6_ Demostrar que los valores de las siguientes expresiones no dependen de x

ⓐ $\underbrace{\int_0^x \frac{1}{1+t^2}\,dt}_{g(x)} + \underbrace{\int_0^{1/x = z(x)} \frac{1}{1+t^2}\,dt}_{h(z(x))} = f(x)$

$g'(x) = \frac{1}{1+x^2}$ $\quad h'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{(-1)}{x^2} = \frac{x^2}{x^2+1} \cdot \frac{(-1)}{x^2} = \frac{-1}{1+x^2}$

$1 + \frac{1}{x^2} = \frac{x^2+1}{x^2}$

$f'(x) = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0 \gg f(x)$ no depende de x

ⓑ $\int_{-\cos(x)}^{\operatorname{sen}(x)} \frac{1}{\sqrt{1-t^2}}\,dt = f(x) = \int_0^{\operatorname{sen}(x)} \frac{1}{\sqrt{1-t^2}}\,dt - \int_0^{-\cos(x)} \frac{1}{\sqrt{1-t^2}}\,dt$

$f'(x) = \frac{\cos(x)}{\sqrt{1-\operatorname{sen}^2(x)}} - \frac{\operatorname{sen}(x)}{\sqrt{1-\cos^2(x)}}$ $\quad 1 = \operatorname{sen}^2(x) + \cos^2(x)$

$\cos^2(x)$ $\quad \operatorname{sen}^2(x)$

$f'(x) = \frac{\cos(x)}{\cos(x)} - \frac{\operatorname{sen}(x)}{\operatorname{sen}(x)} = 0 = f'(x) \Rightarrow f(x)$ no depende de x

Calcular $(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))}$

ⓐ $f(x) = \int_0^x 1 + \operatorname{sen}(\operatorname{sen}(t))\,dt = 0$

$f'(x) = 1 + \operatorname{sen}(\operatorname{sen}(x))$ $\quad f^{-1}(0) = x_0 \;/\; f(x_0) = 0$

$f^{-1}(0) = 0$

$(f^{-1})'(0) = \frac{1}{f'(0)} = \frac{1}{1+\operatorname{sen}(\operatorname{sen}(0))} = 1$

ⓒ $f(x) = \int_0^{e^x \ln(x)} \sqrt{t^2 + e^t}\,dt = 0 \gg e^x \ln(x) = 0 \Rightarrow x = 1 = f^{-1}(0)$

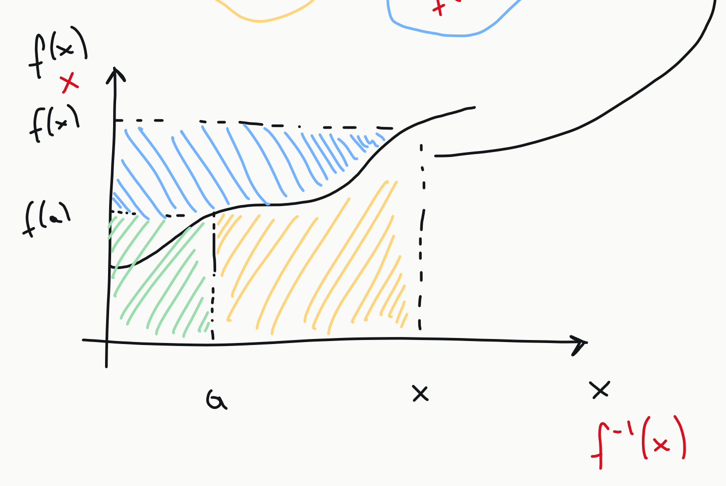
$f'(x) = \left[e^x \ln(x) + \frac{e^x}{x}\right] \sqrt{(e^x \ln(x))^2 + e^{e^x \ln(x)}}$

$(f^{-1})'(0) = \frac{1}{f'(1)} = \frac{1}{\left[e^1\ln(1) + \frac{e^1}{1}\right]\sqrt{(e^1\ln(1))^2 + e^{e^1\ln(1)}}} = \frac{1}{e\sqrt{1}} = \frac{1}{e}$

($e^1\ln(1) = 0$, $\frac{e^1}{1} = e$, $(e^1 \ln(1))^2 = 0$, $e^{e^1\ln(1)} = 1$)

9_ (Examen feb. 2017)

Probar que $\int_a^x f(t)\,dt + \int_{f(a)}^{f(x)} f^{-1}(t)\,dt - x f(x) + a f(a) = 0 = g(x)$




$\underbrace{\int_a^a f(t)\,dt}_{=0} + \underbrace{\int_{f(a)}^{f(a)} f^{-1}(t)\,dt}_{=0} - \underbrace{a f(a) + a f(a)}_{=} = 0$

la función no depende de x evaluo en x = a

$g'(x) = f(x) + f'(x)\underbrace{f^{-1}(f(x))}_{x} - \underbrace{(f(x) + f'(x)x)}_{(x f(x))'} = 0$

Calcular $\int_1^2 \ln(t)\,dt = \int_{1}^{2} f(t)\,dt = -\int_{f(1)=0}^{f(2)=\ln(2)} e^t\,dt + 2 f(2) - 1 f(1)$

$= \ln(2)$ $\quad = \ln(1) = 0$

$\int_1^2 \ln(t)\,dt = -\int_0^{\ln(2)} e^t\,dt + 2\ln|2| = 2\ln(2) - 1$ $\quad \int_a^b f(t)\,dt = F(b) - F(a)$

$= -1$

$\int_0^{\ln(2)} e^t\,dt = e^t \Big|_0^{\ln(2)} = e^{\ln(2)} - e^0 = 2 - 1 = 1$

14_ ⓐ $\int_0^2 x^4 - x^5 + 3\,dx = \left(\frac{x^5}{5} - \frac{x^6}{6} + 3x\right)\Big|_0^2$

$= \frac{2^5}{5} - \frac{2^2}{6} + 6$

ⓓ $\int_1^4 3\sqrt{x} + \sqrt[5]{x}\,dx = 3\int_1^4 x^{1/2} + \int_1^4 x^{1/5}$

$2x^{3/2}\Big|_1^4 + \frac{5x^{6/5}}{6}\Big|_1^4$ $\quad 3\frac{x^{1/2+1}}{1/2+1} = 3/2$ $\quad \frac{x^{6/5}}{6/5}$

$2x^{3/2}$ $\quad \frac{5x^{6/5}}{6}$

$\frac{e^{2x}}{2}$

ⓖ $\int_1^3 e^{2x}\,dx = \frac{e^{2x}}{2}\Big|_1^3 = \frac{e^6}{2} - \frac{e^2}{2}$

$2e^{2x}$

ⓕ $\int_1^3 \frac{dx}{\sqrt{4x+3}} = \frac{\sqrt{4x+3}}{2}\Big|_1^3 = \frac{\sqrt{15}}{2} - \frac{\sqrt{7}}{2}$ $\quad \left(= \frac{1}{\sqrt{4x+3}}\,dx\right)$

$\left(\frac{\sqrt{4x+3}}{2}\right)' = \frac{4}{2 \cdot 2\sqrt{4x+3}} = \frac{2}{\sqrt{4x+3}\,2} = \frac{1}{\sqrt{4x+3}}$