

$$7.2.3) \text{ e) } \int \frac{1}{x^2+x+2} \quad x = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$$

$$\arctg' = \frac{1}{1+x^2} \Rightarrow \int \arctg' = \int \frac{1}{1+x^2} = \arctg(x)$$

$$(x+a)^2 + b^2 = x^2 + 2ax + a^2 + b^2 = x^2 + x + 2$$

$$\begin{cases} 2a=1 \Rightarrow a=1/2 \\ a^2+b^2=2 \rightarrow b^2=2-(1/2)^2=2-1/4=7/4 \end{cases}$$

$$\int \frac{1 dx}{x^2+x+2} = \int \frac{1 dx}{(x+1/2)^2 + 7/4}$$

$$\int \frac{1 dx}{\frac{7}{4} \left(\frac{4}{7} (x+1/2)^2 + 1 \right)} = \frac{2}{\sqrt{7}} \int \frac{1 du}{u^2+1}$$

$\frac{4}{7} (x+1/2)^2 = u^2 \rightarrow u = \sqrt{\frac{4}{7} (x+1/2)^2} = \frac{2}{\sqrt{7}} (x+1/2)$

$$u = \frac{2}{\sqrt{7}} x + \frac{2}{\sqrt{7}} \cdot \frac{1}{2} \rightarrow du = \frac{2}{\sqrt{7}} dx$$

$$\frac{2}{\sqrt{7}} \int \frac{1 du}{u^2+1} = \frac{2}{\sqrt{7}} \arctg(u) = \frac{2}{\sqrt{7}} \arctg\left(\frac{2}{\sqrt{7}} (x+1/2)\right) = \int \frac{1 dx}{x^2+x+2}$$

arctg(u)

$$\text{f) } \int \frac{4x-3}{3x^2+3x+2} dx = \int \frac{2/3(6x+3) - 5}{3x^2+3x+2} dx$$

$du = 6x+3 dx$

$$\frac{2}{3} \int \frac{6x+3}{3x^2+3x+2} dx - \frac{5}{3} \int \frac{1}{x^2+x+2/3} dx$$

$$\frac{2}{3} \int \frac{du}{u} = \frac{2}{3} \ln(3x^2+3x+2)$$

$$x^2+x+\frac{2}{3} = (a+x)^2 + b^2 = a^2 + 2ax + x^2 + b^2$$

$$\begin{cases} 1 = 2a \Rightarrow a=1/2 \\ \frac{2}{3} = a^2 + b^2 \Rightarrow \frac{2}{3} - \frac{1}{4} = b^2 \Rightarrow b = \sqrt{5/12} \end{cases}$$

$$\frac{5}{3} \int \frac{dx}{x^2+x+2/3} = \frac{5}{3} \int \frac{1 dx}{(x+1/2)^2 + 5/12} = \frac{5}{3} \int \frac{1 dx}{\frac{12}{5} \left[\frac{12}{5} (x+1/2)^2 + 1 \right]}$$

$$\frac{12}{5} (x+1/2)^2 = u^2 \Rightarrow u = \sqrt{\frac{12}{5}} (x+1/2) \quad du = \sqrt{\frac{12}{5}}$$

$$\frac{5}{3} \int \frac{\sqrt{12/5} \sqrt{12/5} dx}{\frac{12}{5} (x+1/2)^2 + 1} = \frac{5}{3} \cdot \sqrt{\frac{12}{5}} \int \frac{1}{u^2+1} = \frac{\sqrt{12} \sqrt{5}}{3} \arctg\left(\sqrt{\frac{12}{5}} (x+1/2)\right)$$

$$\int \frac{4x-3}{3x^2+3x+2} = \frac{2}{3} \ln(3x^2+3x+2) + \frac{\sqrt{12} \cdot \sqrt{5}}{3} \arctg\left(\sqrt{\frac{12}{5}} (x+1/2)\right)$$

$$7.2.2) \text{ e) } - \int \frac{(-1)e^x dx}{\sqrt{1-e^x}} \quad \begin{matrix} u = f(x) \\ du = f'(x) dx \end{matrix}$$

$$du = -e^x dx \quad - \int \frac{du}{\sqrt{u}} = -2\sqrt{u} = -2\sqrt{1-e^x}$$

$$\text{g) } \int x\sqrt{1-x^2} dx \quad \begin{matrix} 1-x^2 = u \\ du = -2x dx \end{matrix} \quad \frac{1}{-2} \int -2x dx \sqrt{1-x^2}$$

$$-\frac{1}{2} \int \sqrt{u} du = -\frac{1}{2} \int u^{1/2} du = -\frac{1}{2} \frac{u^{3/2}}{3/2} = -\frac{1}{3} \sqrt{(1-x^2)^3}$$