6. Determinar a,b y/o c para obtener un infinitesimo del mayor orden posible $x \to 0$

decimos que f es un infinitesimo de orden n en a

si $f(x) \to 0$ cuando $x \to a$ y $\lim\limits_{x \to a} \frac{f(x)}{(x-a)^n} = 0$

ⓐ $a(e^x - 1) - bx^2 - x$

$$\lim_{x \to 0} \frac{a(e^x-1) - bx^2 - x}{x^n} = \lim_{x \to 0} \frac{a(\overbrace{1 + x + x^2/2 + x^3/6}^{e^x} - 1) - bx^2 - x}{x^n}$$

$$e^x = e^0 + \frac{e^0 x}{1!} + \frac{e^0 x^2}{2!} + \frac{e^0 x^3}{3!} + \dots = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$\lim_{x \to 0} \frac{ax + \frac{a}{2}x^2 + \frac{a}{6}x^3 - bx^2 - x}{x^n} = \lim_{x \to 0} \frac{\frac{a}{6}x^3 + \left(\frac{a}{2} - b\right)x^2 + (a-1)x}{x^n}$$

$$\lim_{x \to 0} \frac{a}{6}\frac{x^3}{x^n} + \left(\frac{a}{2} - b\right)\frac{x^2}{x^n} + (a-1)\frac{x}{x^n} = \lim_{x \to 0} \frac{a}{6} x^{3-n} + \overbrace{\left(\frac{a}{2} - b\right)}^{=0} \underbrace{x^{2-n}}_{\to 0} + \overbrace{(a-1)}^{=0} \underbrace{x^{1-n}}_{\to 0} = 0$$

$$\left.\begin{aligned} a - 1 = 0 &\Rightarrow \boxed{a = 1} \\ \frac{a}{2} - b = 0 &\Rightarrow \boxed{b = \frac{1}{2}} \end{aligned}\right\} \lim_{x \to 0} \frac{x^{3-n}}{6} = 0 \Rightarrow \boxed{n = 2}$$

$$[tg(x)]' = sec^2(x)$$

$$[tg(x)]'' = 2tg(x)\, sec^2(x)$$

$$[tg(x)]''' = \frac{2 + 4 sen^2(x)}{cos^4(x)}$$

ⓑ $x + a\, sen(x) + b\, tg(x)$ $\qquad \lim\limits_{x \to 0} \frac{x + a\, sen(x) + b\, tg(x)}{x^n}$

$$sen(x) = 0 + x + 0 - \frac{x^3}{3!} \qquad tg(x) = 0 + \overbrace{sec^2(0)}^{=1} x + \underbrace{2tg(0)\, sec^2(0)}_{=0} \frac{x^2}{2!} + \underbrace{\left(\frac{2 + 4 sen^2(0)}{cos^4(0)}\right)}_{2} \frac{x^3}{3!}$$

$$tg(x) = x + \frac{2}{3!}x^3 = x + \frac{x^3}{3}, \quad sen(x) = x - \frac{x^3}{6}$$

$$\lim_{x \to 0} \frac{x + a\left(x - \frac{x^3}{6}\right) + b\left(x + \frac{x^3}{3}\right)}{x^n} = \lim_{x \to 0} \left(\frac{b}{3} - \frac{a}{6}\right) x^{3-n} + (1 + a + b)\, x^{1-n} = 0$$

$$1 + 3b = 0 \Rightarrow \boxed{b = -\frac{1}{3}}$$

$$\frac{b}{3} - \frac{a}{6} = 0 \Rightarrow a = 2b \qquad 1 + a + b = 0 \qquad \boxed{a = -\frac{2}{3}}$$

$$\boxed{n = 3}$$

9. Hallar McLaurin de orden n

ⓐ $\frac{1}{2-x}$

$$\overbrace{(1 + x + x^2 + x^3 + x^4 + \dots)}^{\sum_{k=0}^{\infty} x^k}(1 - x) =$$

$$= 1 + \cancel{x} + \cancel{x^2} + \cancel{x^3} + \dots - \cancel{x} - \cancel{x^2} - \cancel{x^3} - \dots = 1$$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} = 1 + x + x^2 + \dots$$

$$\frac{1}{2} \cdot \frac{1}{1 - \frac{x}{2}} = \frac{1}{2} \cdot \frac{1}{1 - u} = \frac{1}{2} \sum_{i=0}^{\infty} u^i = \boxed{\sum_{i=0}^{\infty} \frac{1}{2}\left(\frac{x}{2}\right)^i} \qquad \frac{x}{2} = u$$

ⓑ $(x^2 + x)\, e^x = \sum\limits_{k=0}^{n} \frac{x^{k+2} + x^{k+1}}{k!}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{k=0}^{n} \frac{x^k}{k!}$$

$$x^2 e^x + x e^x = \underbrace{x^2 \sum_{k=0}^{n} \frac{x^k}{k!}}_{\sum_{k=0}^{n} \frac{x^{k+2}}{k!}} + \underbrace{x \sum_{k=0}^{n} \frac{x^k}{k!}}_{\sum_{k=0}^{n} \frac{x^{k+1}}{k!}} = \sum_{k=0}^{n} \frac{x^{k+2} + x^{k+1}}{k!}$$

Resto de Taylor $\quad R_n(f,a)(x) = f(x) - P_n(f,a)(x)$

$$\lim_{x \to a} \frac{R_n(f,a)(x)}{(x-a)^n} = 0$$

Forma del resto de Lagrange $\quad R_n(f,a)(x) = \frac{f^{(n+1)}(c_x)}{(n+1)!}(x-a)^{n+1}$

$$c_x \in (a, x)$$

1. Dar una aprox. de un numero racional con un error menor que 10^{-4}

ⓐ e $\qquad e^x = \sum\limits_{i=0}^{n} \frac{f^{(i)}(0)}{i!} x^i + R_n(e^x, 0)(x)$

$$e^x - \underbrace{\sum_{i=0}^{n} \frac{f^{(i)}(0)}{i!} x^i}_{P_n(e^x,0)(x)} = R_n(e^x, 0)(x) < 10^{-4}$$

$$e - P_n(e^x, 0)(1) = R_n(e^x, 0)(1) < 10^{-4}$$

$$1 = e^0 = \qquad \frac{f^{(n+1)}(0)}{(n+1)!} 1^{n+1} < 10^{-4}$$

$$\frac{1}{(n+1)!} < 10^{-4} = \frac{1}{10000}$$

$$10\,000 > (n+1)!$$

$$\boxed{n = 7}$$

n	(n+1)!
2	6
3	24
4	120
5	720
6	5040
→ 7	40320