Ejercicio cuestionario:

Igualdad:

$$\int_c^{x^2} f(t)\,dt = \frac{1}{1+x^2} - c$$

$f:[0,+\infty) \to \mathbb{R}$ es cont. y $c \geq 0$

$$\int_c^{x^2} f(t) = \frac{1}{1+x^2} - c \implies \left(\int_c^{x^2} f(t)\,dt\right)' = \left(\frac{1}{1+x^2} - c\right)$$

$$f(x^2)\,2x = \frac{-1}{(1+x^2)^2}(1+x^2)'$$

$$f(x^2)\,2x = \frac{-1}{(1+x^2)^2}(2x)$$

$$\text{Si } x \neq 0 \quad f(x^2) = \frac{-1}{(1+x^2)^2} \quad \forall x \in (0,+\infty)$$

$$y = \sqrt{x} \quad f(y^2) = \frac{-1}{(1+y^2)^2}$$

$$f(\sqrt{x}^2) = \frac{-1}{(1+\sqrt{x}^2)^2}$$

$$f(x) = \frac{-1}{(1+x)^2}$$

$$\implies \boxed{f(x) = \frac{-1}{(1+x)^2}}$$

$$\int \frac{-1}{(1+t)^2}\,dt = \frac{1}{1+t}$$

$$\implies \underbrace{\int_c^{x^2} f(t)\,dt}_{} = \boxed{\frac{1}{1+x^2} - c}$$

$$\implies \int_c^{x^2} \frac{-1}{(1+t)^2}\,dt = \frac{1}{1+t}\Big|_c^{x^2} = \boxed{\frac{1}{1+x^2} - \frac{1}{1+c}}$$

$$\implies \frac{1}{1+x^2} - c = \frac{1}{1+x^2} - \frac{1}{1+c}$$

$$-c = \frac{-1}{1+c}$$

$$c = \frac{1}{1+c} \iff c(1+c) = 1$$

$$c + c^2 - 1 = 0$$

$$c = \frac{-1 \pm \sqrt{1-4(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\implies c = \frac{-1+\sqrt{5}}{2}$$

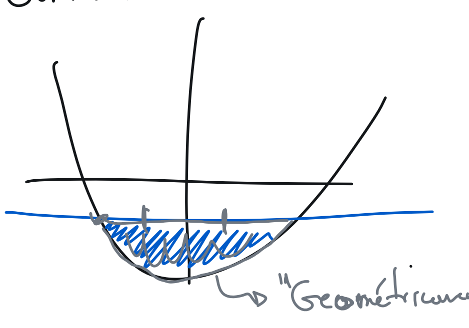
Sección 3 (Práctico 7): Área

Ejercicio 1:

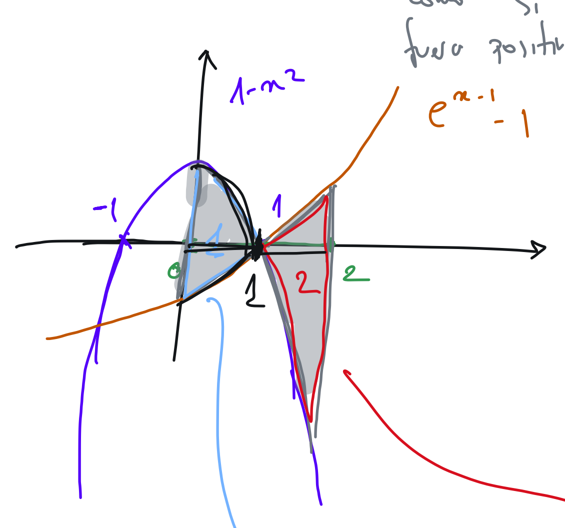
1. (*) Hallar el área encerrada entre los gráficos de las siguientes funciones

