5) $x^5+5x+10^{-12}$ raíces con 4 dígitos de precisión.

$$r_1 = \frac{-5+\sqrt{25-4\times 10^{-12}}}{2} \qquad r_2 = \frac{-5-\sqrt{25-4\times 10^{-12}}}{2}$$

Vamos a hallar r_2 con 4 dígitos de precisión

Si ignoramos el 4×10^{-12}, queda $\frac{-5-5}{2} = -5$. ¿El error de esa aproximación afecta hasta el 4 dígito?

$x := 25 \qquad \bar{x} := 25 - 4\times 10^{-12}$

Vamos a usar el número de condición de F.

$$F(t) = \frac{-5-\sqrt{t}}{2}$$

$$|\varepsilon_y| \approx K_F(x)\,|\varepsilon_x| \qquad K_F(x) := \left|\frac{F'(x)\,x}{F(x)}\right|$$

$y := F(x) = -5$

$\bar{y} := F(\bar{x}) = r_2$

quermos ver cuánto vale $|\bar{y}-y|$

$$F'(t) = \frac{-1}{4\sqrt{t}}$$

$$F'(x) = \frac{-1}{20}$$

$$K_F(x) = \left|\frac{\frac{-1}{20}25}{-5}\right| = \frac{5}{20}$$

$$|\varepsilon_x| = \left|\frac{\bar{x}-x}{x}\right| = \frac{4\times 10^{-12}}{25}$$

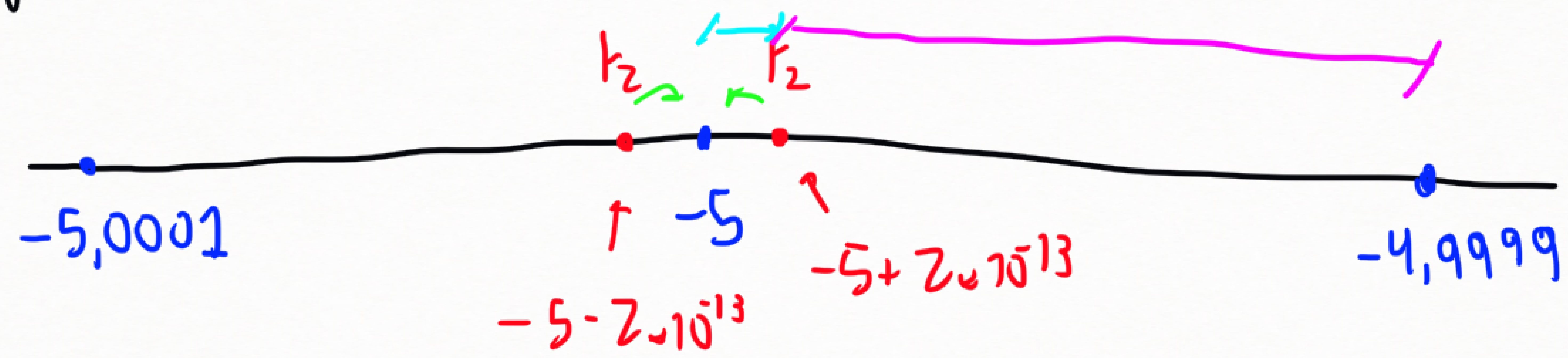
$$|\varepsilon_x| = \frac{4}{25}\times 10^{-12}$$

$$|\varepsilon_y| \approx \frac{5}{20}\times\frac{4}{25}\times 10^{-12} = \frac{1}{25}\times 10^{-12}$$

$$\frac{|\bar{y}-y|}{|y|} \approx \frac{1}{25}\times 10^{-12}$$

$$\Rightarrow \underbrace{|\bar{y}-y|}_{|r_2-(-5)|} \approx \frac{|y|}{25}\times 10^{-12} = \frac{10^{-12}}{5} = 2\times 10^{-13}$$

$|r_2 - |-5|| \approx 2\times 10^{-13}$ esto implica que con 4 dígitos, lo más cercano es -5.



