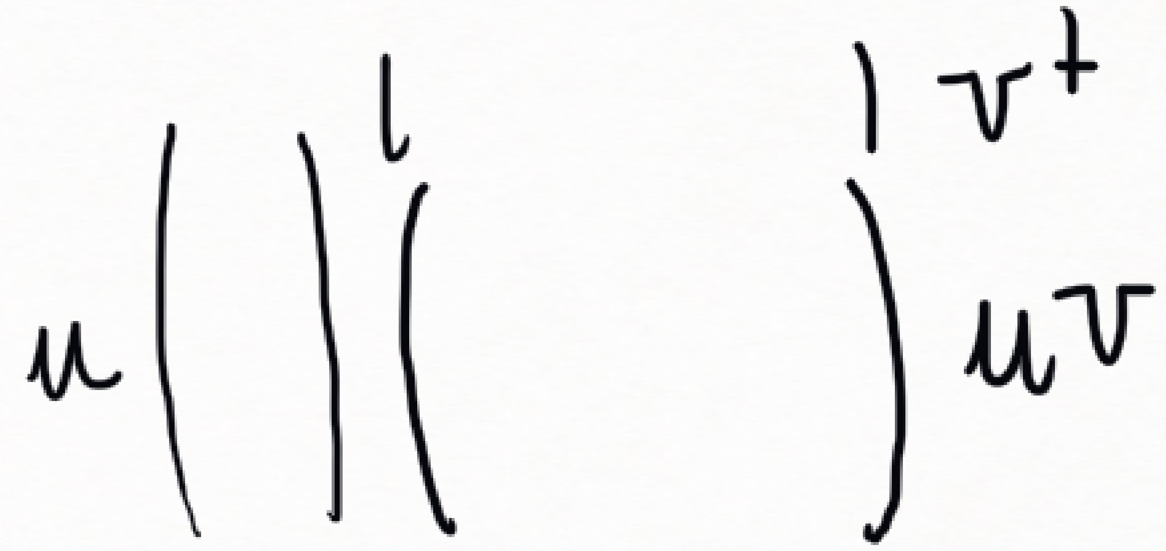
6) $B := A - uv^t$



$uv^t \in M_{n\times n}(\mathbb{R})$

Probar: $B^{-1} = A^{-1} + \alpha\,(A^{-1}u)(v^tA^{-1})$ con cierto $\alpha \in \mathbb{R}$.

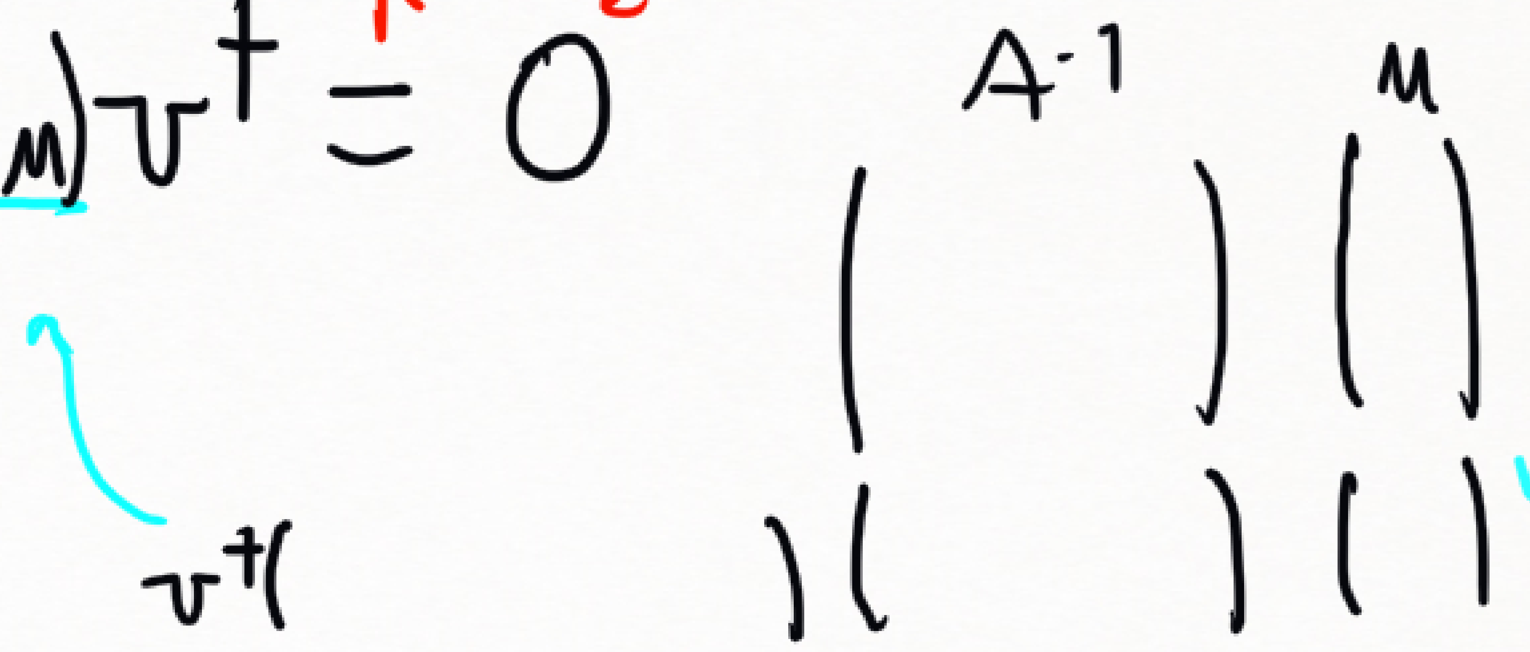
Eso es la inversa $\iff (A^{-1}+\alpha(A^{-1}u)v^tA^{-1})B = Id$.

