-1) + (x+1)x}$$

c) Paso 1: $P_0(x)$ interpola $(-1, 0)$

Paso 2: $P_1(x)$ interpola $(-1,0)$, $(0,3)$

Paso 3: $P_2(x)$ interpola $(-1,0)$, $(0,3)$, $(1,2)$

En cada paso agregamos un punto más de interpolación.

Paso 1: $p_0(x)$ interpola $(-1,0)$

$P_0(x) = 0$

Paso 2: $P_1(x)$ interpola $(-1,0),(0,3)$

ya interpola en $x=-1$

$P_1(x) = P_0(x) + \underbrace{q(x)}_{\text{hallar}} = P_0(x) + c\underbrace{(x+1)}$

en $x=-1$ se anula para no interferir con P_0

Al definirlo de esa forma, interpola $(-1,0)$

$P_1(-1) = P_0(-1) + 0 = P_0(-1) = 0$

Buscamos c para que además interpole $(0,3)$ $\quad P_1(0) = \underbrace{P_0(0)}_{0} + c = c \Rightarrow c = 3$

$$\boxed{P_1(x) = 3(x+1)}$$

Paso 3: Tenemos $P_1(x)$ que interpola $(-1,0)$ y $(0,3)$

Queremos $P_2(x)$ que además interpole $(1,2)$.

