$$P_n(f,a) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

② a_ $f(x) = x^4 - x^3 + 2$, $a=0$, $n=2$

$$P_2(f(x),0) = \sum_{i=0}^{2} \frac{f^{(i)}(0)}{i!} x^i$$

$$\frac{f(0)}{0!}\underbrace{(x-0)^0}_{=1} + \frac{f'(0)}{1!}\underbrace{(x-0)^1}_{x} + \frac{f''(0)}{2!}(x-0)^2 \qquad (0! = 1)$$

$f'(x) = 4x^3 - 3x^2$

$f''(x) = 12x^2 - 6x$

$$f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$f(0) = 2$ $\quad f''(0) = 0$

$f'(0) = 0$

$$\boxed{P_2(f(x),0) = 2}$$

$f(x) = x^4 - x^3 + 2$, $a=1$, $n=2$

$f(1) = 2$

$f'(1) = 1$

$f''(1) = 6$

$$P_2(f(x),1) = 2 + (x-1) + \frac{6}{2}(x-1)^2$$

($\frac{6}{2}(x-1)^2 = 3x^2 - 6x + 3$)

$$\boxed{P_2(f(x),1) = 4 - 5x + 3x^2}$$

① McLaurin de orden 4 asociado a f es $3 - 5x + 4x^2 - x^3 - 2x^4$

Calcular $f(0), f'(0), f''(0), f'''(0)$ y $f''''(0)$

$$P_4(f,0) = 3 - 5x + 4x^2 - x^3 - 2x^4$$

$$P_4(f,0) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \frac{f''''(0)}{24}x^4$$

$$3 - 5x + 4x^2 - x^3 - 2x^4 = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \frac{f''''(0)}{24}x^4$$

$$\begin{cases} 3 = f(0) \\ -5 = f'(0) \\ 4 = \frac{f''(0)}{2} \Rightarrow f''(0) = 8 \end{cases}$$

$$-1 = \frac{f'''(0)}{6} \Rightarrow f'''(0) = -6$$

$$-2 = \frac{f''''(0)}{24} \Rightarrow f''''(0) = -48$$

② b_ $f(x) = \operatorname{sen}(x)$, $a = \pi$, $n = 6$

$f(x) = \operatorname{sen}(x)$	$f(\pi) = 0$
$f'(x) = \cos(x)$	$f'(\pi) = -1$
$f''(x) = -\operatorname{sen}(x)$	$f''(\pi) = 0$
$f'''(x) = -\cos(x)$	$f'''(\pi) = 1$
$f^{(4)}(x) = \operatorname{sen}(x)$	$f^{(4)}(\pi) = 0$
$f^{(5)}(x) = \cos(x)$	$f^{(5)}(\pi) = -1$
$f^{(6)}(x) = -\operatorname{sen}(x)$	$f^{(6)}(\pi) = 0$

$$\boxed{P_6(\operatorname{sen}(x),\pi) = -1(x-\pi) + \frac{1}{6}(x-\pi)^3 - \frac{1}{120}(x-\pi)^5}$$

e_ $f(x) = e^x = f'(x) = f''(x) = \ldots$ $\qquad f(0) = 1 = f'(0) = \ldots$

$a=0$, $n=4$

$$P_4(e^x,0) = 1 + \frac{1}{1!}(x-0)^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4$$

$$\boxed{P_4(e^x,0) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}}$$

$f(x) = e^x$, $a=1$, $n=4$

$f(1) = f'(1) = f''(1) = \ldots = e$

$$P_4(e^x,1) = e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \frac{e}{24}(x-1)^4$$

ⓘ $f(x) = \int_1^{x^2} e^{-t^2}\,dt$ $\qquad a=1 \qquad n=3$

$f'(x) = 2xe^{-x^4}$ $\qquad f'(1) = \frac{2}{e}$ $\qquad f(1) = 0$

$f''(x) = 2e^{-x^4} - 4x^3e^{-x^4}2x$ $\qquad f''(1) = \frac{2}{e} - \frac{8}{e} = -\frac{6}{e}$

