(10) $A = \begin{pmatrix} 2 & 3 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ $v_1 = (2, 2, 0)$ $v_2 = (3, -1, 0)$ $v_3 = (0, -2, 1)$

GS: Construye un conjunto ortonormal $\{w_1, w_2, w_3\}$ que genera lo mismo que $\{v_1, v_2, v_3\}$.

Def: $\{w_1, w_2, w_3\}$ es ortonormal $\Leftrightarrow$ (1) Ortogonal: $\forall i \neq j$ $\overbrace{\langle w_i, w_j \rangle}^{\text{prod escalar}} = 0$ (los 3 son perpendiculares entre sí.)

(2) $\forall i \in \{1, 2, 3\}$ $\underbrace{\|w_i\|}_{\text{norma}} = 1$ (longitud $= 1$)

- En general usamos el $\langle , \rangle$ y la $\| \ \|$ usuales.

$$\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1 y_1 + x_2 y_2 + x_3 y_3$$

$$\|(x_1, x_2, x_3)\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

- GS es un algoritmo iterativo/recursivo

Paso 1: $v_1 \mapsto w_1$ tq $\|w_1\| = 1$

Paso 2: $v_2 \mapsto$ agregar w_2 tq $\{w_1, w_2\}$ sea ortonormal

Paso 3: $v_3 \mapsto$ agregar w_3 tq $\{w_1, w_2, w_3\}$ sea ortonormal

$v_1 = (2,2,0) \quad v_2 = (3,-1,0) \quad v_3 = (0,-2,1)$

Paso 1: $v_1 = (2,2,0)$. $\quad \boxed{w_1 = \dfrac{v_1}{\|v_1\|}}$

$\|v_1\| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$$\boxed{w_1 = \frac{1}{2\sqrt{2}}(2,2,0) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)}$$

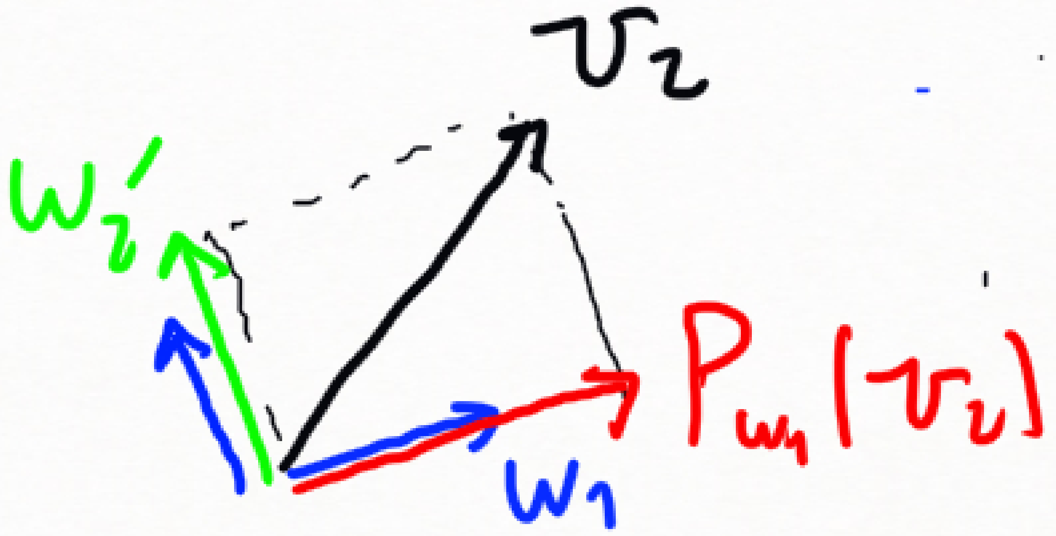
$v_1 = \|v_1\|\, w_1$

$v_2 = \|w_2'\|\, w_2 + \langle v_2, w_1\rangle w_1$

Paso 2: $v_2 \mapsto w_2$ tq $\{w_1, w_2\}$ ortonormal

1) $\langle w_1, w_2\rangle = 0 \qquad$ 2) $\|w_2\| = 1$

Primero construimos un w_2' tq $\langle w_1, w_2'\rangle = 0$

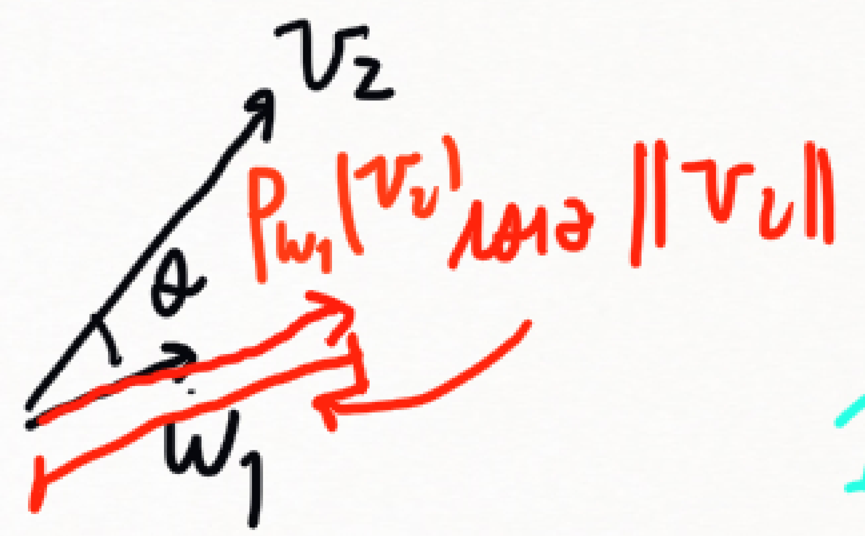


$$w_2' = v_2 - P_{w_1}(v_2) = (3,-1,0) - \sqrt{2}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = (2,-2,0)$$
$$= v_2 - \langle v_2, w_1\rangle w_1$$