$$\left(A^{-1}+\alpha(A^{-1}u)(v^tA^{-1})\right)(A-uv^t) = \underbrace{A^{-1}A}_{Id} - A^{-1}uv^t + \alpha(A^{-1}u)(v^t\underbrace{A^{-1})A}_{Id} - \alpha(A^{-1}u)(v^tA^{-1})uv^t$$

$$= Id - A^{-1}uv^t + \alpha A^{-1}uv^t - \alpha A^{-1}uv^tA^{-1}uv^t$$

$$= Id \iff -A^{-1}uv^t + \alpha A^{-1}uv^t - \alpha A^{-1}u\underline{(v^tA^{-1}u)}v^t = 0 \; *$$

$\swarrow$ matriz nula



el resultado es un número

$$-\underline{A^{-1}u}v^t + \alpha\,\underline{A^{-1}u}v^t - \alpha\overbrace{(v^tA^{-1}u)}^{\in\mathbb{R}}\,\underline{A^{-1}u}v^t$$

$$= \left(-1+\alpha-\alpha(v^tA^{-1}u)\right)A^{-1}uv^t$$

si $-1+\alpha-\alpha(v^tA^{-1}u) = 0$, se cumple la igualdad $*$ $\Rightarrow -1+\alpha(1-v^tA^{-1}u) = 0$

$$\iff \boxed{\alpha = \frac{1}{1-v^tA^{-1}u}}$$

la sugerencia dice que $v^tA^{-1}u \neq 1$

$$B = A - uv^t$$

$$\Rightarrow B(A^{-1}u) = AA^{-1}u - u\underbrace{v^tA^{-1}u}_{x}$$

$$\Rightarrow B(A^{-1}u) = u - xu = (1-x)u$$

$$\Rightarrow (\underbrace{BA^{-1}}_{\text{invertible}})u = (1-x)u$$

invertible
(producto de invertibles es invertible)

si C es invertible, los sist $Cx = b$ tienen solución única.
$\Rightarrow Cx = \vec{0}$ solo tiene solución $x = \vec{0}$

$\Rightarrow$ • Si $u \neq \vec{0}$, entonces $(BA^{-1})u \neq \vec{0}$

$\Rightarrow (1-x)u \neq \vec{0} \Rightarrow \underline{x \neq 1}$ ✓

• por otra parte, si $u = \vec{0}$, $v^tA^{-1}u = 0 \neq 1$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \longrightarrow B = \begin{pmatrix} a_{11} & w_1 & a_{13} \\ a_{21} & w_2 & a_{23} \\ a_{31} & w_3 & a_{33} \end{pmatrix}$$

La idea es hallar u, v tq $B = A - uv^t$ y usar la parte anterior.