- $|-5+2\times10^{-13} - (-5)| = 2\times10^{-13}$
- $|-5+2\times10^{-13} - (-4{,}9999)| = |-0{,}0001 + 2\times10^{-13}| = 0{,}00009999999998$

$$t_1 = \frac{-5 + \sqrt{25 - 4\times 10^{-12}}}{2} \qquad F(t) = \frac{-5 + \sqrt{t}}{2}$$

$x = 25 \qquad \bar{x} = 25 - 4\times 10^{-12}$ No podemos hacer lo mismo porque

$y = F(x) = 0 \qquad \bar{y} = F(\bar{x}) = t_1 \qquad \varepsilon_y = \dfrac{|\bar{y} - y|}{|y|}$ $|y| = 0$

Vamos a estimar $|t_1 - 0| = |F(\bar{x}) - F(x)|$ sin usar errores relativos.

$$|\underbrace{F(\bar{x})}_{t_1} - \underbrace{F(x)}_{0}| \approx |F'(x)|\,|\bar{x} - x| = \frac{1}{20}\times 4\times 10^{-12} = \frac{10^{-12}}{5} = 2\times 10^{-13}$$

Taylor de orden 1 alrededor de x evaluado en $\bar{x}$.

$$F(\bar{x}) \approx F(x) + F'(x)(\bar{x} - x)$$

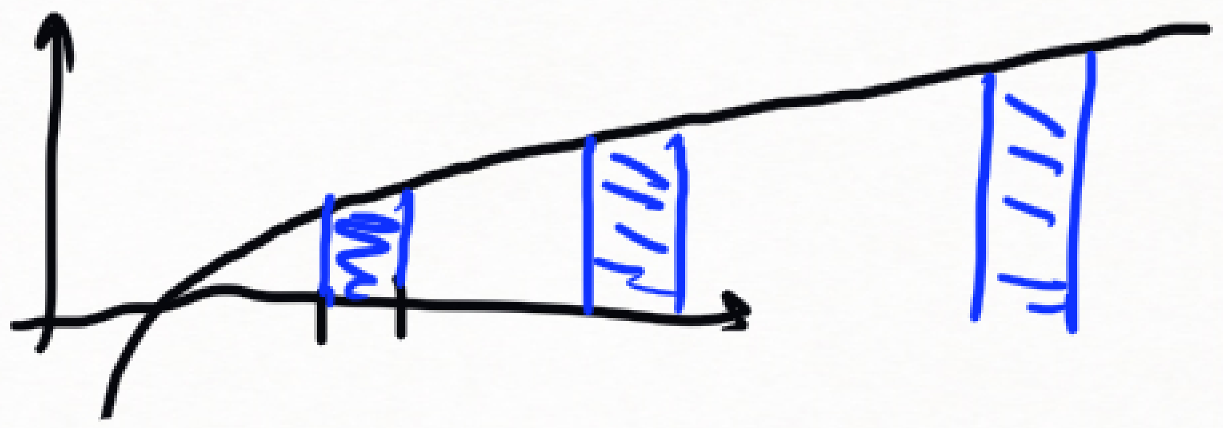
$$F'(t) = \frac{1}{4\sqrt{t}}$$

$$F'(x) = \frac{1}{4\sqrt{25}} = \frac{1}{20}$$

$$|\bar{x} - x| = 4\times 10^{-12}$$

Se concluye igual que con el anterior. Con 4 dígitos de precisión $t_1 = 0$.

8 $\displaystyle \lim_{n\to\infty} \int_n^{n+1} \log(x)\,dx = \infty$



$$\int_n^{n+1} \log(x)\,dx = x\log(x) - x \Big|_n^{n+1} = \underbrace{(n+1)\log(n+1) - (n+1)} - \underbrace{(n\log(n) - n)}$$

