Fracciones simples: $\int \frac{p(x)}{q(x)}$

$= (r_1)(r_2)(r_3)$

p, q polinomios

$gr(p) < gr(q)$

$\int \frac{p(x)}{q(x)} = \int \frac{a}{r_1} + \int \frac{a}{r_2} + \cdots \int \frac{a}{r_n}$

(Sección 2)

Ejercicio 3:

a) $\int \frac{x}{(x+1)(x+2)(x+3)} dx$

$\rightarrow \frac{x}{(x+1)(x+2)(x+3)} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{x+3}$ ¿a, b, c?

$\frac{x}{(x+1)(x+2)(x+3)} = \frac{a(x+2)(x+3) + b(x+1)(x+3) + c(x+1)(x+2)}{(x+1)(x+2)(x+3)}$

$= \frac{a(x^2+5x+6) + b(x^2+4x+3) + c(x^2+3x+2)}{(x+1)(x+2)(x+3)}$

$= \frac{ax^2 + 5ax + 6a + bx^2 + 4bx + 3b + cx^2 + 3cx + 2c}{(x+1)(x+2)(x+3)}$

$\frac{x}{(x+1)(x+2)(x+3)} = \frac{(a+b+c)x^2 + (5a+4b+3c)x + (6a+3b+2c)}{(x+1)(x+2)(x+3)}$

$\leftrightarrow x = \underbrace{(a+b+c)}_{=0} x^2 + \underbrace{(5a+4b+3c)}_{=1} x + \underbrace{(6a+3b+2c)}_{=0}$

$\leftrightarrow \begin{cases} a+b+c=0 \\ 5a+4b+3c=1 \\ 6a+3b+2c=0 \end{cases}$

$c = -a-b$

$5a+4b+3(-a-b) = 1$

$2a + b = 1 \rightarrow b = 1-2a$

$c = -a-b = -a-(1-2a)$

$= a-1$

$6a+3(1-2a)+2(a-1) = 0$

$\cancel{6a} + 3 - \cancel{6a} + 2a - 2 = 0$

$2a = -1$

$\boxed{a = -1/2}$

$\boxed{c = -\frac{1}{2} - 1 = -\frac{3}{2}}$

$\boxed{b = 1 - 2(-1/2) = 1+1 = 2}$

$\Rightarrow \boxed{\frac{x}{(x+1)(x+2)(x+3)} = \frac{-1/2}{x+1} + \frac{2}{x+2} - \frac{3/2}{x+3}}$ ✓

Ojo: no siempre anda

Ej: $\frac{x}{(x+1)^2(x+3)}$ ✗

$\frac{x}{(x^2+1)(x-3)}$ ✗

"Tapadito":

$\frac{x}{(x+1)(x+2)\cancel{(x+3)}} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{x+3}$

$a = \frac{-1}{(-1+2)(-1+3)} = \frac{-1}{2}$ ✓

$b = \frac{-2}{(-2+1)(-2+3)} = \frac{-2}{(-1)(1)} = 2$ ✓

$c = \frac{-3}{(-3+1)(-3+2)} = \frac{-3}{-2(-1)} = \frac{-3}{2}$ ✓

$= \frac{-3}{2(x+3)}$

$\int \frac{x}{(x+1)(x+2)(x+3)} dx = \int \frac{-1/2}{x+1} + \frac{2}{x+2} - \frac{3/2}{x+3} dx$

$= -\frac{1}{2} \int \frac{1}{x+1} dx + 2 \int \frac{1}{x+2} dx - \frac{3}{2} \int \frac{1}{x+3} dx$

$= -\frac{1}{2} \log|x+1| + 2\log|x+2| - 3\log|x+3| + C$

d) $\int \frac{6x^3}{x^3-6x^2+11x-6} dx = \int \frac{6(x^3-6x^2+11x-6)}{x^3-6x^2+11x-6} \ominus \frac{(-36x^2+66x-36)}{x^3-6x^2+11x-6} dx$

