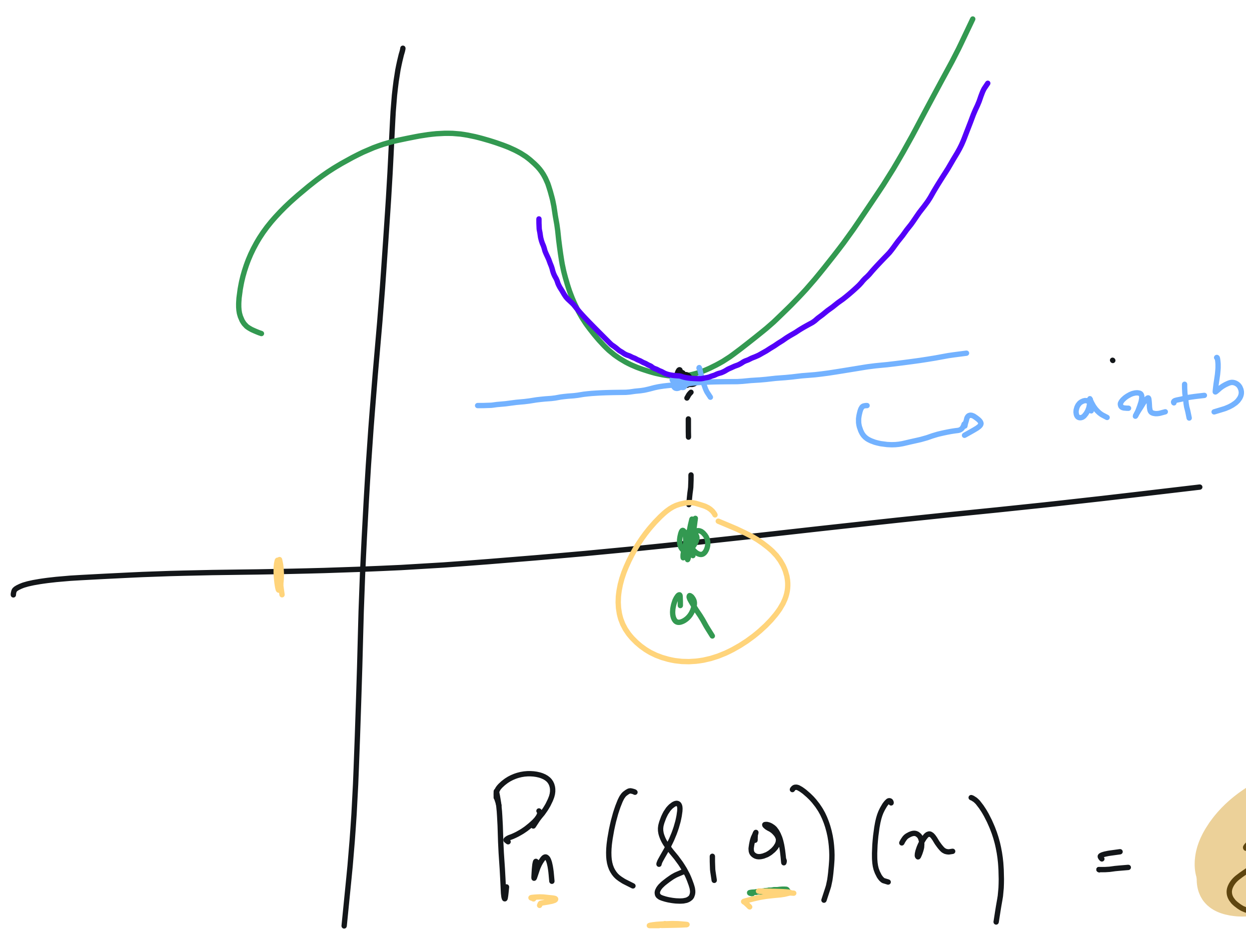
Polinomio de Taylor:



↳ $ax+b$

$(f'(a)(x-a)+f(a))$

Son números

$$P_n(f,a)(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

↳ El polinomio de Taylor de orden n centrado en a

1. (*) El polinomio de McLaurin de orden 4 asociado a una cierta función f es $3-5x+4x^2-x^3-2x^4$. Calcular $f(0)$, $f'(0)$, $f''(0)$, $f'''(0)$, $f^{(4)}(0)$.

$$P_4(f,0)(x) = \boxed{3-5x+4x^2-x^3-2x^4}$$

$$= \boxed{f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{\underbrace{4!}_{4}}x^4}$$

$\Rightarrow$

$3 = f(0)$ ✓

$-5 = f'(0)$ ✓

$4 = \frac{f''(0)}{2} \longrightarrow f''(0) = 4 \times 2 = 8$

$-1 = \frac{f^{(3)}(0)}{3!} \longrightarrow f^{(3)}(0) = (-1)\,3! = -6$

$-2 = \frac{f^{(4)}(0)}{4!} \longrightarrow f^{(4)}(0) = (-2)(4!)$

$= (-2)(24)$

$= -48$

Ejercicio 2: Calcular el polinomio de Taylor

a) $f(x) = x^4 - x^3 + 2$, $a=0$ y $a=1$, $n=2$

$$P_2(f,0)(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$f(0) = 2$

$f'(x) = 4x^3 - 3x^2 \Rightarrow f'(0) = 0$

$f''(x) = 12x^2 - 6x \Rightarrow f''(0) = 0$

$$\Rightarrow P_2(f,0)(x) = 2 + \cancel{0 \cdot x} + \cancel{\frac{0}{2} \cdot x^2} = 2$$