   a) $f(x) = e^{x-1} - 1$ y $g(x) = 1 - x^2$ en el intervalo $[0,2]$

Comentario

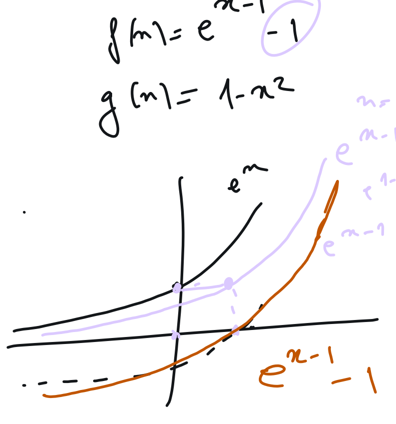


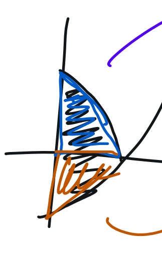
"Geométricamente, si nos piden área, haremos la cuenta como si el signo fuera positivo."



$f(x) = e^{x-1} - 1$

$g(x) = 1 - x^2$





$$\int_0^1 g(t)\,dt$$

$$-\int_0^1 f(t)\,dt$$

$$\int_0^1 g(t) - f(t)\,dt$$

$$\int_0^1 g(t) - f(t)\,dt + \int_1^2 f(t) - g(t)\,dt$$

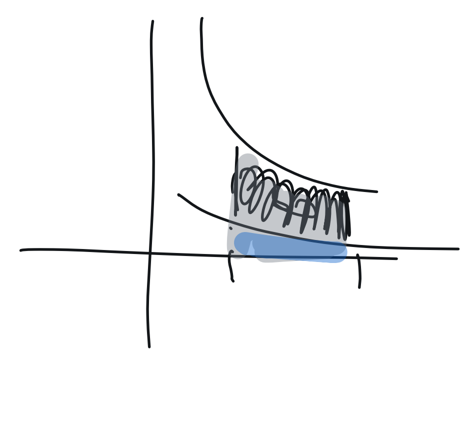
$$= \int_0^1 1 - t^2 - (e^{t-1} - 1)\,dt + \int_1^2 e^{t-1} - 1 - (1-t^2)\,dt$$

$$= \int_0^1 1 - t^2 - e^{t-1} + 1\,dt + \int_1^2 e^{t-1} - 1 - 1 + t^2\,dt$$

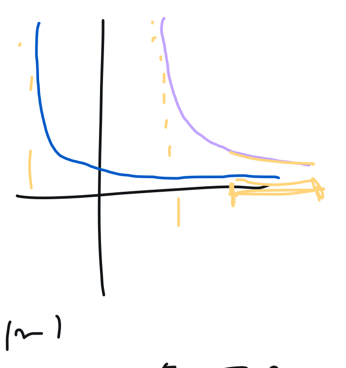
$$= \int_0^1 2 - t^2 - e^{t-1}\,dt + \int_1^2 e^{t-1} + t^2 - 2\,dt$$

$$= \left(2t - \frac{t^3}{3} - e^{t-1}\right)\Big|_0^1 + \left(e^{t-1} + \frac{t^3}{3} - 2t\right)\Big|_1^2$$

$$= \ldots$$

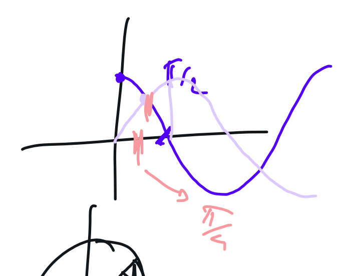


$$\frac{1}{x+1} \qquad \frac{1}{x-1}$$

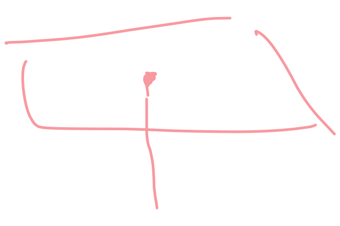


c) $f(x) = \sin(x)$

$g(x) = \cos(x)$ $\quad [0, \frac{\pi}{2})$



Ejercicio 4, Sección 5:



Claramente para una figura simétrica (cuadrado, rectangulo, circunferencia y un triangulo equilatero) el centro de gravedad coincide con el centro geométrico de la figura.