$$= F(n+1) - F(n)$$

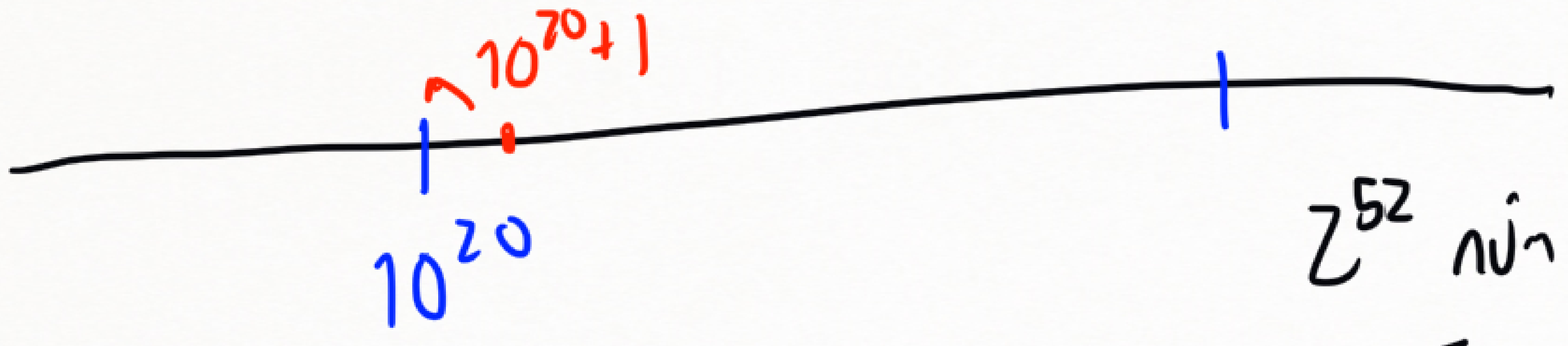
con $F(t) = t\log(t) - t$

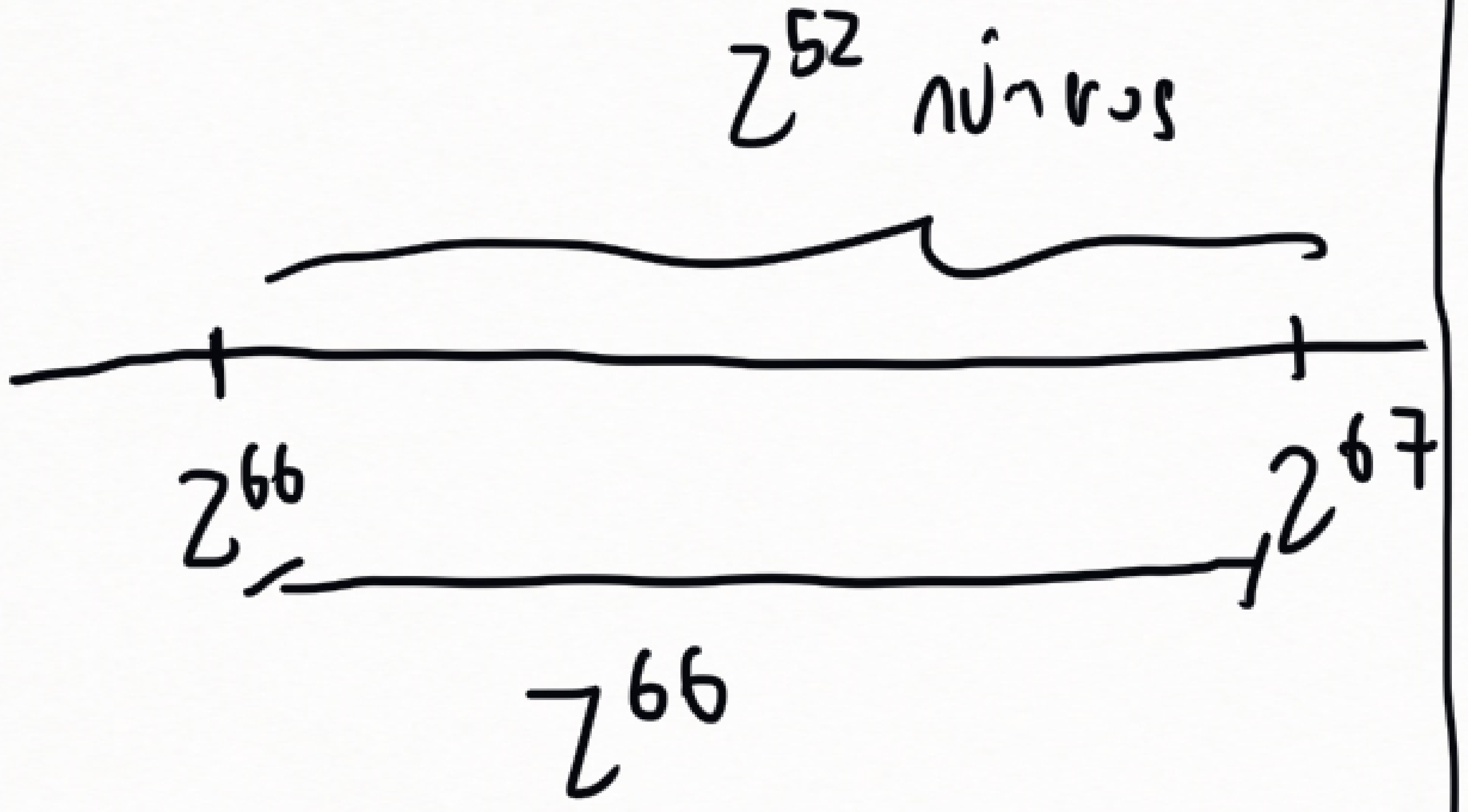
$$\int \underset{F'}{1} \cdot \underset{g}{\log(x)}\,dx = \underset{F}{x}\,\underset{g}{\log(x)} - \int \underset{F}{x}\,\underset{g'}{\frac{1}{x}}\,dx = x\log(x) - x$$