Ejemplo: $P_3(\cancel{4x^4} + 3x^3 + 2x - 1, 0)(x)$

$= 3x^3 + 2x - 1$

$P_2(4x^4 + 3x^3 + 2x - 1, 0)(x)$

$= 2x - 1$

NO son el mismo

Ej: $P_3(4(x-1)^4 + \boxed{(x-1)^3 - 2(x-1)^2 + 1}, 1)(x)$

$= (x-1)^3 - 2(x-1)^2 + 1$

$$P_2(x^4-x^3+2,1)(x)$$
$$= f(1)+f'(1)(x-1)+\frac{f''(2)}{2}(x-1)^2$$

$f(1)=1^4-1^3+2=2$

$f'(x)=4x^3-3x^2 \rightsquigarrow f'(1)=4\cdot 1^3-3\cdot 1^2=1$

$f''(x)=12x^2-6x \rightsquigarrow f'(1)=12\cdot 1^2-6\cdot 1=6$

$$\Rightarrow P_2(x^4-x^3+2,\underline{1})(x)$$
$$=2+1(x-1)+\frac{6}{2}(x-1)^2$$

$$=2+(x-1)+3(x-1)^2$$

Ejercicio 3 : $f(x)=x\cos(x)-\sin(x)$

a) Hallar el polinomio de McLaurin de orden 5.

$$P_5(f,0)(x)=f(0)+f'(0)x+\dots+\frac{f^{(5)}(0)}{5!}x^5$$

$f(0)=0\cdot\cos(0)-\sin(0)=0$

$$f'(x) = \left(x\cos(x) - \sin(x)\right)' = \cancel{\cos(x)} + x\left(-\sin(x)\right) - \cancel{\cos(x)}$$

$$= -x\sin(x)$$

$$\Rightarrow f'(0) = 0$$

$$f^{(2)}(x) = \left(-x\sin(x)\right)' = (-1)\sin(x) - x\left(\cos(x)\right)$$

$$= -\sin(x) - x\cos(x)$$

$$\Rightarrow f^{(2)}(0) = -\sin(0) - 0\cdot\cos(0) = 0$$

$$f^{(3)}(x) = \left(-\sin(x) - x\cos(x)\right)' = -\cos(x) - \cos(x) - x\left(-\sin(x)\right)$$

$$= -2\cos(x) + x\sin(x)$$

$$\Rightarrow f^{(3)}(0) = -2\underbrace{\cos(0)}_{=1} + 0\cdot\sin(0) = -2$$

$$f^{(4)}(x) = \left(-2\cos(x) + x\sin(x)\right)'$$

$$= -2\left(-\sin(x)\right) + \sin(x) + x\cos(x)$$

$$= 3\sin(x) + x\cos(x)$$

$$\Rightarrow f^{(4)}(0) = 3\sin(0) + 0\cdot\cos(0) = 0$$

$$f^{(5)}(x) = \left(3\sin(x) + x\cos(x)\right)'$$

$$= 3\cos(x) + \cos(x) + x\left(-\sin(x)\right)$$

$$= 4\cos(x) - x\sin(x)$$

$$\Rightarrow f^{(5)}(0) = 4\cos(0) - 0\cdot\sin(0)$$

$$\boxed{= 4}$$

$$P_5\left(\underline{x\cos(x) - \sin(x)}, 0\right)(x) =$$

$$0 + 0\cdot x + \frac{0}{2}\cdot x^2 - \frac{2}{3!}x^3 + \frac{0}{4!}x^4 + \frac{4}{5!}x^5$$

$$= \boxed{-\frac{1}{3}x^3 + \frac{1}{30}x^5}$$

$$-\frac{1}{3}x^3$$

no tiene extremo relativo en 0

Prop: $P_n(f+g, a)(x)$

$$= P_n(f, a)(x) + P_n(g, a)(x)$$

$\Downarrow$

$x\cos(x) - \sin(x)$

tampoco tiene extremo relativo en 0.

c) Calcular el límite:

$$\lim_{x\to 0^+} \frac{f(x)+x^3/3}{x^5} = \lim_{x\to 0^+} \frac{P_5\left(f(x)+\frac{x^3}{3},0\right)(x)}{x^5}$$

$$= \lim_{x\to 0^+} \frac{P_5(f,0)(x) + P_5\left(\frac{x^3}{3},0\right)(x)}{x^5}$$

$-\frac{x^3}{3} =$

$$= \lim_{x\to 0^+} \frac{\cancel{-\frac{1}{3}x^3} + \frac{1}{30}x^5 + \cancel{\frac{x^3}{3}}}{x^5}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{30}\cdot \cancel{x^5}}{\cancel{x^5}} = \boxed{\frac{1}{30}}$$