$$u \begin{pmatrix} -w_1 + a_{12} \\ -w_2 + a_{22} \\ -w_3 + a_{32} \end{pmatrix} \overset{v^t}{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}} \quad \underbrace{\begin{pmatrix} 0 & a_{12} - w_1 & 0 \\ 0 & a_{22} - w_2 & 0 \\ 0 & a_{32} - w_3 & 0 \end{pmatrix}}_{uv^t}$$

$$A - uv^t = \begin{pmatrix} a_{11} & w_1 & a_{13} \\ a_{21} & w_2 & a_{23} \\ a_{31} & w_3 & a_{33} \end{pmatrix} = B$$

$\Longrightarrow$ usamos la Fórmula $B^{-1} = A^{-1} + \alpha (A^{-1}u)(v^t A^{-1})$

con $\alpha = \dfrac{1}{1 - v^t A^{-1} u}$

$u = (-w_1 + a_{12}, -w_2 + a_{22}, -w_3 + a_{33})$

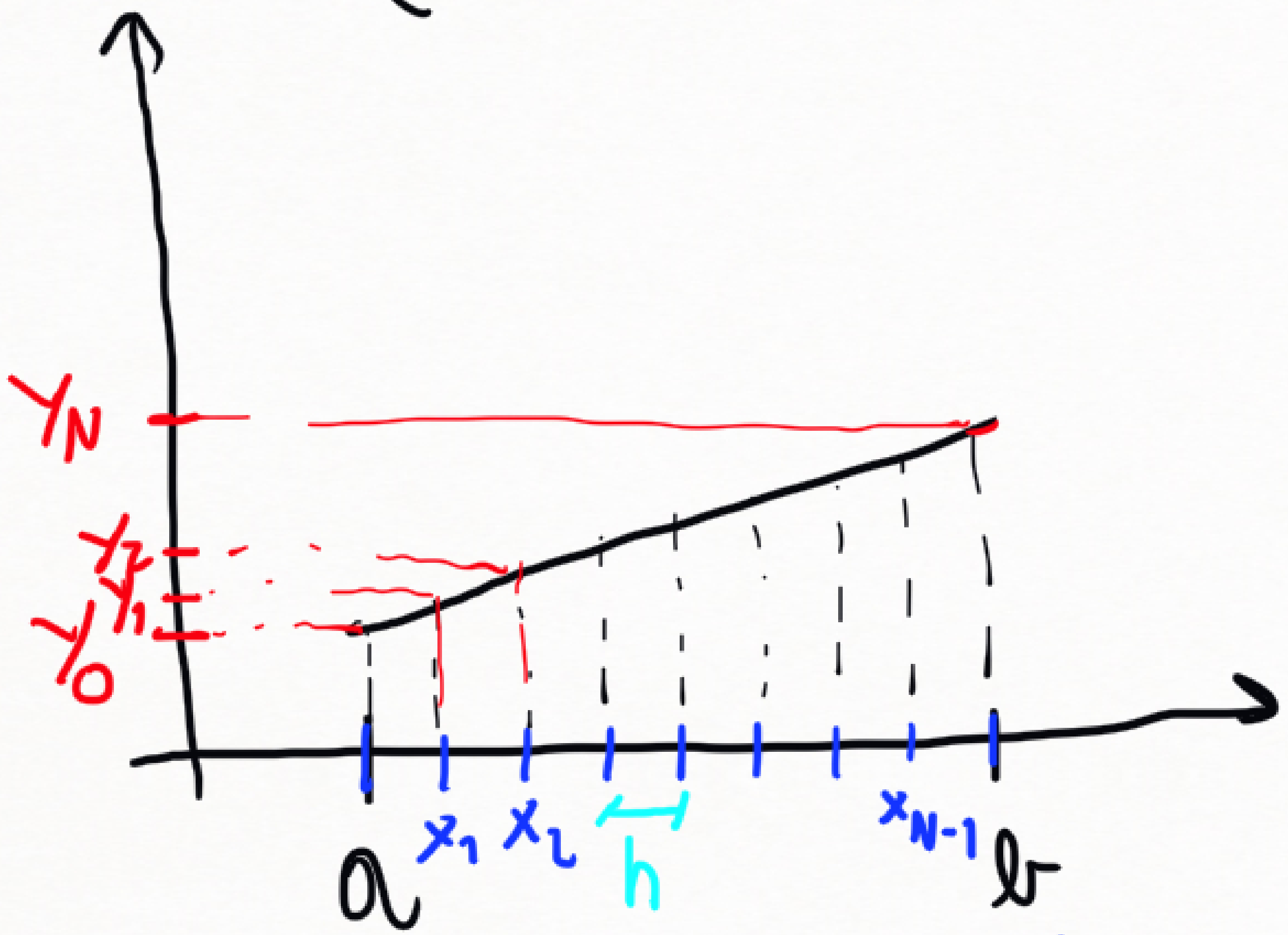
$v = (0, 1, 0)$

7

$$\begin{cases} y''(x) + g(x)\, y(x) = F(x) \\ y(a) = \alpha \qquad y(b) = \beta \end{cases}$$

$y : [a, b] \to \mathbb{R}$

sol de la ec.



