$a, b \in \mathbb{Z}$ no nulos

$mcm(a,b) = \min\{x \in \mathbb{Z}^+ : a|x \text{ y } b|x\}$

$$mcm(a,b) = \frac{|ab|}{mcd(a,b)}$$

si a y b son coprimos : $mcm(a,b) = \frac{|ab|}{mcd(a,b)} = |ab|$

$d = mcd(a,b)$

$\Rightarrow a = da^*, \ b = db^*$ con a^* y b^* coprimos

$ab = da^* db^*$

$$\frac{ab}{mcd(a,b)} = \frac{\cancel{d}a^* db^*}{\cancel{d}} = a^* d b^*$$

<u>Ejercicio 7</u>

c. $a + b = 1271$ y $mcm(a,b) = 330 \cdot mcd(a,b)$.

buscamos $a, b \in \mathbb{N}$ que verifiquen $\begin{cases} a+b = 1271 \\ mcm(a,b) = 330\, mcd(a,b) \end{cases}$

$d = mcd(a,b)$

$\Rightarrow a = da^*$ y $b = db^*$ con a^* y b^* coprimos

* $a + b = 1271$

$da^* + db^* = 1271$

$\boxed{d(a^* + b^*) = 1271}$

$d = mcd(a,b)$

$\Rightarrow \begin{cases} d|a \rightsquigarrow a = dq \\ d|b \rightsquigarrow b = dq' \end{cases}$

* $mcm(a,b) = \frac{ab}{mcd(a,b)} = \frac{\cancel{d}a^* db^*}{\cancel{d}} = a^* b^* d$

$mcm(a,b) = 330\, mcd(a,b)$

$\Rightarrow a^* b^* d = 330\, d$

$\Rightarrow \boxed{a^* b^* = 330}$

Tenemos:

* a^* y b^* coprimos
* $d(a^*+b^*) = 1271 \rightarrow a^*+b^* \mid 1271$
* $a^* b^* = 330 \rightarrow a^* \mid 330$ y $b^* \mid 330$

1271

2	11
3	13
5	17
7	

31

Divisores de 1271:

$1271 \cdot 1 = 1271$

$\underset{d}{31} \cdot \underset{\substack{(30+11) \\ a^*+b^*}}{41} = 1271$

Divisores de 330:

$$330 = 2 \cdot 165 = 2 \cdot 3 \cdot 55 = 2 \cdot 3 \cdot 5 \cdot 11$$

$330 = 330 \cdot 1$ ✗ $\quad 330 = 66 \cdot 5$ ✗

$330 = 2 \cdot 165$ ✗ $\quad 330 = 22 \cdot 15$ ✗

$330 = 6 \cdot 55$ ✗ $\quad 330 = 10 \cdot 33$ ✗

$330 = 30 \cdot 11$ ✓ $\quad 330 = 110 \cdot 3$ ✗

$30 + 11 = 41$

y 41 es divisor de 1271

$$a^* = 30, \ b^* = 11 \ \text{ y } d = 31$$

① $a^* = 30$, $b^* = 11$ y $d = 31$

$a = d a^* = 31 \cdot 30 = 930$

$b = d b^* = 31 \cdot 11 = 341$

② $a^* = 11$, $b^* = 30$ y $d = 31$

$a = 341$

$b = 930$

Algoritmo de Euclides extendido

$a, b \in \mathbb{Z}$

$\to$ buscamos $mcd(a,b)$ y $x, y \in \mathbb{Z}$ tales que $ax + by = mcd(a,b)$

Ejercicio 1

b) $a = 455$, $b = 1235$

Forma 1:

$$1235 = 2 \cdot \underline{455} + \underline{325} \longrightarrow 325 = 1235 - 2 \cdot 455$$

$$455 = \underline{325} + \underline{130} \longrightarrow 130 = 455 - 325$$

$$325 = 2 \cdot \underline{130} + \underline{65} \longrightarrow 65 = \underbrace{325} - 2 \cdot \underbrace{130}$$

$$130 = 2 \cdot 65 + \underline{0}$$

entonces $mcd(1235, 455) = 65$

$$\begin{aligned} 65 &= 325 - 2 \cdot 130 \\ &= 325 - 2(455 - 325) \\ &= 325 - 2 \cdot 455 + 2 \cdot 325 \\ &= 3 \cdot \underbrace{325} - 2 \cdot 455 \\ &= 3(1235 - 2 \cdot 455) - 2 \cdot 455 \\ &= 3 \cdot 1235 - 6 \cdot 455 - 2 \cdot 455 \\ &= 3 \cdot 1235 - 8 \cdot 455 \end{aligned}$$

$$\boxed{65 = 3 \cdot 1235 - 8 \cdot 455}$$

$b = 1235 = 65 b^* \rightsquigarrow b^* = \frac{1235}{65} = 19$

$a = 455 = 65 a^* \rightsquigarrow a^* = \frac{455}{65} = 7$

Forma 2: $mcd(1235, 455) = ?$

$mcd(5,2) = 1$

$5 - 2\cdot 2 = 1$ Bezout

$5\cdot 3 - 2\cdot 7 = 1$

$$B_0 = \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

* $1235 = 2\cdot \underline{455} + \underline{325}$

$\longrightarrow 325 = 1235 - 2\cdot 455$

$$B_1 = \begin{pmatrix} 455 \\ 325 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

* $455 = \underline{325} + \underline{130}$

$\longrightarrow 130 = 455 - 325$

$$B_2 = \begin{pmatrix} 325 \\ 130 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 455 \\ 325 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

* $325 = 2\cdot \underline{130} + \underline{65}$

$\longrightarrow 65 = 325 - 2\cdot 130$

$$B_3 = \begin{pmatrix} 130 \\ 65 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 325 \\ 130 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 3 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

* $130 = 2\cdot 65 + \underline{0}$

entonces $65 = mcd(1235, 455)$

$$\begin{pmatrix} 130 \\ 65 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} 1235 \\ 455 \end{pmatrix}$$

$$65 = 3\cdot 1235 - 8\cdot 455$$