Se soluciona reescribiendo.

$$(n+1)\log(n+1) - (n+1) - n\log(n) + n =$$

$$= n\log\left(\frac{n+1}{n}\right) - 1 + \log(n+1)$$





$10^{20} \in [2^{66}, 2^{67})$

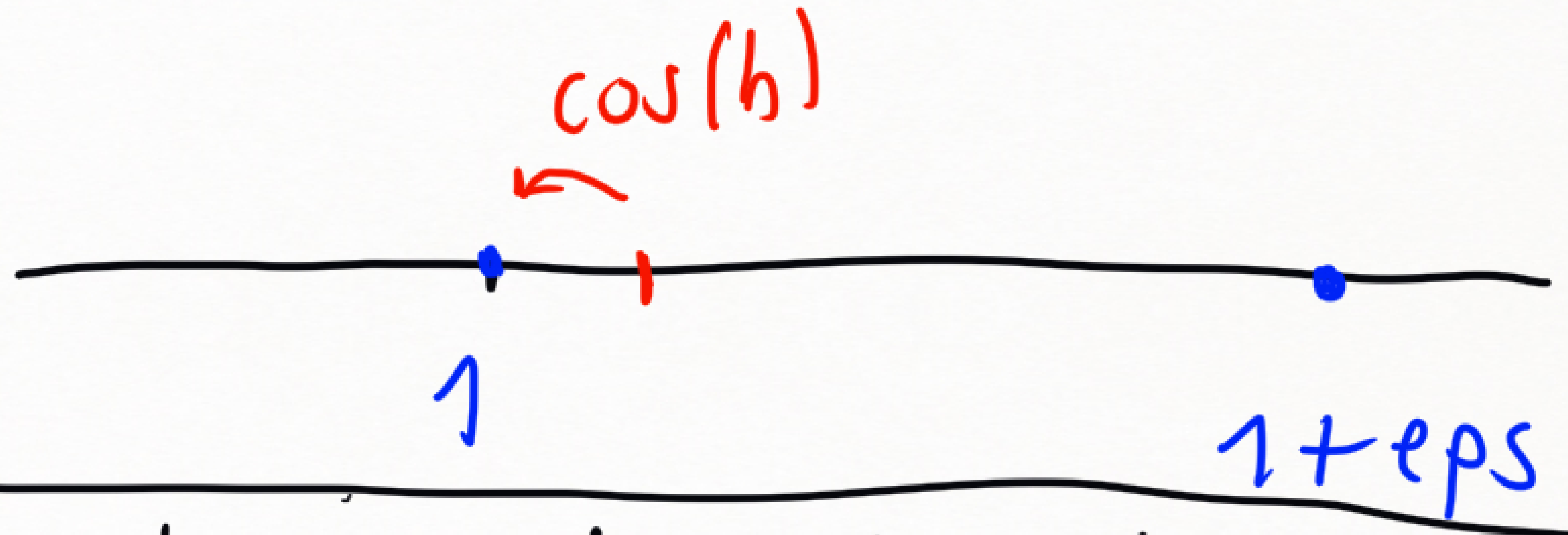
sep: $\dfrac{2^{66}}{2^{52}} = 2^{14}$

$$\frac{e^x}{e^x+1} \xrightarrow{x\to\infty} 1$$

$$\frac{e^x}{e^x+1} = \frac{e^x \cdot e^{-x}}{(e^x+1)\cdot e^{-x}} = \frac{1}{1+e^{-x}}$$

