Práctico 2, Sección 1: Axiomas de cuerpo ordenado.

Ejercicio 2: Determinar los números que verifican las inecuaciones.

Parte a) $\underbrace{\frac{2-x}{1+x}}_{A} \le \underbrace{\frac{1+x}{2-x}}_{B} \longleftrightarrow \frac{2-x}{1+x} - \frac{1+x}{2-x} \le 0$

$A \le B$

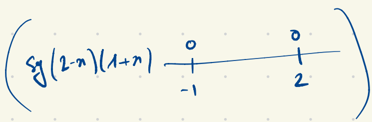
$A - B \le B - B = 0$

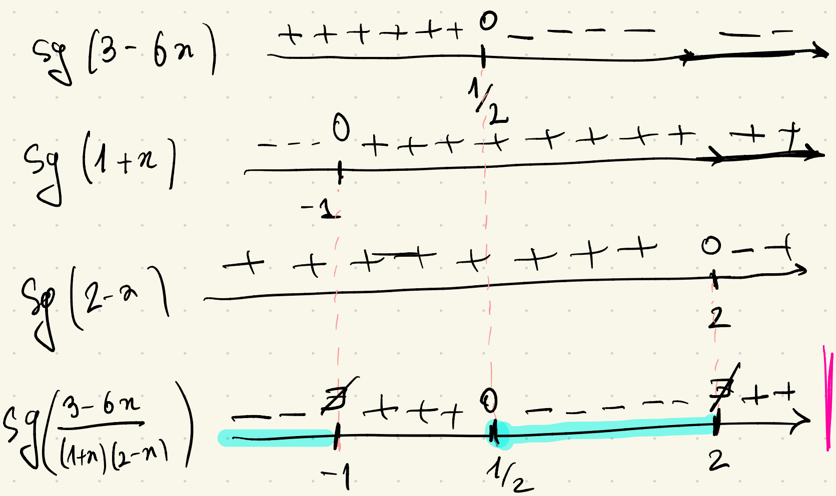
$(2-x)^2 = 4-4x+x^2$ $\qquad = 1+2x+x^2 = (1+x)^2$

$$\longleftrightarrow \frac{(2-x)(2-x) - (1+x)(1+x)}{(1+x)(2-x)} \le 0$$

$$\longleftrightarrow \frac{4-4x+x^2-\left((1+2x+x^2)\right)}{(1+x)(2-x)} \le 0$$

$$\longleftrightarrow \frac{3-6x}{(1+x)(2-x)} \le 0$$





$\Rightarrow \boxed{(-\infty,-1) \cup [\tfrac{1}{2}, 2)}$

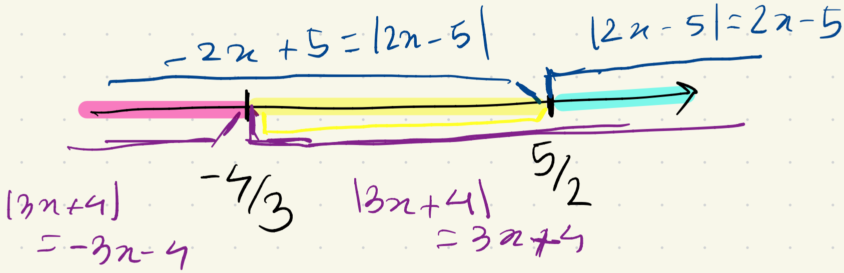
$$2x-5=0 \longleftrightarrow x=\frac{5}{2}$$

i) $|2x-5| < |3x+4|$

$$|2x-5| = \begin{cases} 2x-5 & \text{si } x \geq 5/2 \\ -2x+5 & \text{si } x < 5/2 \end{cases}$$

$$3x+4=0 \longleftrightarrow x=\frac{-4}{3}$$

$$|3x+4| = \begin{cases} 3x+4 & \text{si } x \geq \frac{-4}{3} \\ -3x-4 & \text{si } x < -\frac{4}{3} \end{cases}$$



Resolvemos en cada tramo:

1) Cuando $x \in (-\infty, -4/3)$:

$$\begin{cases} |2x-5| = -2x+5 \\ |3x+4| = -3x-4 \end{cases} \Rightarrow |2x-5| < |3x+4|$$

$$\longleftrightarrow -2x+5 < -3x-4$$

$$\longleftrightarrow x < -9$$

$\boxed{\text{Queda: } (-\infty, -9)}$

2) $[-4/3, 5/2) \ni x$

En este caso:

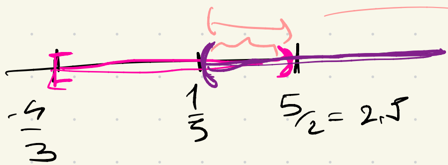
$$\begin{cases} |2x-5| = -2x+5 \\ |3x+4| = 3x+4 \end{cases} \Rightarrow |2x-5| < |3x+4|$$

$$\Longleftrightarrow -2x+5 < 3x+4$$

$$\Longleftrightarrow 1 < 5x$$

$$\Longleftrightarrow \frac{1}{5} < x \quad \left(x \in \left(\frac{1}{5}, +\infty\right)\right)$$

Nos queda: $\overbrace{[-4/3, 5/2)} \cap \overbrace{\left(\frac{1}{5}, +\infty\right)} = \boxed{\left(\frac{1}{5}, \frac{5}{2}\right)}$



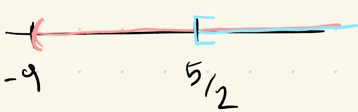
3) Veamos que pasa cuando $x \in \left[\frac{5}{2}, +\infty\right)$

$$\begin{cases} |2x-5| = 2x-5 \\ |3x+4| = 3x+4 \end{cases} \Rightarrow |2x-5| < |3x+4|$$

$$\Longleftrightarrow 2x-5 < 3x+4$$

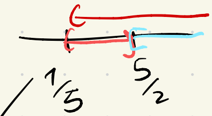
$$\Longleftrightarrow -9 < x$$

$x \in (-9, +\infty)$



Nos quedo:

$$\left[\frac{5}{2}, +\infty\right) \cap (-9, +\infty) = \boxed{\left[\frac{5}{2}, +\infty\right)}$$

Los x que verifican la inecuación son: 

$$(-\infty,-4)\cup\underbrace{\left(\frac{1}{5},\frac{5}{2}\right)\cup\left[\frac{5}{2},+\infty\right)}_{=\left(\frac{1}{5},+\infty\right)}$$

Ejercicio 4: $a,b\in\mathbb{R}$ tales que: $0<a<b$.

$$A:=\frac{a+b}{2},\quad G:=\sqrt{ab},\quad H:=\frac{2}{\frac{1}{a}+\frac{1}{b}}$$

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Por ejemplo: si $a=1$, $b=2\Rightarrow$

$$\begin{cases} A=\dfrac{1+2}{2}=\dfrac{3}{2}\\ G=\sqrt{1\cdot 2}=\sqrt{2}\\ H=\dfrac{2}{\frac{1}{1}+\frac{1}{2}}=\dfrac{2}{3/2}=\dfrac{4}{3}\end{cases}$$

Demostrar:

$$a<H<G<A<b$$

Habría que probar: $a<H$, $H<G$, $G<A$, $A<b$.

Probamos que $A<b$: $\left(a<b\to\frac{a}{2}<\frac{b}{2};\ \frac{a}{2}+\frac{b}{2}<\frac{b}{2}+\frac{b}{2}\right)$

$$\frac{a+b}{2}<\frac{b+b}{2}=\frac{2b}{2}=b$$

↳ $a<b$

$$\Rightarrow\frac{a+b}{2}<b$$

$$\Rightarrow\boxed{A<b}$$

Probemos que $G < A$:

Queremos probar que $\sqrt{ab} < \frac{a+b}{2}$

$$A = \frac{a+b}{2} = \sqrt{\left(\frac{a+b}{2}\right)^2} = \sqrt{\frac{a^2+2ab+b^2}{4}}$$

Quiero ver que:

$a^2+b^2 > 2ab$

$\longleftrightarrow a^2-2ab+b^2 > 0$

$\longleftrightarrow (a-b)^2 > 0$

$$> \sqrt{\frac{2ab+2ab}{4}}$$

$$= \sqrt{\frac{4ab}{4}} = \sqrt{ab} = G$$

$$\Rightarrow \boxed{A > G}$$