$a = x_0 \qquad b = x_N$

$$h = \frac{b-a}{N}$$

$y_i \quad i = 0:N \qquad y_i = y(x_i)$

$y_0 = y(x_0) = y(a) = \alpha$

$y_N = y(x_N) = y(b) = \beta$

$y_1, y_2, \ldots, y_{N-1}$ incógnitas

conocidos $\begin{cases} x_i = a + ih & i = 0:N \\ x_N = a + Nh = a + N\frac{b-a}{N} = b \end{cases}$

ecuación en x_i: $y''(x_i) + g(x_i)\, y(x_i) = F(x_i)$

$\boxed{y''(x_i)} + g(x_i)\, y_i = F(x_i)$ conocidos

necesitamos aproximarlo en base a las incógnitas

$$\Delta(h) := \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = \frac{y(x_i+h) \cancel{- 2y(x_i)} + y(x_i-h)}{h^2}$$

usando Taylor vamos a probar que el error es $O(h^2)$

Vamos a probar con Taylor de orden 4, donde vamos a usar la Fórmula del resto de Lagrange en el 4to orden.

$$y(x_i+h) = \cancel{y(x_i)} + \cancel{y'(x_i)h} + \frac{1}{2}y''(x_i)h^2 + \cancel{\frac{1}{3!}y^{(3)}(x_i)h^3} + \overbrace{\frac{1}{4!}y^{(4)}(c)h^4}^{\text{resto de Lagrange}} \qquad c \in [x_i, x_i+h]$$

$$y(x_i-h) = \cancel{y(x_i)} - \cancel{y'(x_i)h} + \frac{1}{2}y''(x_i)h^2 - \cancel{\frac{1}{3!}y^{(3)}(x_i)h^3} + \frac{1}{4!}y^{(4)}(d)h^4 \qquad d \in [x_i-h, x_i]$$

$$\Delta(h) = \frac{y''(x_i)h^2 + \frac{1}{4!}(y^{(4)}(c) + y^{(4)}(d))h^4}{h^2} = y''(x_i) + \frac{1}{4!}(y^{(4)}(c) + y^{(4)}(d))h^2$$

$$|e(h)| = |\Delta(h) - y''(x_i)| = \frac{1}{4!}|y^{(4)}(c) + y^{(4)}(d)|\,h^2 \approx \frac{2}{4!}|y^{(4)}(x_i)|\,h^2 \qquad O(h^2)$$

$c \approx x_i$, $d \approx x_i$, $y \in C^4$ (derivada 4ta continua)

c) x_i : $y''(x_i) + \underbrace{g(x_i)}_{g_i} y_i = \underbrace{F(x_i)}_{F_i}$ números que podemos calcular

sustituimos por la aproximación

$$y''(x_i) + g_i y_i = F_i \rightarrow \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + g_i y_i = F_i$$

$$\iff y_{i-1} + (h^2 g_i - 2) y_i + y_{i+1} = h^2 F_i$$ ecuación lineal en los y_i incog

tenemos una de esas ecuaciones para cada i

$\Rightarrow$ sistema de ecuaciones que al resolverlo tenemos los y_i.

Las incógnitas son y_i $i = 1:N-1$

$i=1$ $\quad \overset{\alpha}{y_0} + (h^2 g_1 - 2) y_1 + y_2 = h^2 F_1 \rightarrow (h^2 g_1 - 2) y_1 + y_2 = h^2 F_1 - \alpha$

$i=2$ $\quad y_1 + (h^2 g_2 - 2) y_2 + y_3 = h^2 F_2$

$\vdots$

$i=N-1$ $\quad y_{N-2} + (h^2 g_{N-1} - 2) y_{N-1} + \overset{\beta}{y_N} = h^2 F_{N-1}$

$\rightarrow y_{N-2} + (h^2 g_{N-1} - 2) y_{N-1} = h^2 F_{N-1} - \beta$

$y_0 = \alpha \quad y_N = \beta$