$f'''(x) = -8e^{-x^4} - 32x^3e^{-x^4} + 4x^3e^{-x^4}8x^4$ $\qquad f'''(1) = -\frac{8}{e} - \frac{32}{e} + \frac{32}{e} = -\frac{8}{e}$

$$\boxed{P_3(f(x),1) = \frac{2}{e}(x-1) - \frac{3}{e}(x-1)^2 - \frac{4}{3e}(x-1)^3}$$

③ $f(x) = x\cos(x) - \operatorname{sen}(x)$

a_ McLaurin de orden 5 ($a=0$)

$f'(x) = \cos(x) - x\operatorname{sen}(x) - \cos(x) = -x\operatorname{sen}(x)$

$f''(x) = -\operatorname{sen}(x) - x\cos(x)$

$f'''(x) = -\cos(x) - \cos(x) + x\operatorname{sen}(x) = -2\cos(x) + x\operatorname{sen}(x)$

$f^{(4)}(x) = 2\operatorname{sen}(x) + \operatorname{sen}(x) + x\cos(x) = 3\operatorname{sen}(x) + x\cos(x)$

$f^{(5)}(x) = 3\cos(x) + \cos(x) - x\operatorname{sen}(x) = 4\cos(x) - x\operatorname{sen}(x)$

$f(0) = 0 \quad f'(0) = 0 \quad f''(0) = 0$

$f'''(0) = -2 \quad f^{(4)}(0) = 0 \quad f^{(5)}(0) = 4$

$$P_5(f(x),0) = -\frac{2x^3}{3!} + \frac{4x^5}{5!} = -\frac{x^3}{3} + \frac{x^5}{30}$$

b_ ¿f presenta extremo relativo en $x=0$? $\qquad f'(0) = 0$ punto crítico

$f''(0) = 0$ punto de inflexion

∄ extremo relativo en $x=0$

c_ $$\lim_{x\to 0^+} \frac{f(x) + \frac{x^3}{3}}{x^5} \quad \left(f(x) = -\frac{x^3}{3} + \frac{x^5}{30}\right) = \lim_{x\to 0^+} \frac{-\frac{x^3}{3} + \frac{x^5}{30} + \frac{x^3}{3}}{x^5} = \lim_{x\to 0^+} \frac{x^5}{30x^5} = \frac{1}{30}$$

4_ Calcular

a_ $$\lim_{x\to 0} \frac{x - \log(1+x)}{x^2}$$

$$P_2(\log(1+x),0) = x - \frac{1}{2}x^2$$

$$(\log(1+x))' = \frac{1}{1+x}$$

$$(\log(1+x))'' = \frac{-1}{(1+x)^2}$$

$$\lim_{x\to 0} \frac{x - (x - x^2/2)}{x^2} = \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$$

b_ $$\lim_{x\to 0} \frac{\log(1+x) - \operatorname{sen}(x)}{x^2 + 4x^3}$$

$$P_3(\log(1+x),0) = x - \frac{x^2}{2} + \frac{2}{6}x^3$$

$$(\log(1+x))''' = -\frac{2(1+x)(-1)}{((1+x)^2)^2} = \frac{2}{(1+x)^3}$$

$$P_3(\operatorname{sen}(x),0) = 1x - \frac{1}{6}x^3$$

$\operatorname{sen}(0) = 0$

$\cos(0) = 1$

$-\operatorname{sen}(0) = 0$

$-\cos(0) = -1$

$$\frac{x^3}{3} + \frac{x^3}{6} = \frac{2x^3 + x^3}{6} = \frac{3x^3}{6} = \frac{x^3}{2}$$

$$\lim_{x\to 0} \frac{\left(x - \frac{x^2}{2} + \frac{x^3}{3}\right) - \left(x - \frac{x^3}{6}\right)}{x^2 + 4x^3} = \lim_{x\to 0} \frac{-\frac{x^2}{2} + \frac{3x^3}{6}}{x^2 + 4x^3}$$

$$-\frac{x^2}{2} + \frac{x^3}{2}$$

$$\lim_{x\to 0} \frac{-x^2 + x^3}{2(x^2 + 4x^3)} = \lim_{x\to 0} \frac{x^2(-1 + x)}{2x^2(1 + 4x)} = \lim_{x\to 0} \frac{-1 + x}{2(1 + 4x)} = -\frac{1}{2}$$