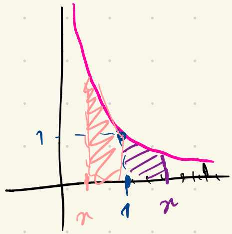
Logaritmo: $x>0$, definimos $\log(x)=\displaystyle\int_1^x \frac{1}{t}\,dt$

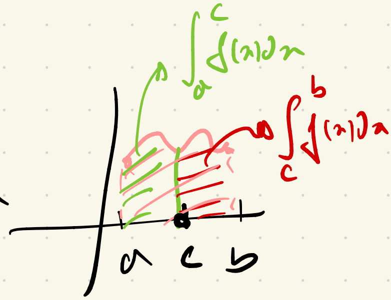


Ejemplo: $x=1/2$:

$$\log\left(\tfrac{1}{2}\right)=\int_1^{1/2}\frac{1}{t}\,dt = -\int_{1/2}^{1}\frac{1}{t}\,dt$$

Propiedades de la integral:

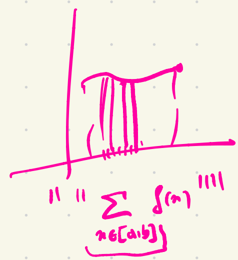
1) $$\underbrace{\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx}$$



Aditividad de la integral.

2) · $$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

· $$\underbrace{\int_a^b \lambda f(x)\,dx = \lambda\int_a^b f(x)\,dx}$$

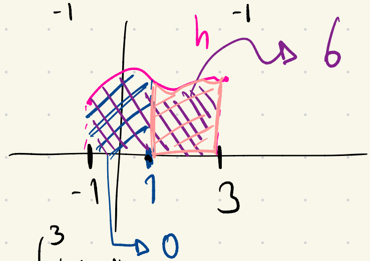


Linealidad de la integrable.

Sección 6, Cálculo de integrales:

Ejercicio 1: a) Sea $h:\mathbb{R}\to\mathbb{R}$ / $\int_{-1}^{1} h(t)dt = 0$ y $\int_{-1}^{3} h(t)dt = 6$.

Calcular $\int_{1}^{3} h(t)dt$.



$$\underbrace{\int_{-1}^{3} h(t)dt}_{6} = \int_{-1}^{1} h(t)dt + \int_{1}^{3} h(t)dt$$

$$6 = 0 + \int_{1}^{3} h(t)dt \Rightarrow \boxed{\int_{1}^{3} h(t)dt = 6}$$

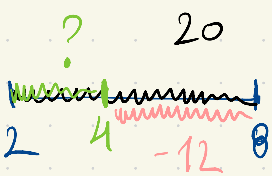
Ejercicio 3: Sea $f:[2,8]\to\mathbb{R}$ integrable tal que

$\int_{2}^{8} f(x)dx = 20$ y $\boxed{\int_{8}^{4} f(x)dx = 12}$ $\to$ $-\int_{4}^{8} f(x)dx = 12$

$\int_{4}^{8} f(x)dx = -12$

a) Calcular $\int_{2}^{4} f(x)dx$

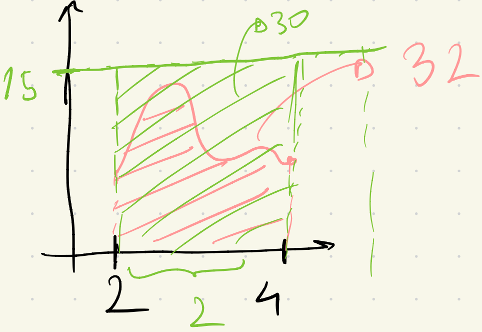
b) Probar que existen $c, d \in [2,4]$ tales que $f(c) \geq 15$ y $f(d) \leq 17$.



$$20 = ? + (-12)$$

$$\Rightarrow \int_{2}^{4} f(x)dx = 20 + 12 = 32$$

$$\Rightarrow \boxed{\int_{2}^{4} f(x)\, dx = 32}$$



Supongamos por absurdo que $\nexists\ c \in [2,4]$ /

$$f(c) \geq 15.$$

$$\Rightarrow f(x) < 15 \quad \forall x \in [2,4]$$

$$\Rightarrow \underbrace{\int_2^4 f(x)\,dx}_{32} \leq \underbrace{\int_2^4 15\,dx}_{30}$$

$$\Rightarrow 32 \leq 30$$

Esto es absurdo

Prop: Si $f(x) \leq g(x) \quad \forall x \in [a,b]$

$$\Rightarrow \int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

Por lo tanto, $\exists\ c \in [2,4]$ tal que $f(c) \geq 15$.

