Integración por partes:

→ $(fg)' = f'g + fg'$ (Regla del producto)

$\int (fg)' = \int f'g + fg'$

"=" $fg = \int f'g + \int fg'$

$\Rightarrow \boxed{\int f'g = fg - \int fg'}$

Sección 2.

Ej 1: a) $\int x \sin(x)\, dx$ ← Primitiva de $x\sin(x)$.

$\int x \sin(x)\, dx = -\cos(x)x - \int -\cos(x)\cdot 1\, dx$ (Partes)

g, f'

$f'(x) = \sin(x) \to f(x) = -\cos(x)$

$g(x) = x \to g'(x) = 1$

$= -\cos(x)x + \int \cos(x)\, dx$

$= -\cos(x)x + \sin(x)$

$\Rightarrow \boxed{\int x\sin(x)\, dx = -\cos(x)x + \sin(x) (+C)}$

Verificación: $(-\cos(x)x + \sin(x))'$

$= \sin(x)x - \cos(x)\cdot 1 + \cos(x)$

$= \sin(x)x$ ✓

$\left(\int x^\alpha = \frac{x^{\alpha+1}}{\alpha+1} \quad (\alpha \neq -1)\right)$

b) $\int x\log(x)\, dx = fg - \int fg'$

f', g

$f'(x) = x \to f(x) = \frac{x^2}{2}$

$g(x) = \log(x) \to g'(x) = 1/x$

$= \frac{x^2}{2}\log(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\, dx$

$= \frac{x^2}{2}\log(x) - \frac{1}{2}\int x\, dx$

$= \frac{x^2}{2}\log(x) - \frac{1}{2}\cdot\frac{x^2}{2} (+C)$

d) $\int \sin^2(x)\, dx = \int \sin(x)\sin(x)\, dx$

f', g

$f'(x) = \sin(x) \to f(x) = -\cos(x)$

$g(x) = \sin(x) \to g'(x) = \cos(x)$

$= -\cos(x)\sin(x) - \int -\cos(x)\cos(x)$

$= -\cos(x)\sin(x) + \int \cos^2(x)$

$\Rightarrow \int \sin^2(x) = -\cos(x)\sin(x) + \int \cos^2(x)\, dx$

$= -\cos(x)\sin(x) + \int 1 - \sin^2(x)\, dx$

$= -\cos(x)\sin(x) + \int 1\, dx - \int \sin^2(x)$

$\to x$

$1 = \sin^2(x) + \cos^2(x)$

$\Rightarrow \cos^2(x) = 1 - \sin^2(x)$

$\int \sin^2(x)\, dx + \int \sin^2(x)\, dx = -\cos(x)\sin(x) + x$

$2\int \sin^2(x)\, dx = -\cos(x)\sin(x) + x$

$\Rightarrow \int \sin^2(x)\, dx = \frac{1}{2}(-\cos(x)\sin(x) + x)$ ✓

f) $\int \log(x)\, dx = \int 1\cdot\log(x)\, dx = x\log(x) - \int x\cdot\frac{1}{x}\, dx$

f', g

$= x\log(x) - \int 1\, dx$

$= x\log(x) - x + C$ ✓

$f'(x) = 1 \to f(x) = x$

$g(x) = \log(x) \to g'(x) = 1/x$

$\Rightarrow \boxed{\int \log(x)\, dx = x\log(x) - x}$

Método de sustitución:

$(f(u(x)))' = f'(u(x))\, u'(x)$ ← Regla de la cadena

$\int (f(u(x)))' = \int f'(u(x))\, u'(x)\, dx$

$f(u(x)) = \int f'(u(x))\, u'(x)\, dx$ ←

u, u, du $\quad u(x) = \ldots$

$\int f(u)\, du = f(u)$

Ej 2:

a) $\int e^x \sin(e^x)\, dx = \int \sin(u)\, du = -\cos(u) = -\cos(e^x)$

du, u

$u = e^x$

$du = e^x dx$

$\Rightarrow \int e^x \sin(e^x)\, dx = -\cos(e^x) + C$

Verificación: $(-\cos(e^x))' = \sin(e^x)\, e^x$ ✓

b) $\int \frac{e^x}{1+e^{2x}}\, dx = \int \frac{e^x\, dx}{1+(e^x)^2}$

$(e^x)^2$

$= \int \frac{1}{1+(e^x)^2}\cdot e^x dx$

du

→ $u = e^x$

→ $du = e^x dx$

$(e^x)' dx$

$= \int \frac{1}{1+u^2}\, du$

$\boxed{(\text{arctg}(x))' = \frac{1}{1+x^2}}$

$= \text{arctg}(u)$

$= \text{arctg}(e^x)$

$\Rightarrow \boxed{\int \frac{e^x}{1+e^{2x}}\, dx = \text{arctg}(e^x) + C \quad C \in \mathbb{R}}$

f) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\int \frac{\cos\sqrt{x}}{2\sqrt{x}}\, dx$

du

$u = \sqrt{x}$

$du = (\sqrt{x})'\, dx = \frac{1}{2\sqrt{x}}\, dx$

$= 2\int \cos u\, du$

$= 2(\sin(u))$

$= 2\sin(\sqrt{x})$

$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\int \cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\, dx$

du

$= 2\int \cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\, dx$

u

$= 2\int \cos u\, du = 2\sin(u)$

$= 2\sin(\sqrt{x})$

$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\sin(\sqrt{x}) + C \quad C \in \mathbb{R}$

d) $\int \frac{x^2}{\sqrt{x^3-1}}\, dx$

$u = x^3 - 1$ ✓

g) $\frac{1}{-2}\int x\sqrt{1-x^2}\, dx$

$u = 1 - x^2$

$du = -2x\, dx$ ✓

j) $\int \tan(x)\, dx = \int \frac{\sin(x)}{\cos(x)}\, dx$

$= -\int \frac{1}{\cos(x)}\cdot(-\sin(x))\, dx$

du

$u = \cos(x)$

$du = -\sin(x)\, dx$

$= -\int \frac{1}{u}\, du$

$= -\log(u) + C, \quad C \in \mathbb{R}$..

k) $\int \frac{1}{\cos(x)}\, dx = \int \frac{\cos(x)}{\cos^2(x)}\, dx$

$= \int \frac{\cos(x)}{1-\sin^2(x)}\, dx = \int \frac{1}{1-u^2}\, du$

$= \int \frac{1}{(1+u)(1-u)}\, du$

$u = \sin(x)$

$du = \cos(x)\, dx$

Se puede resolver usando fracciones simples

$1 - u^2 = (1+u)(1-u)$