7.2 -

1. a - $\int x\,\mathrm{sen}(x)\,dx$ ($u = x$, dv)

$u = x \to du = 1$

$dv = \mathrm{sen}(x) \to v = -\cos(x)$

$$\int u\,dv = uv - \int v\,du$$

I : Inversas trigonométricas
L : Logarítmicas
A : Algebraicas
T : Trigonométricas
E : Exponenciales

(u arriba, dv abajo)

$$\int x\,\mathrm{sen}(x)\,dx = x(-\cos(x)) - \int(-\cos(x)) = -x\cos(x) + \int \cos(x) = \mathrm{sen}(x) - x\cos(x) + C$$

($\int \cos(x) = \mathrm{sen}(x) + C$)

1. c - $\int x e^x = x e^x - \int e^x = x e^x - e^x + C$ ($\int e^x = e^x + C$)

$u = x \to du = 1$

$dv = e^x \to v = e^x$

1. d - $\int \mathrm{sen}^2(x)\,dx = \int \mathrm{sen}(x)\,\mathrm{sen}(x) =$ (u, dv)

$u = \mathrm{sen}(x) \to du = \cos(x)$ $\qquad 1 = \mathrm{sen}^2(x) + \cos^2(x)$

$dv = \mathrm{sen}(x) \to v = -\cos(x)$

$$= \mathrm{sen}(x)(-\cos(x)) - \int \cos(x)(-\cos(x)) = -\mathrm{sen}(x)\cos(x) + \int \cos^2(x)$$

$$\underbrace{\int \mathrm{sen}^2(x)}_{A} = -\mathrm{sen}(x)\cos(x) + \underbrace{\int 1}_{x + C} - \underbrace{\int \mathrm{sen}^2(x)}_{A}$$

$$2A = -\mathrm{sen}(x)\cos(x) + x \to A = \int \mathrm{sen}^2(x) = \frac{x - \mathrm{sen}(x)\cos(x)}{2}$$

1. f - $\int \log(x) = \int 1 \cdot \log(x)$ ($u = \log(x)$, $dv = 1$) $\qquad u = \log \to du = 1/x$

$dv = 1 \to v = x$

$$= x\log(x) - \int \frac{1}{x}\cdot x = x\log(x) - x + C$$

($\frac{1}{x}\cdot x = 1$, $\int 1 = x + C$)

1. g - $\int \cos(2x)\,\mathrm{sen}(3x)$ ($dv = \cos(2x)$, $u = \mathrm{sen}(3x)$)

$u = \mathrm{sen}(3x) \to du = 3\cos(3x)$

$dv = \cos(2x) \to v = \frac{\mathrm{sen}(2x)}{2}$

$$\int \cos(2x)\,\mathrm{sen}(3x) = \frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} - \frac{3}{2}\int \cos(3x)\,\mathrm{sen}(2x)$$

$u = \mathrm{sen}(2x) \to du = 2\cos(2x)$

$dv = \cos(3x) \to v = \frac{\mathrm{sen}(3x)}{3}$

$$\frac{\mathrm{sen}(2x)\,\mathrm{sen}(3x)}{3} - \frac{2}{3}\int \cos(2x)\,\mathrm{sen}(3x)$$

$$\int \cos(2x)\,\mathrm{sen}(3x) = \frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} - \frac{3}{2}\left(\frac{\mathrm{sen}(2x)\,\mathrm{sen}(3x)}{3} - \frac{2}{3}\int \cos(2x)\,\mathrm{sen}(3x)\right)$$

$$\int \cos(2x)\,\mathrm{sen}(3x) = \frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} - \frac{\mathrm{sen}(2x)\,\mathrm{sen}(3x)}{2} + \int \cos(2x)\,\mathrm{sen}(3x)$$

$u = \cos(3x) \to du = -3\,\mathrm{sen}(3x)$

$dv = \mathrm{sen}(2x) \to v = -\frac{\cos(2x)}{2}$

$$-\frac{\cos(3x)\cos(2x)}{2} - \int \frac{3\,\mathrm{sen}(3x)\cos(2x)}{2}$$

$$\int \cos(2x)\,\mathrm{sen}(3x) = \frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} - \frac{3}{2}\left(-\frac{\cos(3x)\cos(2x)}{2} - \frac{3}{2}\int \mathrm{sen}(3x)\cos(2x)\right)$$

$$\underbrace{\int \cos(2x)\,\mathrm{sen}(3x)}_{A} = \frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} + \frac{3}{4}\cos(3x)\cos(2x) + \underbrace{\frac{9}{4}\int \mathrm{sen}(3x)\cos(2x)}_{\frac{9}{4}A}$$

$$A - \frac{9}{4}A = \frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} + \frac{3}{4}\cos(3x)\cos(2x) = -\frac{5}{4}A$$

$$A = \int \cos(2x)\,\mathrm{sen}(3x) = -\frac{4}{5}\left(\frac{\mathrm{sen}(3x)\,\mathrm{sen}(2x)}{2} + \frac{3}{4}\cos(3x)\cos(2x)\right)$$

2. a - $\int e^x\,\mathrm{sen}(e^x)\,dx$ (e^x: $f(x)$; $\mathrm{sen}(e^x)$: $g(f(x))$) $\qquad \int f(x)\,dx = \int u\,du$

$u = f(x)$

$du = f'(x)\,dx$

$u = e^x \to du = e^x\,dx$

$$\int \mathrm{sen}(u)\,du = \int \mathrm{sen}(e^x)\,e^x\,dx$$

($u = e^x$, $du = e^x\,dx$)

$$\int \mathrm{sen}(u)\,du = -\cos(u) = -\cos(e^x) = \int e^x\,\mathrm{sen}(e^x)\,dx$$

2. b - $\int \frac{e^x}{1 + e^{2x}}\,dx$ $\qquad \int \mathrm{arctg}'(x) = \int \frac{1}{1 + x^2} \Rightarrow \mathrm{arctg}(x) = \int \frac{1}{1+x^2}\,dx$

$$\int \frac{e^x\,dx}{1 + (e^x)^2}$$

$u = e^x$

$du = e^x\,dx$

$$\int \frac{1}{1 + u^2}\,du = \mathrm{arctg}(u) = \mathrm{arctg}(e^x)$$

2. d - $\frac{3}{3}\int \frac{x^2}{\sqrt{x^3 - 1}}\,dx$ $\qquad u = x^3 - 1$

$du = 3x^2\,dx$

$$\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{2\sqrt{u}}{3} = \frac{2}{3}\sqrt{x^3 - 1}$$

2. j - $\int \mathrm{tg}(x)\,dx = \int \frac{\mathrm{sen}(x)}{\cos(x)}\,dx \cdot \frac{(-1)}{(-1)}$ $\qquad u = \cos(x)$

$du = -\mathrm{sen}(x)\,dx$

$$\int \mathrm{tg}(x)\,dx = -\int \frac{1}{u}\,du = -\mathrm{Ln}(u) = -\mathrm{Ln}(\cos(x))$$