$f:\mathbb{R}\to\mathbb{R} \mid \quad f(x) = x^2 - \int_0^{2x} e^{t^3}\,dt$

¿Taylor de orden 2 en 0?

$$P_2(f,0)(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$$f(0) = \underbrace{0^2}_{=0} - \int_0^0 e^{t^3}\,dt = 0 \quad \checkmark$$

$$f'(x) = \left(x^2 - \int_0^{\overset{g(x)}{2x}} e^{t^3}\,dt\right)'$$

$$= 2x - (2)\,e^{(2x)^3}$$

$$= 2x - 2e^{8x^3}$$

$$\left(\int_0^{g(x)} f(t)\,dt\right)' = g'(x)\,f(g(x))$$

$$\Rightarrow f'(0) = \overbrace{2\cdot 0}^{=0} - 2\,\underbrace{e^{\overset{=0}{8\cdot 0^3}}}_{1}$$

$$= -2(1) = -2$$

$$f''(x) = \left(2x - 2e^{8x^3}\right)'$$

$$= 2 - 2e^{8x^3}\cdot(8x^3)'$$

Regla de la cadena

$$= 2 - 2e^{8x^3}\cdot 24x^2$$

$$= 2 - 48\, e^{8x^3} x^2$$

$$\Rightarrow f''(0) = 2 - \underbrace{48\, e^{8 \cdot 0^3} \cdot 0}_{= 0} = 2$$

$$\Rightarrow P_2(f, 0)(x) = 0 - 2x + \frac{2}{2} x^2$$

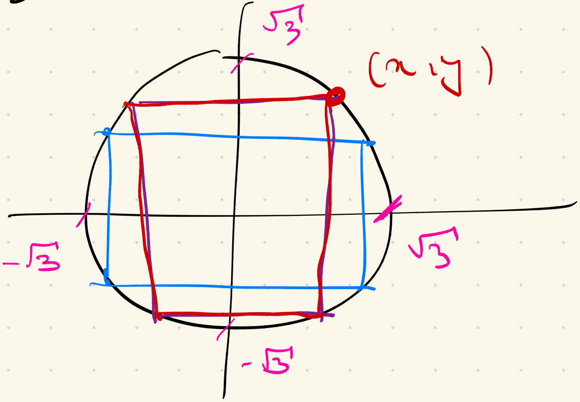
$$\boxed{= -2x + x^2}$$

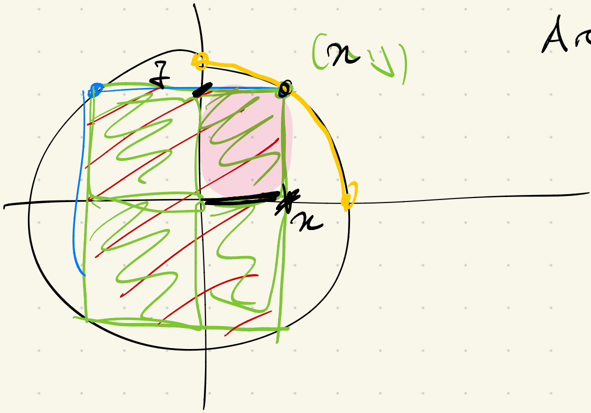
Circunferencia C de ecuación $x^2 + y^2 = 3$

$x^2 + y^2 = r^2$

$A = \{ \text{Area}(R) : R$ rectángulo inscripto en $C \}$

¿Máximo de A?





$$\text{Area} = 4xy$$

Como $x^2 + y^2 = 3$

$$y^2 = 3 - x^2$$

$$y = \sqrt{3 - x^2}$$

$$\Rightarrow \text{Área} = \overbrace{4 \cdot x\sqrt{3-x^2}}^{f(x)}$$

↳ solo depende de x

$$f'(x) = \left(4x\sqrt{3-x^2}\right)'$$

$$= (4x)'\sqrt{3-x^2} + 4x\left(\sqrt{3-x^2}\right)'$$

$$= 4\sqrt{3-x^2} + 4x\left(\left(3-x^2\right)^{1/2}\right)'$$

$$= 4\sqrt{3-x^2} + 4x \cdot \frac{1}{2}\left(3-x^2\right)^{\frac{1}{2}-1}\left(3-x^2\right)'$$

