Fracciones simples (gr(den) > gr(num))

① Factores lineales distintos $\frac{x+3}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$$\frac{x+3}{(x+1)(x+2)} = \frac{A(x+2)}{(x+1)(x+2)} + \frac{B(x+1)}{(x+1)(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

$$x+3 = A(x+2)+B(x+1) \rightarrow x+3 = (A+B)x + 2A+B \begin{cases} 1 = A+B \rightarrow B = 1-A = -1 \\ 3 = 2A+B \rightarrow 3 = 2A+1-A \\ 3-1 = A = 2 \end{cases}$$

$$\frac{x+3}{(x+1)(x+2)} = \frac{2}{(x+1)} - \frac{1}{(x+2)}$$

② Factores lineales repetidos $\frac{x^2+3x+1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

$$x^2+3x+1 = A(x+1)^2 + B(x+1) + C = Ax^2 + 2Ax + A + Bx + B + C$$

$$\left.\begin{cases} 1 = A \\ 3 = 2A+B \\ 1 = A+B+C \end{cases}\right\} B = 1 \Big\} C = -1 \qquad \frac{x^2+3x+1}{(x+1)^3} = \frac{1}{(x+1)} + \frac{1}{(x+1)^2} + \frac{(-1)}{(x+1)^3}$$

③ Factores cuadráticos distintos $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$1 = Ax^2 + A + Bx^2 + Cx \begin{cases} 0 = A+B \\ 0 = C \\ 1 = A \end{cases} \Big\} B = -1$$

3.a) $\int \frac{x}{(x+1)(x+2)(x+3)}\,dx = \int \frac{A}{x+1} + \int \frac{B}{x+2} + \int \frac{C}{x+3}$

$$A\,(\text{raíz} = -1) = \left.\frac{x}{(x+2)(x+3)}\right|_{x=-1} = \frac{-1}{1\cdot 2} = \frac{-1}{2}$$

$$B\,(\text{raíz} = -2) = \left.\frac{x}{(x+1)(x+3)}\right|_{x=-2} = \frac{-2}{(-1)\,1} = 2$$

$$C\,(\text{raíz} = -3) = \left.\frac{x}{(x+1)(x+2)}\right|_{x=-3} = \frac{-3}{(-2)(-1)} = -\frac{3}{2}$$

$$\int \frac{x}{(x+1)(x+2)(x+3)} = \int \frac{-1/2}{x+1} + \int \frac{2}{x+2} + \int \frac{-3/2}{x+3} = -\frac{1}{2}\ln|x+1| + 2\ln|x+2| - \frac{3}{2}\ln|x+3|$$

3.b) $\int \frac{x}{x^3-6x^2+11x-6}\,dx$

$$(x-1)(x^2-5x+6)$$

	1	-6	11	-6
1		1	-5	6
	1	-5	6	0

$$x = \frac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 3 \\ 2 \end{cases}$$

$$\int \frac{x}{(x-1)(x-2)(x-3)} = A\int \frac{1}{x-1} + B\int \frac{1}{x-2} + C\int \frac{1}{x-3}$$

$$A = \frac{1}{(-1)(-2)} = \frac{1}{2} \qquad B = \frac{2}{1(-1)} = -2 \qquad C = \frac{3}{2\cdot 1} = \frac{3}{2}$$

$$\frac{1}{2}\int \frac{1}{x-1} - 2\int \frac{1}{x-2} + \frac{3}{2}\int \frac{1}{1-3} = \frac{\ln|x-1|}{2} - 2\ln(x-2) + \frac{3}{2}\ln(x-3)$$

c) $\int \frac{x^4}{x^3-6x^2+11x-6}$

$$D = dC + R \qquad \begin{array}{cc} D & d \\ R & C \end{array}$$

$$\begin{array}{l} x^4 \quad \big| x^3-6x^2+11x-6 \\ -x^4+6x^3-11x^2+6x \quad \big| x+6 \\ \hline 6x^3-11x^2+6x \\ -6x^3+36x^2-66x+36 \\ \hline 25x^2-60x+36 \end{array}$$

$$\int \frac{x^4}{x^3-6x^2+11x-6} = \int \frac{(x^3-6x^2+11x-6)(x+6)+(25x^2-60x+36)}{x^3-6x^2+11x-6}$$

$$\int \frac{(x^3-6x^2+11x-6)(x+6)}{(x^3-6x^2+11x-6)} + \int \frac{25x^2-60x+36}{x^3-6x^2+11x-6}$$

$$\int x+6 + \int \frac{25x^2-60x+36}{x^3-6x^2+11x-6} = \frac{x^2}{2} + 6x + \frac{121}{2}\ln(x+1) - 256\ln(x+2) + \frac{441}{2}\ln(x+3)$$

$$(x+1)(x+2)(x+3)$$

$$\int \frac{A}{(x+1)} + \int \frac{B}{x+2} + \int \frac{C}{x+3} = \frac{121}{2}\ln(x+1) - 256\ln(x+2) + \frac{441}{2}\ln(x+3)$$

$$A = \frac{25+60+36}{1\cdot 2} = \frac{121}{2} \qquad B = \frac{25\cdot 4+120+36}{(-1)\,1} = -256 \qquad C = \frac{25\cdot 9+180+36}{(-2)(-1)} = \frac{441}{2}$$

d) $\int \frac{6x^3}{x^3-6x^2+11x-6}$

$$\begin{array}{l} 6x^3 \quad \big| x^3-6x^2+11x-6 \\ -6x^3+36x^2-66x+36 \quad \big| 6 \\ \hline 36x^2-66x+36 \end{array}$$

$$\int \frac{(x^3-6x^2+11x-6)6+(36x^2-66x+36)}{x^3-6x^2+11x-6} = \int 6 + \int \frac{36x^2-66x+36}{(x-1)(x-2)(x-3)}$$

$$A\int \frac{1}{x-1} + B\int \frac{1}{x-2} + C\int \frac{1}{x-3} = 3\ln|x-1| - 48\ln|x-2| + 81\ln|x-3|$$

$$A = \frac{36-66+36}{(-1)(-2)} = 3 \qquad B = \frac{36\cdot 4-66\cdot 2+36}{1\,(-1)} = -48$$

$$C = \frac{36\cdot 9-66\cdot 3+36}{2} = 81$$

$$\int \frac{6x^3}{x^3-6x^2+11x-6} = 6x + 3\ln|x-1| - 48\ln|x-2| + 81\ln|x-3|$$