No hay problemas

$$\frac{\cos(h)-1}{h} \to \frac{0}{h} = 0$$



$$\frac{\cos(x+h)-\cos(x)}{h}$$

$x=0$

$$\frac{\cos(h)-1}{h}$$

$$\frac{\cos(x+h)-\cos(x)}{h} \overset{*}{=} \frac{\cos(x)\cos(h)-\operatorname{sen}(x)\operatorname{sen}(h)-\cos(x)}{h}$$ ✓

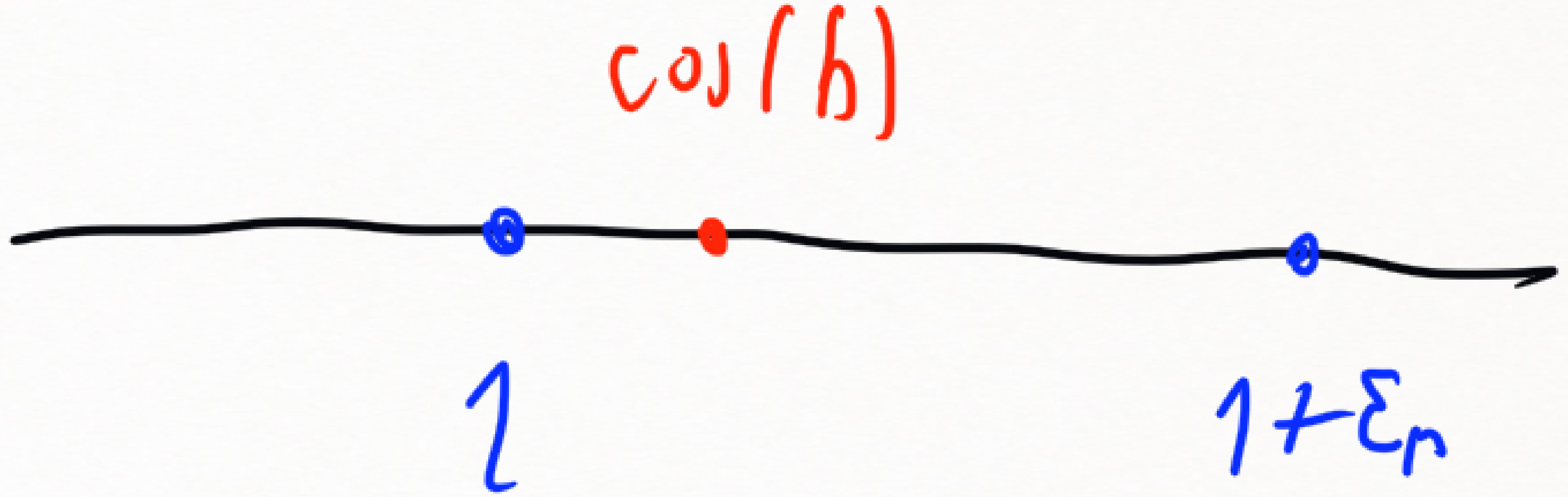
$$= \cos(x)\left(\frac{\cos(h)-1}{h}\right) - \operatorname{sen}(x)\frac{\operatorname{sen}(h)}{h}$$

$$* \quad \frac{\cos(h)-1}{h}\cdot\frac{\cos(h)+1}{\cos(h)+1} = \frac{\cos^2(h)-1}{h(\cos(h)+1)} = \frac{\operatorname{sen}^2(h)}{h(\cos(h)+1)}$$

$$\frac{\cos(h) - 1}{h} \approx \frac{\left(1 + \frac{h^2}{2}\right) - 1}{h}$$

$\frac{h^2}{2} < \varepsilon_M$

no pasa nada



el h de abajo va a ser 0 solo si $h <$ realmin.