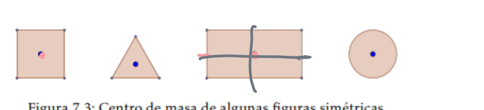
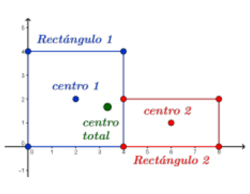


Figura 7.3: Centro de masa de algunas figuras simétricas

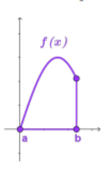
Si se pegan 2 rectangulos como en el de la figura entonces el centro de gravedad es centro total.



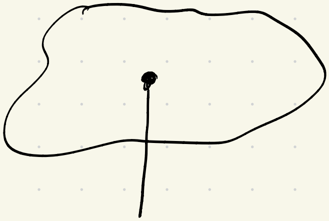
Para una superficie como la de la figura el centro de gravedad es (M_x, M_y), donde

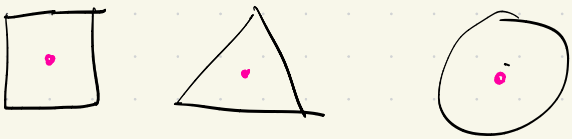
$$M_y = \frac{\int_a^b f(x)^2\,dx}{M} \quad \text{y} \quad M_x = \frac{\int_a^b f(x)x\,dx}{M} \quad \text{con} \quad M = \int_a^b f(x)\,dx$$

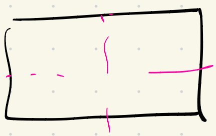
Bosquejar un argumento sobre esta fórmula apartir del caso de los rectángulos.

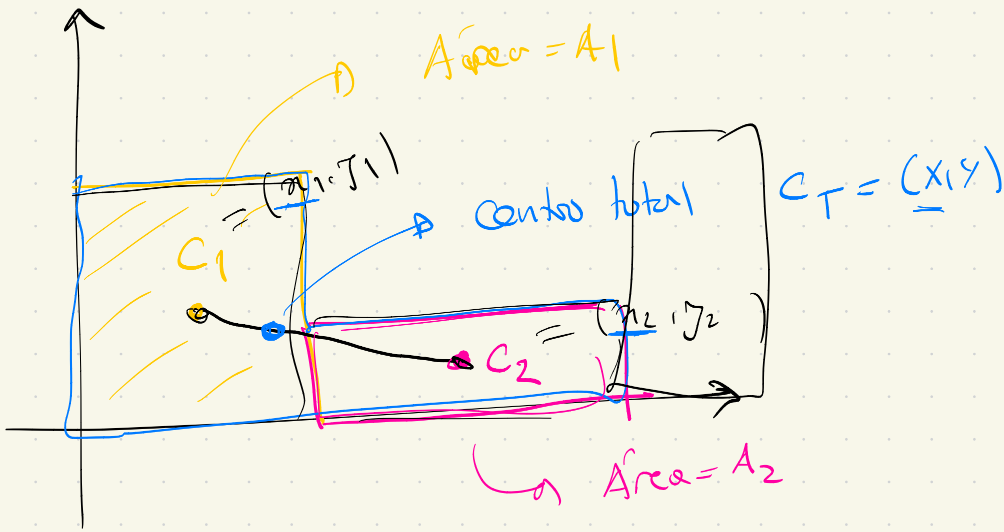


a) Hallar el centro de gravedad de la superficie comprendida bajo una arcada de la sinusoide ($f(x) = \sin(x)$)









$$\text{Centro total} = \frac{A_1 C_1 + A_2 C_2}{A_1 + A_2}$$

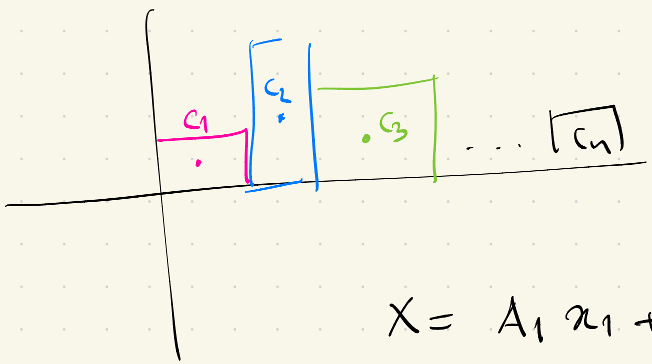
$$C = (X, Y)$$

$\longrightarrow C_1 = (x_1, y_1)$

$\longrightarrow C_2 = (x_2, y_2)$

$$X = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$Y = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$