$6x^3 - 36x^2 + 66x - 36$

$= \int 6 dx + \int \frac{36x^2-66x+36}{x^3-6x^2+11x-6} dx$

$= 6x + \int \frac{36x^2-66x+36}{x^3-6x^2+11x-6}$

Podemos usar fracciones

	1	-6	11	-6
1		1	-5	6
	1	-5	6	0

$\Rightarrow x^3-6x^2+11x-6$

$= (x-1)(x^2-5x+6)$

$= (x-1)(x-2)(x-3)$

Usamos fracciones simples.

i) $\int \frac{1}{(x^2+1)(x^2+2)} dx$

$\frac{1}{(x^2+1)(x^2+2)} = \frac{ax+b}{x^2+1} + \frac{cx+d}{x^2+2}$

$= \frac{(ax+b)(x^2+2) + (cx+d)(x^2+1)}{(x^2+1)(x^2+2)}$

$= \frac{ax^3 + 2ax + bx^2 + 2b + cx^3 + cx + dx^2 + d}{(x^2+1)(x^2+2)}$

$\frac{1}{(x^2+1)(x^2+2)} = \frac{\underbrace{(a+c)}_{=0}x^3 + \underbrace{(b+d)}_{=0}x^2 + \underbrace{(2a+c)}_{=0}x + \underbrace{(2b+d)}_{=1}}{(x^2+1)(x^2+2)}$

$\begin{cases} a+c=0 \\ b+d=0 \\ 2a+c=0 \\ 2b+d=1 \end{cases}$

$c = -a$

$d = -b$

$2a - a = 0 \rightarrow \boxed{a=0} \rightarrow \boxed{c=0}$

$2b - b = 1$ $\boxed{b=1} \rightarrow \boxed{d=-1}$

$\Rightarrow \frac{1}{(x^2+1)(x^2+2)} = \frac{1}{x^2+1} + \frac{-1}{x^2+2}$

$= \frac{1}{x^2+1} - \frac{1}{x^2+2}$

$\int \frac{1}{(x^2+1)(x^2+2)} dx = \int \frac{1}{x^2+1} dx - \int \frac{1}{x^2+2}$ $= 2\left(\frac{x^2}{2}+1\right)$

$= \text{arctg}(x) - \int \frac{1}{2\left(\frac{x^2}{2}+1\right)}$ $\frac{x^2}{2} = \left(\frac{x}{\sqrt{2}}\right)^2$

$= \text{arctg}(x) - \frac{1}{2} \int \frac{1}{\frac{x^2}{2}+1}$

$u = \frac{x}{\sqrt{2}}$

$du = \frac{1}{\sqrt{2}} dx$

$= \text{arctg}(x) - \frac{1}{2} \int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1} dx$

$= \text{arctg}(x) - \frac{1}{2} \cdot \sqrt{2} \int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1} \cdot \frac{1}{\sqrt{2}} dx$

$= \text{arctg}(x) - \frac{1}{\sqrt{2}} \int \frac{1}{u^2+1} du$ $\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$

$= \text{arctg}(x) - \frac{1}{\sqrt{2}} \text{arctg}(u)$

$\boxed{= \text{arctg}(x) - \frac{1}{\sqrt{2}} \text{arctg}\left(\frac{x}{\sqrt{2}}\right)}$

e) $\int \frac{1}{x^2+x+2} dx$

$x = \frac{-1 \pm \sqrt{1-4(2)}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$ ✗ no tiene raíces reales.

$x^2+x+2 = (x-\alpha)^2 + \beta$

$x^2+x+2 = x^2 - 2\alpha x + \alpha^2 + \beta$

$1 = -2\alpha \rightarrow \alpha = -1/2$

$2 = \alpha^2 + \beta = \left(-\frac{1}{2}\right)^2 + \beta$

$= \frac{1}{4} + \beta \Rightarrow \beta = 2 - \frac{1}{4}$

$= \frac{3}{4}$

$\int \frac{1}{x^2+2x+2} = \int \frac{1}{(x+1/2)^2 + \frac{3}{4}}$