Binomio conjugado:
$$(a-b)(a+b)=a^2-b^2$$

Sección 4.3, ejercicio 1)

j) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$$(a-b)(a+b)=a^2-b^2$$

$$=\lim_{x\to 0}\left(\frac{\overbrace{\sqrt{1+x}}^{a}-\overbrace{\sqrt{1-x}}^{b}}{x}\right)\cdot\left(\frac{\overbrace{\sqrt{1+x}}^{a}+\overbrace{\sqrt{1-x}}^{b}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$=\lim_{x\to 0}\frac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{x(\sqrt{1+x}+\sqrt{1-x})}=\lim_{x\to 0}\frac{1+x-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}$$

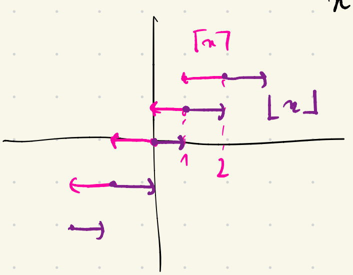
$$=\lim_{x\to 0}\frac{\cancel{1}+x-\cancel{1}+x}{x(\sqrt{1+x}+\sqrt{1-x})}=\lim_{x\to 0}\frac{2\cancel{x}}{\cancel{x}(\sqrt{1+x}+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{2}{\sqrt{1+x}+\sqrt{1-x}}=\frac{2}{1+1}=\frac{2}{2}=1$$

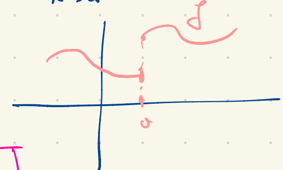
$$\Rightarrow \boxed{\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=1}$$

Ejercicio 2:

a) $\displaystyle\lim_{x\to 0} \lceil x\rceil-\lfloor x\rfloor$



Límites laterales:
$$\lim_{x\to a^+} f(x);\ \lim_{x\to a^-} f(x).$$



$$\lim_{x\to 0^+} \lceil x\rceil-\lfloor x\rfloor$$
$$=1-0=1$$
$$\Rightarrow \boxed{\lim_{x\to 0^+} \lceil x\rceil-\lfloor x\rfloor=1}$$

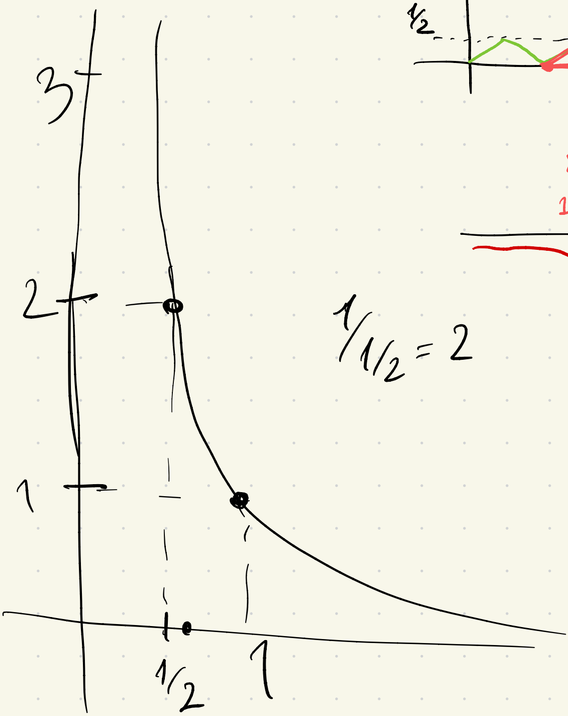
$\displaystyle\lim_{x\to a} f(x)$ existe si y solo si $\exists \displaystyle\lim_{x\to a^-} f(x)$, $\displaystyle\lim_{x\to a^+} f(x)$ y $\displaystyle\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$.

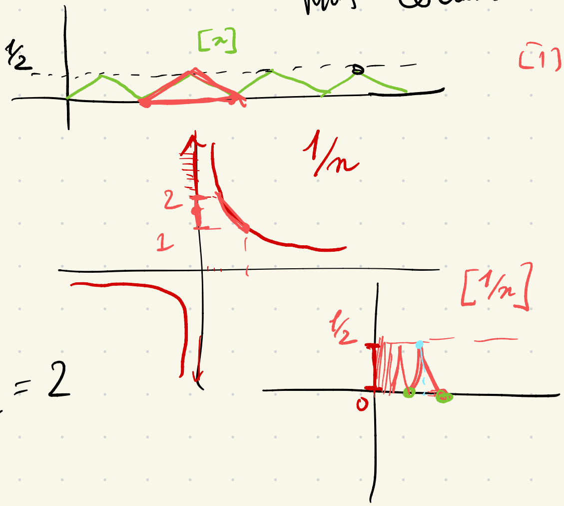
$$\lim_{x\to 0^-} \lceil x\rceil-\lfloor x\rfloor=0-(-1)=0+1=1$$
$$\Rightarrow \boxed{\lim_{x\to 0^-} \lceil x\rceil-\lfloor x\rfloor=1}$$