$C_1 = (x_1, y_1)$

$C_2 = (x_2, y_2)$

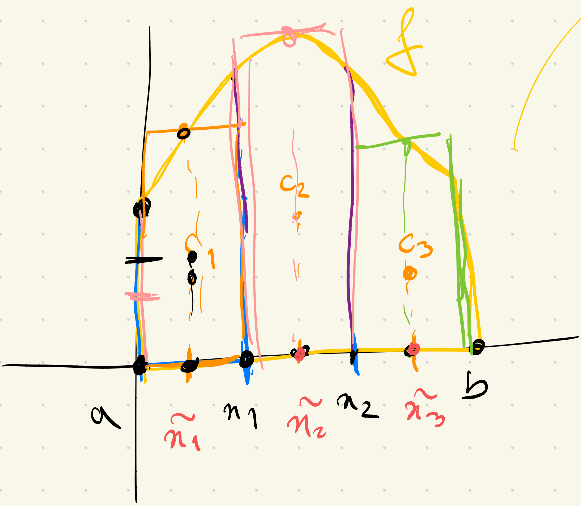
$C_3 = (x_3, y_3)$

$$X = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$Y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$X = \frac{A_1 x_1 + \dots + A_n x_n}{A_1 + \dots + A_n} \qquad Y = \frac{A_1 y_1 + \dots + A_n y_n}{A_1 + \dots + A_n}$$

Centro total $C = (X, Y)$



Centro de gravedad $= (M_x, M_y)$

donde $M_y = \dfrac{\int_a^b f(x)^2 \, dx}{M}$

$M_x = \dfrac{\int_a^b f(x) x \, dx}{M}$,

donde $M = \int_a^b f(x) \, dx$

$C_1 = \left(\tilde{x}_1, \dfrac{f(\tilde{x}_1)}{2}\right)$, $C_2 = \left(\tilde{x}_2, \dfrac{f(\tilde{x}_2)}{2}\right)$

... $C_n = \left(\tilde{x}_n, \dfrac{f(\tilde{x}_n)}{2}\right)$

Centro de gravedad $C = (X, Y)$

$$X = \frac{A_1 \tilde{x}_1 + A_2 \tilde{x}_2 + \dots + A_n \tilde{x}_n}{A_1 + \dots + A_n}$$

$A_1 + \dots + A_n \xrightarrow[n \to \infty]{} \int_a^b f(x) \, dx$

$$Y = \frac{A_1 \frac{f(\tilde{x}_1)}{2} + \cdots + A_n \frac{f(\tilde{x}_n)}{2}}{A_1 + \cdots + A_n}$$

$A_1 + \cdots + A_n \longrightarrow \int_a^b f(x)dx$

$$A_1 = \text{Base} \times \text{Altura} = \frac{(b-a)}{n} f(\tilde{x}_1)$$

$$\vdots$$

$$A_n = \frac{(b-a)}{n} f(\tilde{x}_n)$$

$f(x)x \quad \frac{(b-a)}{n}$

$f(x)x$

$$\Rightarrow X = \frac{A_1 \tilde{x}_1 + \cdots + A_n \tilde{x}_n}{A_1 + \cdots + A_n}$$

$$= \frac{\frac{(b-a)}{n} f(\tilde{x}_1)\tilde{x}_1 + \frac{(b-a)}{n} f(\tilde{x}_2)\tilde{x}_2 + \cdots + \frac{(b-a)}{n} f(\tilde{x}_n)\tilde{x}_n}{A_1 + \cdots + A_n}$$

$n \to +\infty$

$$\frac{\int_a^b f(x)x}{\int_a^b f(x)\,dx}$$