$= -1/2$

$$= 4\sqrt{3-x^2} + 2x \cdot \frac{1}{\sqrt{3-x^2}}(-2x)$$

$$= 4\sqrt{3-x^2} - \frac{4x^2}{\sqrt{3-x^2}}$$

denominador común ↳

$$= \frac{4\sqrt{3-x^2} \cdot \sqrt{3-x^2} - 4x^2}{\sqrt{3-x^2}}$$

$$= \frac{4(3-x^2) - 4x^2}{\sqrt{3-x^2}}$$

$$= \frac{12 - 4x^2 - 4x^2}{\sqrt{3-x^2}}$$

$$= \frac{12 - 8x^2}{\sqrt{3-x^2}} = 0$$

$$\longleftrightarrow \quad 12 - 8x^2 = 0$$

$$8x^2 = 12$$

$$x^2 = \frac{12}{8} = \frac{3}{2}$$

$$x = \sqrt{\frac{3}{2}}$$

$sg(f')$

0		$\sqrt{\frac{3}{2}}$		$\sqrt{3}$
	$+ + + +$	0	$- - -$	$\nexists$

$\Rightarrow \sqrt{\frac{3}{2}}$ se da el máximo

$$\text{Área} = 4xy = 4 \cdot \sqrt{\frac{3}{2}} \cdot \sqrt{\frac{3}{2}} = 4 \cdot \frac{3}{2} = \frac{12}{2} = 6$$

$$y = \sqrt{3-x^2} = \sqrt{3 - \left(\sqrt{\frac{3}{2}}\right)^2}$$

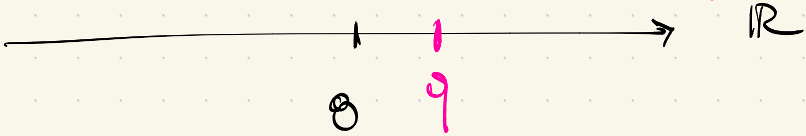
$$= \sqrt{3 - \frac{3}{2}} = \sqrt{\frac{6-3}{2}} = \sqrt{\frac{3}{2}}$$

$$R_n(x) = \frac{f^{(n+1)}(c_x)}{(n+1)!}(x-a)^{n+1}$$

$\leftarrow$ Error de Taylor

$\sqrt{9} = 3$

nos conviene hacer el Taylor en 9



$$f(a) + f'(a)(x-a) + \dots + \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^n$$

$$f(x) = \sqrt{x} \Rightarrow f(9) = \sqrt{9} = 3$$

$$f^{(1)}(x) = \left((x)^{1/2}\right)' = \frac{1}{2}(x)^{-1/2}$$

$$f^{(2)}(x) = \left(\frac{1}{2}(x)^{-1/2}\right)' = \frac{1}{2} \cdot \left(-\frac{1}{2}\right)(x)^{-3/2}$$

$$f^{(3)}(x) = \left(\frac{1}{2}\left(-\frac{1}{2}\right)(x)^{-3/2}\right)'$$

$$\frac{-3}{2} - 1 = \frac{-3}{2} - \frac{2}{2}$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{-3}{2}\right)(x)^{-5/2} \qquad = -\frac{5}{2}$$

$$f^{(n)}(x) = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\cdots \underbrace{(x)^{-\frac{(2n-1)}{2}}}_{\frac{1}{x^{(2n-1)/2}} = \frac{1}{\sqrt[2]{x^{2n-1}}}}$$

$$|R_n(8)| < 10^{-4}$$

$8 \qquad c_8 \qquad 9$

$$'' \quad \left|\frac{f^{(n+1)}(c_8)}{(n+1)!}\underbrace{(8-9)^{n+1}}_{(-1)^{n+1}}\right|$$

$$= \left|\frac{f^{(n+1)}(c_8)}{(n+1)!}\right| \leq \left|f^{(n+1)}(8)\right| < 10^{-4}$$

$$\frac{1}{\left(\sqrt[2]{8}\right)^{2n-1}}$$

$$\sqrt{8} < 10$$

$$f(x) = \int_{\sin(x)}^{x} e^t\sqrt{t^2+1}\,dt \qquad ¿\, f'(x)\,?$$

Fórmulas

$$F(x) = \int_{g(x)}^{h(x)} f(t)\,dt \Rightarrow F'(x) = h'(x) f(h(x)) - g'(x) f(g(x))$$