$P_2(x) := P_1(x) + c\underbrace{(x+1)x}$ se anula en $-1, 0$

Automáticamente interpola $(-1,0)$ y $(0,3)$

$P_2(-1) = P_1(-1) + 0 = P_1(-1) = 0$ ✓

$P_2(0) = P_1(0) + 0 = P_1(0) = 3$ ✓

Vamos a hallar c para que interpole $(1,2)$

$P_2(x) = 3(x+1) + c(x+1)x$

$P_2(1) = 6 + 2c = 2 \iff c = -2$

$$\boxed{P_2(x) = 3(x+1) - 2(x+1)x}$$

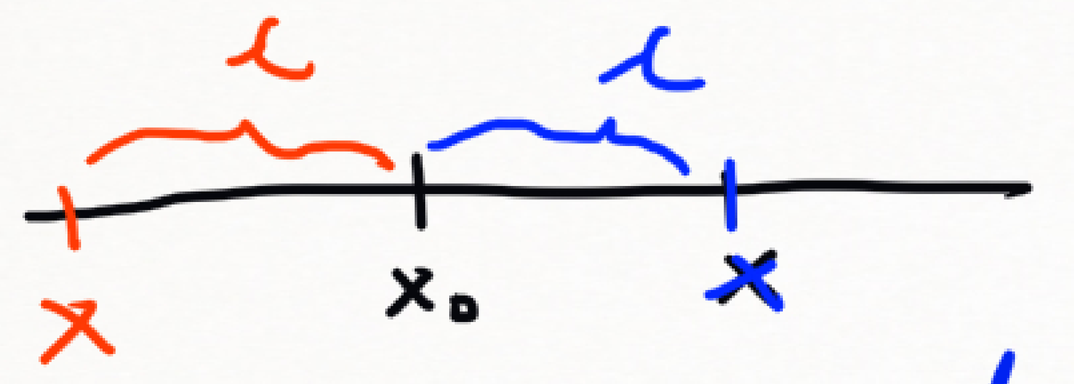
Resto de Lagrange

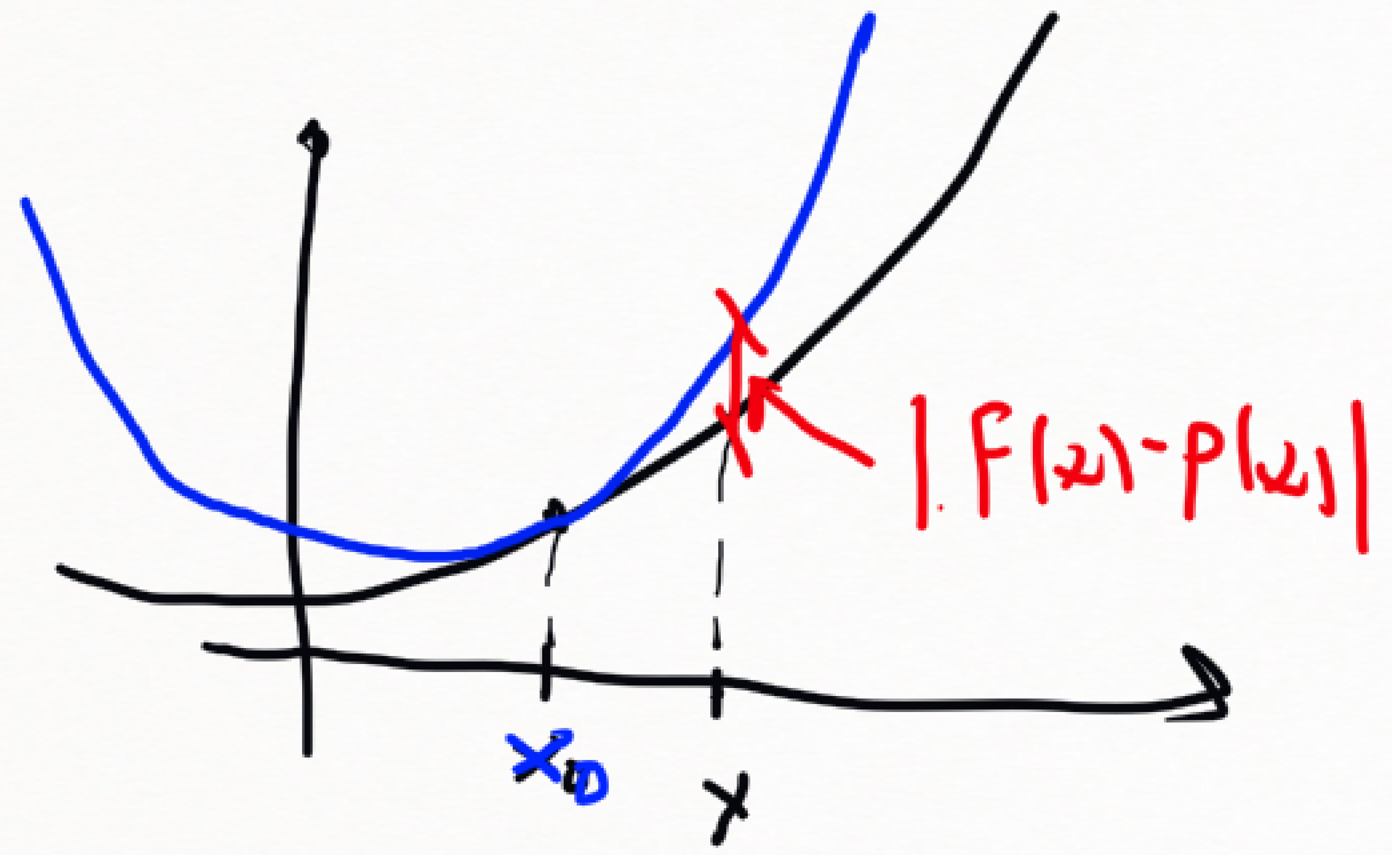
$f:[a,b]\to\mathbb{R}$, $x_0\in(a,b)$

$p(x)$ Taylor de orden n.