$\left(\int f(x)+g(x)\,dx = \int f(x) + \int g(x) \right)$

$\left(\int \lambda f(x)\,dx = \lambda \int f(x)\,dx \right)$

Ejercicio 4.

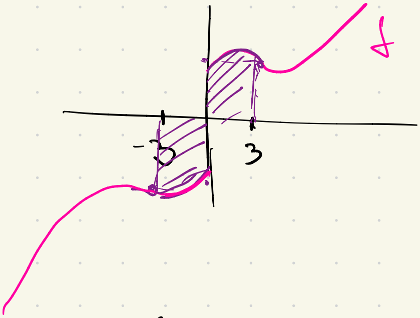
a) $$\int_0^2 2x^2 + x - 3\,dx = \int_0^2 2x^2\,dx + \underbrace{\int_0^2 x\,dx}_{\frac{2^2-0^2}{2} = 2} + \int_0^2 -3\,dx = 2\int_0^2 x^2\,dx + 2 + (2-0)(-3)$$

$$= 2\left(\frac{2^3-0^3}{3}\right) + 2 - 6 = 2\left(\frac{8}{3}\right) - 4$$

$$= \frac{16}{3} - 4 = \frac{16-12}{3} = \frac{4}{3}$$

$$\Rightarrow \boxed{\int_0^2 2x^2 + x - 3\,dx = \frac{4}{3}}$$

Función Impar: $\underbrace{f(-x) = -f(x)} \quad \forall x \in \mathbb{R}$.



$\to f(-3) = -f(3)$

$f(-7) = -f(7)$

Ejemplo: $\star\ f(x) = x$

$$f(-x) = -x = -f(x)$$

Prop: Si f es impar

$$\Rightarrow \int_{-a}^{a} f(x)\,dx = 0.$$

$\star\ f(x) = x^3$:

$$f(-x) = (-x)^3 = (-1)^3 x^3$$
$$= -1\, x^3 = -f(x)$$

$(\sqrt{ab} = \sqrt{a}\sqrt{b})$

Ejercicio 8)

$$\int_a^b \sqrt{x}\,dx = \frac{2}{3}\left(b^{3/2} - a^{3/2}\right)$$

a) $$\int_3^4 \underbrace{\sqrt{3x}}_{\sqrt{3}\sqrt{x}}\,dx = \int_3^4 \sqrt{3}\sqrt{x}\,dx = \sqrt{3}\int_3^4 \sqrt{x}\,dx = \sqrt{3}\left(\frac{2}{3}\left(4^{3/2} - 3^{3/2}\right)\right)$$

$(x^{1/2} = \sqrt{x})$

$(x^{m/n} = \sqrt[n]{x^m})$

$$= \frac{2\sqrt{3}}{3}\left(\sqrt[2]{4^3} - \sqrt[2]{3^3}\right)$$

$$= \frac{2}{\sqrt{3}}\left(\sqrt[2]{64} - \sqrt[2]{27}\right)$$

$$\frac{2\cancel{\sqrt{3}}}{\sqrt{3}\cancel{\sqrt{3}}} = \frac{2}{\sqrt{3}}$$

e) $$\int_5^7 \sqrt{x-3}\,dx = \int_{2+3}^{4+3} \sqrt{x-3}\,dx = \int_2^4 \sqrt{x}\,dx = \frac{2}{3}\left(4^{3/2} - 2^{3/2}\right)$$

$$= \frac{2}{3}\left(\sqrt[2]{64} - \sqrt[2]{8}\right)$$

$$\boxed{\text{Cambio variable: } \int_a^b f(t)\,dt = \int_{a+p}^{b+p} f(t-p)\,dt.}$$

→ Queremos aplicar esto para $f(x) = \sqrt{x}$.

$f(x-p) = \sqrt{x-p}$

$p = 3$