Derivacion $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Capitulo 6

6.1 Calculos elementales

① a_ $f(x) = x^2 \to f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} 2x + h = 2x$

b_ $f(x) = x^3$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$(x^3)' = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3xh^2 + 3x^2h + h^3 - x^3}{h} = \lim_{h \to 0} 3xh + 3x^2 + h^2 = 3x^2$

c_ $f(x) = \sqrt{x}$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$

$f'(x) = \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})} = \frac{1}{2\sqrt{x}} = (\sqrt{x})'$

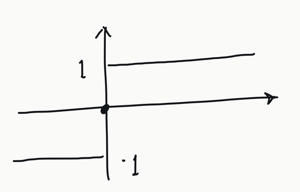
d_ $f(x) = 1/x$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

$f'(x) = \lim_{h \to 0} \frac{x - (x+h)}{h(x+h)x} = \lim_{h \to 0} \frac{-h}{h(x+h)x} = \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{x^2}$

② a_ $f(x) = \text{Signo}(x) \begin{cases} 1 & \text{si } x > 0 \\ 0 & \text{si } x = 0 \\ -1 & \text{si } x < 0 \end{cases}$

f es derivable ⇒ f es continua

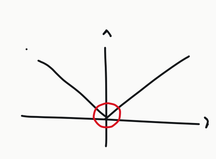
f no es cont. ⇒ f no es derivable



$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$

$\left.\begin{array}{l} \lim_{x \to 0^+} \text{sing}(x) = 1 \\ \lim_{x \to 0^-} \text{sign}(x) = -1 \end{array}\right\}$ no es cont. en 0 ⇒ no es derivable

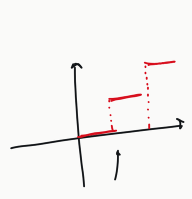
b_ $f(x) = |x| \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{si } x < 0 \end{cases}$



$\lim_{x \to 0^+} |x| = 0 = \lim_{x \to 0^-} |x|$ es cont.

$\left.\begin{array}{l} \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = 1 \\ \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^-} \frac{-(x+h) - (-x)}{h} = \lim_{h \to 0} \frac{-x-h+x}{h} = -1 \end{array}\right\}$ $\nexists \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ no es derivable

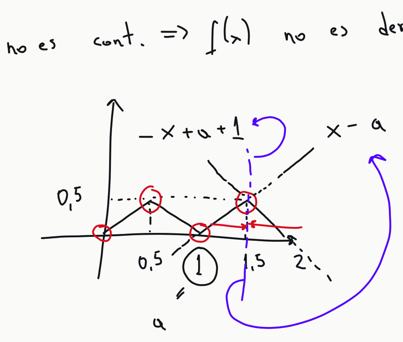
c_ $f(x) = \lfloor x \rfloor$



tiene discontinuidades (saltos) en los enteros

⇒ $\forall a \in \mathbb{Z}$ $f(x)$ no es cont. ⇒ $f(x)$ no es derivable

d_ $f(x) = [x]$



$\forall a \in \mathbb{Z}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\left.\begin{array}{l} \lim_{h \to 0^+} \frac{\overbrace{-x-h+a+1}^{f(x+h)} \overbrace{+x-a-1}^{-f(x)}}{h} = -1 \\ \lim_{h \to 0^-} \frac{\overbrace{x+h-a}^{f(x+h)} \overbrace{-x+a}^{-f(x)}}{h} = 1 \end{array}\right\}$ no es derivable

⑨ a_ $f(x) = \begin{cases} ax + b & \text{si } x \leq 1 \\ 1/x & \text{si } x > 1 \end{cases}$

$\left.\begin{array}{l} \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^+} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - x - h}{h(x+h)x} = \frac{-1}{x^2} \\ \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^-} \frac{a(x+h) + b - ax - b}{h} = a \end{array}\right\}$ $a = \left.\frac{-1}{x^2}\right|_{x=1}$ $\boxed{a = -1}$

$\left.\begin{array}{l} \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} = 1 \\ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} ax + b = a + b \end{array}\right\}$ $1 = a + b \to \boxed{b = 2}$

b_ $f(x) = \begin{cases} 2x + 2 & \text{si } x \leq a \\ 2x^2 & \text{si } x > a \end{cases}$

$\left.\begin{array}{l} \lim_{x \to a^+} 2x^2 = 2a^2 \\ \lim_{x \to a^-} 2x + 2 = 2a + 2 \end{array}\right\}$ $a^2 - a - 1 = 0 \to a = \frac{1 \pm \sqrt{5}}{2}$

$\left.\begin{array}{l} \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^+} \frac{2(x+h)^2 - 2x^2}{h} = \lim_{h \to 0^+} \frac{4xh + h^2}{h} = 4x \\ \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^-} \frac{2(x+h) + 2 - 2x - 2}{h} = 2 \end{array}\right\}$ $2 \neq 4a$ $\forall a \begin{cases} \frac{1+\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{cases}$ no es derivable

6.2 Recta tangente

$y = m(x - x_0) + y_0 \longrightarrow m = f'(x_0)$

④ a_ $f(x) = x^2$, $p = (3, 9)$ $\left.\right\}$ $x_0 = 3$, $y_0 = 9$ $\quad f'(x) = 2x$, $m = f'(3) = 6$

$\Rightarrow y = 6(x - 3) + 9 \quad (f(x) = 6x - 9)$

b_ $f(x) = \cos(x)$, $p = (\frac{\pi}{2}, 0)$ $\left.\right\}$ $x_0 = \frac{\pi}{2}$ $\quad f'(x) = -\text{sen}(x)$, $m = f'(\pi/2) = -1$

$\Rightarrow y = -(x - \frac{\pi}{2}) = \frac{\pi}{2} - x$

d_ $f(x) = \sqrt{x}$, $p = (4, 2)$ $\left.\right\}$ $m = f'(4) = 1/4$, $f'(x) = 1/2\sqrt{x}$ $\quad \Rightarrow y = \frac{1}{4}(x - 4) + 2$