$$f(x)-p(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)^{n+1}$$

donde c depende de x





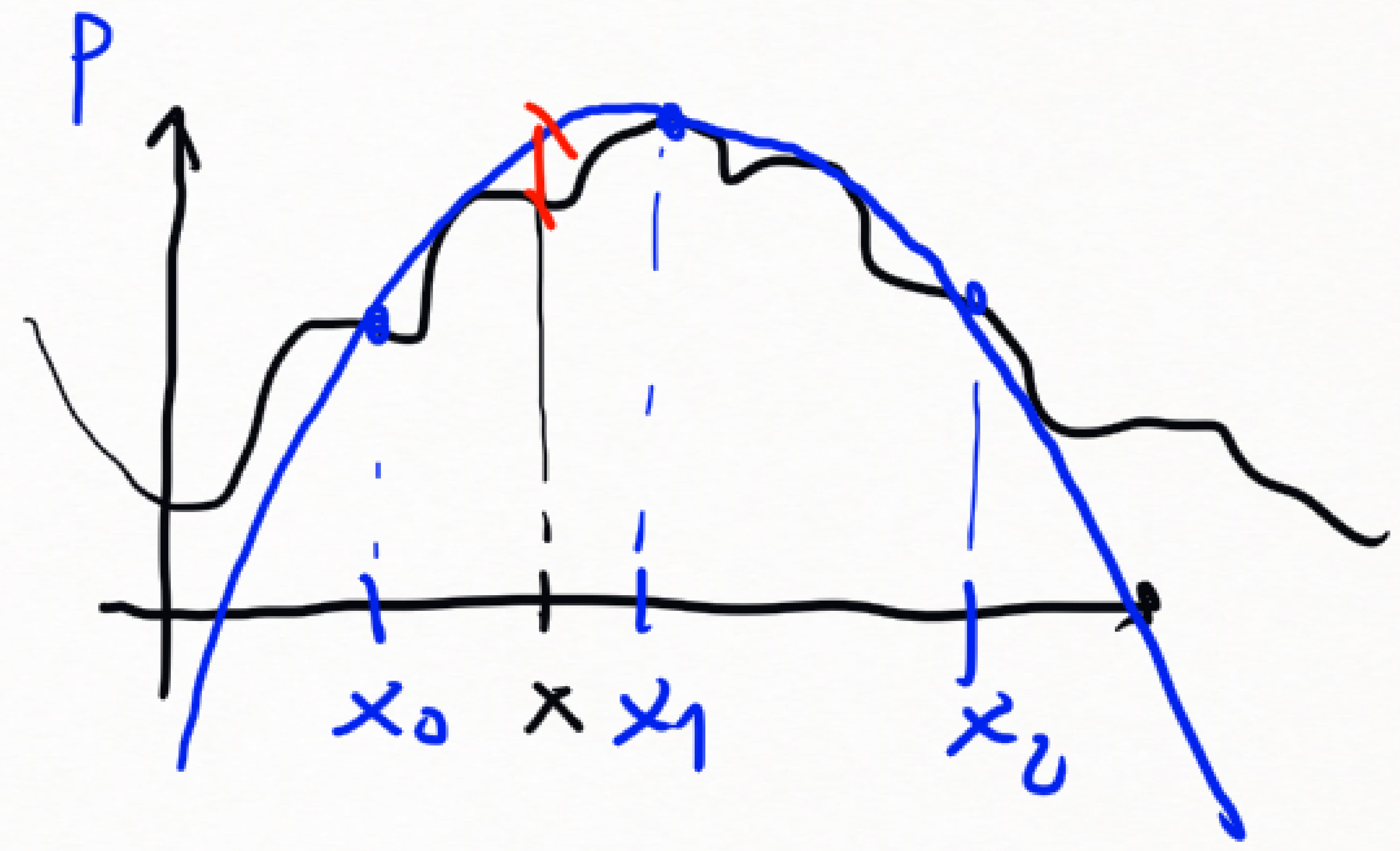
Error de interpolación

$f:[a,b]\to\mathbb{R}$, $x_0,x_1,\dots,x_n\in[a,b]$

$p(x)$ pol. interpolante de f por $x_0,\dots,x_n$

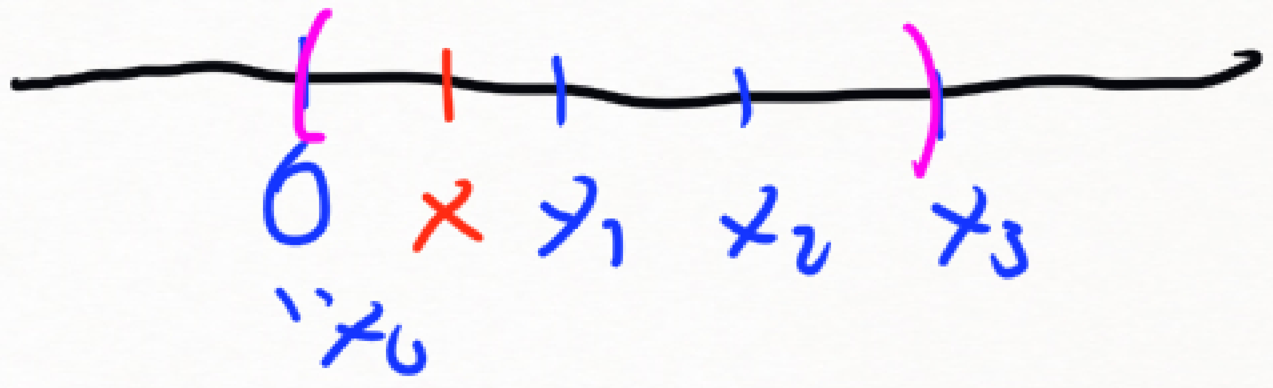
$$f(x)-p(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)$$

$c\in(a,b)$



6) $F(x) = sen(x) \qquad x_0 = 0 \quad x_1 = \frac{\pi}{6} \quad x_2 = \frac{\pi}{4} \quad x_3 = \frac{\pi}{3}$

y ver qué pasó con el error en $x = \pi/8$



c) $$sen\left(\frac{\pi}{8}\right) - p\left(\frac{\pi}{8}\right) = \frac{\overbrace{sen^{(4)}(c)}^{sen(c)}}{4!} \overbrace{\left(\frac{\pi}{8}-0\right)\left(\frac{\pi}{8}-\frac{\pi}{6}\right)\left(\frac{\pi}{8}-\frac{\pi}{4}\right)\left(\frac{\pi}{8}-\frac{\pi}{3}\right)}^{negativo}$$

$$\left|sen\left(\frac{\pi}{8}\right) - p\left(\frac{\pi}{8}\right)\right| = \frac{\overbrace{|sen(c)|}^{\leq 1}}{4!} \frac{\pi}{8}\left(\frac{\pi}{6}-\frac{\pi}{8}\right)\left(\frac{\pi}{4}-\frac{\pi}{8}\right)\left(\frac{\pi}{3}-\frac{\pi}{8}\right) \leq \frac{1}{4!}\frac{\pi}{8}\left(\frac{\pi}{6}-\frac{\pi}{8}\right)\left(\frac{\pi}{4}-\frac{\pi}{8}\right)\left(\frac{\pi}{3}-\frac{\pi}{8}\right)$$

$$\approx 5{,}5 \times 10^{-4}$$

$$sen\left(\frac{\pi}{8}\right) = \sqrt{\frac{1-\cos(\pi/4)}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$c \in (0, \pi/3) \qquad sen(c) > 0$