$$= \int_0^{h(x)} f(t)\,dt - \int_0^{g(x)} f(t)\,dt$$

$$|f^{(n+1)}(x)| \leq 1$$

$$|R_n(x)| = \left| \frac{\cancel{f^{(n+1)}(c_n)}}{(n+1)!} x^{n+1} \right|$$

$$f(x) = \sin(x)$$

$$f^{(n+1)}(x) = \begin{cases} \pm\sin(x) \\ \pm\cos(x) \end{cases}$$

$$\leq 1$$

$$\leq \left| \frac{f^{(n+1)}(c_2)}{(n+1)!} 2^{n+1} \right|$$

$$\leq \frac{2^{n+1}}{(n+1)!} < 0{,}1$$

$n = 1$ $\quad \frac{2^2}{2!} = \frac{4}{2} = 2$ X

$n = 2$ $\quad \frac{2^3}{3!} = \frac{8}{6} =$ X

$n = 3 =$

...

f cont. $\quad f(1)=1$

$$\lim_{x\to 1} \frac{F(x)-F(1)}{x-1} = F'(1)$$

$$\lim_{x\to 1} \frac{\int_{1/x}^{x} f(t)\,dt}{x-1}$$

$$= \lim_{x\to 1} \frac{\int_{1/x}^{x} f(t)\,dt - \int_{1/1}^{1} f(t)\,dt}{x-1}$$

$$F(x) = \int_{1/x}^{x} f(t)\,dt \qquad \lim_{x\to 1} \frac{F(x) - F(1)}{x-1}$$

$$= F'(1)$$

$\exists\ c \in (a,b)$

$$(f(b)-f(a))\,g'(c) = (g(b)-g(a))\,f'(c)$$

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

$\neq$ $\neq 0$

(E) Si $g' > 0 \Rightarrow \exists\, \partial$ / ...

$g' > 0 \Rightarrow g$ es creciente

$g(b)-g(a) \neq 0$

$g'(c) \neq 0$

$$\int_c^{x^2} f(t)\,dt = \frac{1}{1+x^2} - c$$

Derivo de ambos lados:

$$\left(\int_c^{x^2} f(t)\,dt\right)' = (x^2)'\,f(x^2) - \cancel{\underbrace{(c)'}_{=0} f(c)}$$

$$= 2x\, f(x^2)$$

$\frac{1}{x}$ $\quad x^2+1$

$$\left(\frac{1}{1+x^2} - C\right)' = \left(\frac{1}{1+x^2}\right)'$$

$$= 2x \cdot \frac{-1}{(1+x^2)^2}$$

$$\Rightarrow \cancel{2x}\, f(x^2) = \cancel{2x} \left(\frac{-1}{(1+x^2)^2}\right)$$

$$f(\underbrace{x^2}_{y}) = \frac{-1}{(1+\underbrace{x^2}_{y})^2}$$

$$f(y) = \frac{-1}{(1+y)^2}$$

$$f(x) = \frac{-1}{(1+x)^2}$$

$$\underbrace{\int_c^{x^2} f(t)\,dt}= \frac{1}{1+x^2} - c$$

$$\int_c^{x^2} \frac{-1}{(1+t)^2}\,dt = \frac{1}{1+x^2} - c$$

$$\int \frac{-1}{(1+t)^2}\,dt = \int \frac{-1}{u^2}\,du$$

$u = 1+t$

$du = dt$

$$= \frac{1}{u} = \frac{1}{1+t}$$

$$\left(\frac{1}{1+t}\right)\Bigg|_c^{x^2} = \frac{\cancel{1}}{1+x^2} - \frac{1}{1+c}$$

$$= \frac{\cancel{1}}{1+x^2} - c$$

$$-c = \frac{-1}{1+c}$$

$$-c(1+c) = -1$$

$$-c - c^2 = -1$$

$$0 = c^2 + c - 1$$

$$c = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1(-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$$c > 0 \Rightarrow \quad c = \frac{-1+\sqrt{5}}{2} \quad \checkmark$$