$w_2' = (2,-2,0)$

$\langle v_2, w_1\rangle = \dfrac{3}{\sqrt{2}} - \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$$\boxed{w_2 = \frac{w_2'}{\|w_2'\|} = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)}$$



$\langle w_1, v_2\rangle = \cos\theta\, \|v_2\|\, \|w_1\|$
$= \cos\theta \cdot \|v_2\|$

$$P_{w_1}(v_2) = \underbrace{\langle w_1, v_2\rangle}_{\text{longitud}}\ \underbrace{w_1}_{\text{dirección norma 1}}$$

$w_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \quad w_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right) \quad w_3 = (0,0,1)$

Paso 3: $v_3 = (0,-2,1)$

Primero w_3' tq $\langle w_3', w_1\rangle = 0$
$\langle w_3', w_2\rangle = 0$

y después $w_3 = \frac{w_3'}{\|w_3'\|}$

$$w_3' = v_3 - P_{w_1}(v_3) - P_{w_2}(v_3)$$
$$= v_3 - \langle v_3, w_1\rangle w_1 - \langle v_3, w_2\rangle w_2$$
$$= (0,0,1)$$

hacer cuenta

$$\boxed{w_3 = \frac{w_3'}{\|w_3'\|} = (0,0,1)}$$

$\{w_1, w_2, w_3\}$ base ortonormal de $\mathbb{R}^3$. Esto implica que la matriz Q que consiste en colgar sus vectores es ortogonal.

Prop: Q ortogonal $\iff$ sus columnas son bon de $\mathbb{R}^n$

$$Q = (w_1 | w_2 | w_3) = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Para la matriz R, despejamos v_1, v_2 y v_3 en base a w_1, w_2 y w_3.

$v_1 = \underline{\|v_1\|}\, w_1$
$v_2 = \underline{\|w_2'\|}\, w_2 + \underline{\langle v_2, w_1\rangle}\, w_1$
$v_3 = \underline{\|w_3'\|}\, w_3 + \underline{\langle v_3, w_1\rangle}\, w_1 + \underline{\langle v_3, w_2\rangle}\, w_2$

Forman la matriz triangular R

$$v_3 = (0,-2,1) \qquad \langle v_3, w_1\rangle = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

$$\langle v_3, w_2\rangle = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$v_1 = \|v_1\|\, w_1$$

$$v_2 = \langle v_2, w_1\rangle w_1 + \|w_2'\|\, w_2$$

$$v_3 = \langle v_3, w_1\rangle w_1 + \langle v_3, w_2\rangle w_2 + \|w_3'\|\, w_3$$

$$R$$

$$\begin{pmatrix} \|v_1\| & \langle v_2, w_1\rangle & \langle v_3, w_1\rangle \\ 0 & \|w_2'\| & \langle v_3, w_2\rangle \\ 0 & 0 & \|w_3'\| \end{pmatrix}$$

$$\left(w_1 \,\middle|\, w_2 \,\middle|\, w_3 \right) \qquad \left(\|v_1\|\, w_1 \,\middle|\, \langle v_2, w_1\rangle w_1 + \|w_2'\|\, w_2 \,\middle|\, \langle v_3, w_1\rangle w_1 + \langle v_3, w_2\rangle w_2 + \|w_3'\|\, w_3 \right)$$

$$Q \qquad = \left(v_1 \,\middle|\, v_2 \,\middle|\, v_3 \right) = A$$

$F(x) = \frac{1}{1+x}$ $x_0 = 0$ $n = 3$

El polinomio de Taylor (depende de los 3 datos) es el polinomio de grado ≤ 3 que mejor aproxima a la Función F en un entorno de $x_0 = 0$

Los coefs del polinomio dependen de las derivadas de grado ≤ 3 de F en el punto x_0.

$$P(x) = F(x_0) + F'(x_0)(x-x_0) + \frac{F''(x_0)}{2!}(x-x_0)^2 + \frac{F'''(x_0)}{3!}(x-x_0)^3$$

en este caso:

$$F(x) = \frac{1}{1+x} = (1+x)^{-1}$$

$$F'(x) = -(1+x)^{-2}$$

$$F''(x) = 2(1+x)^{-3} \quad F'''(x) = -6(1+x)^{-4}$$

$F(0) = 1$

$F'(0) = -1$

$F''(0) = 2$ $F'''(0) = -6$

$$\boxed{P(x) = 1 - x + x^2 - x^3}$$