Como los límites laterales coinciden, obtenemos que:

$$\boxed{\lim_{x\to 0} \lceil x\rceil - \lfloor x\rfloor = 1}$$

f) $\lim_{x\to 0} \left[\frac{1}{x}\right]$. Recordar que: $[x]$ = "distancia al entero más cercano"





$$\frac{1}{1/2} = 2$$

$[1/x]$

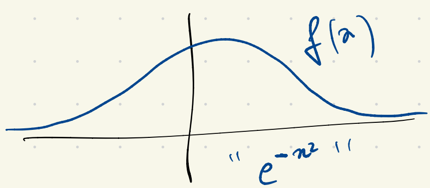
$[1/1] = 0$

$[1/(1/2)] = [2] = 0$

$[3/2] = 1/2$

$$\Rightarrow \boxed{\nexists \lim_{x\to 0} [1/x]}$$

Ejemplo:



$\lim_{x\to 0} f(1/x) = 0$

$\lim_{x\to 0^+} f(\underbrace{1/x}_{\to +\infty}) = 0$

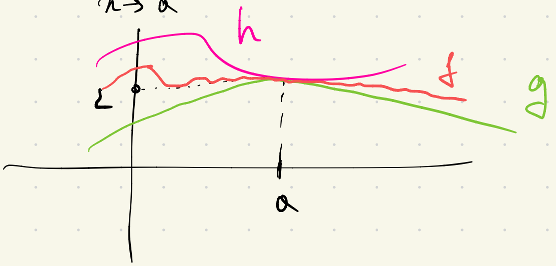
$\lim_{x\to 0^-} f(\underbrace{1/x}_{\to -\infty}) = 0$

Proposición: $\lim_{x \to a} f(x) = 0$, g acotada en un entorno de a

$$\Rightarrow \lim_{x \to 0} f(x) g(x) = 0$$

Teorema del Sanguche: Si $g(x) \leq f(x) \leq h(x)$

$$\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L \Rightarrow \lim_{x \to a} f(x) = L$$



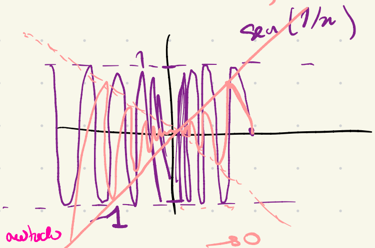
Ejercicio 5.

acotada entre $[-1, 1]$

$(\sin(x) \in [-1,1] \ \forall x \in \mathbb{R})$

a) $$\lim_{x \to 0} x \sin(1/x) = 0$$

$x \to 0$



está acotado

$\to 0$

b) $$\lim_{x \to 0} \frac{\sin(x)^2}{x} = \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \sin(x) = 0$$

Recordemos: $|\sin(x)| \leq |x| \ \forall x \in \mathbb{R}$

$$\frac{|\sin(x)|}{|x|} \leq 1 \ \forall x$$

$$\left|\frac{\sin(x)}{x}\right| \leq 1 \ \forall x \Rightarrow \frac{\sin(x)}{x} \in [-1, 1] \ \forall x$$

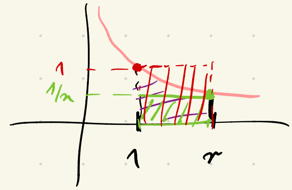
es decir, está acotado.

$$\Rightarrow \lim_{x \to 0} \frac{\sin(x)^2}{x} = 0.$$

Ejercicio 6:

a) $\lim\limits_{x\to 1} \dfrac{\log(x)}{x-1}$

$$\log(x) = \int_1^x \frac{1}{t}\,dt$$



$$\boxed{\frac{x-1}{x} \le \log(x) \le x-1}$$

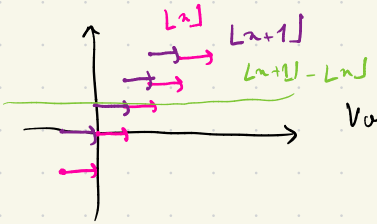
$$\frac{1}{x} \le \frac{\log(x)}{x-1} \le 1 \quad (x>1)$$

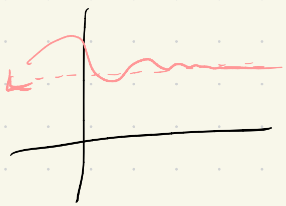
$\downarrow_{x\to 1}$

$$1 \le \lim_{x\to 1^+} \frac{\log(x)}{x-1} \le 1$$

Ejercicio 6:

a) $\lim\limits_{x\to +\infty} \lfloor x+1\rfloor - \lfloor x\rfloor$





Vale: $\lfloor x+1\rfloor = \lfloor x\rfloor + 1$

$$\Rightarrow \lim_{x\to+\infty} \lfloor x+1\rfloor - \lfloor x\rfloor = \lim_{x\to+\infty} \cancel{\lfloor x\rfloor} + 1 - \cancel{\lfloor x\rfloor} = 1$$

Ejercicio 7:

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

a) $$\lim_{x\to+\infty} \frac{\overset{=\sqrt{x^2}}{x}}{\sqrt{x^2+x+1}} = \lim_{x\to+\infty} \frac{\sqrt{x^2}}{\sqrt{x^2+x+1}}$$

$$= \lim_{x\to+\infty} \sqrt{\frac{x^2}{x^2+x+1}}$$

$$= \lim_{x\to+\infty} \sqrt{\underbrace{\frac{x^2}{x^2}}_{=1} \cdot \frac{1}{1+\underbrace{1/x}_{\to 0}+\underbrace{1/x^2}_{\to 0}}} = \sqrt{1} = 1$$