7.1 Teo. Fundamental

1_ Derivar sin calcular la integral

$$\int_0^x f(t)dt = F(x) \rightarrow F'(x) = f(x)$$

a_ $f(x) = \int_1^x \underbrace{\sqrt{t^2-t+1}}_{h(t)}\, dt = H(x)$

$f'(x) = H'(x) = h(x) = \sqrt{x^2-x+1}$

b_ $f(x) = \int_x^3 \sqrt{t^2-t+1}\, dt$

$\int_3^x \sqrt{t^2-t+1}\, dt = -f(x)$

$\sqrt{x^2-x+1} = -f'(x)$

$\Rightarrow f'(x) = -\sqrt{x^2-x+1}$

$$\int_0^{g(x)} f(t)dt = F(g(x)) \rightarrow F'(g(x))g'(x) = f(g(x))g'(x)$$

c_ $f(x) = \int_{\cos(x)}^3 \sqrt{t^2-t+1}\, dt$

$-f(x) = \int_3^{\cos(x)} \sqrt{t^2-t+1}\, dt$ ($\cos(x) = g(x)$) $\qquad g'(x) = -\operatorname{sen}(x)$

$-f'(x) = \sqrt{\cos^2(x) - \cos(x) + 1}\,(-\operatorname{sen}(x))$

$f'(x) = \operatorname{sen}(x)\sqrt{\cos^2(x) - \cos(x) + 1}$

d_ $f(x) = \int_{x^2}^2 \frac{t^7}{1+t^4}\, dt = H(g(x))$ ($x^2 = g(x)$, $\frac{t^7}{1+t^4} = h(t)$)

$f'(x) = H'(g(x))g'(x) = h(g(x))g'(x)$

$-f(x) = \int_2^{x^2} \frac{t^7}{1+t^4}\, dt \rightarrow -f'(x) = \frac{(x^2)^7}{1+(x^2)^4} \cdot 2x$

$f'(x) = -\frac{2x^{15}}{1+x^8}$

f. $f(x) = \int_{\cos(x)}^{\operatorname{sen}(x)} \frac{t^7}{1+t^4}\, dt = \int_{\cos(x)}^0 \frac{t^7}{1+t^4}\, dt + \int_0^{\operatorname{sen}(x)} \frac{t^7}{1+t^4}\, dt$

$f(x) = \int_0^{\operatorname{sen}(x)} \frac{t^7}{1+t^4}\, dt - \int_0^{\cos(x)} \frac{t^7}{1+t^4}\, dt$

$f'(x) = \frac{\operatorname{sen}^7(x)}{1+\operatorname{sen}^4(x)} \cdot \cos(x) - \frac{\cos^7(x)}{1+\cos^4(x)}(-\operatorname{sen}(x))$

$f'(x) = \frac{\operatorname{sen}^7(x)\cos(x)}{1+\operatorname{sen}^4(x)} + \frac{\cos^7(x)\operatorname{sen}(x)}{1+\cos^4(x)}$

2_ $\int_0^1 \sqrt{t^2-t+1}\, dt = \frac{1}{8}(4+\ln(27))$

Derivar en $x=1$ $\qquad$ a_ $f(x) = \int_0^{x^2} \sqrt{t^2-t+1} + x\, dt$

$\int_0^{x^2} \sqrt{t^2-t+1}\, dt + \int_0^{x^2} x\, dt = f(x)$ ($x^2 = g(x)$) $\qquad \int_0^{x^2} x\, dt = x^3$

TFC $\qquad x\int_0^{x^2} dt = x.(x^2-0) = x^3$ $\qquad \partial/\partial x$

$f'(x) = \sqrt{x^4-x^2+1} . \underbrace{2x}_{g'(x)} + 3x^2$

$f'(1) = \sqrt{1-1+1} . 2 + 3 = 5$

c_ $f(x) = \int_x^0 x^2\sqrt{t^2-t+1}\, dt = \underbrace{-x^2}_{g(x)}\underbrace{\int_0^x \sqrt{t^2-t+1}\, dt}_{h(x)}$

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$g'(x) = -2x \qquad h'(x) = \sqrt{x^2-x+1}$

$f'(x) = -2x\int_0^x \sqrt{t^2-t+1}\, dt - x^2\sqrt{x^2-x+1}$

$f'(1) = -2\underbrace{\int_0^1 \sqrt{t^2-t+1}\, dt}_{\frac{1}{8}(4+\ln(27))} - 1\sqrt{1-1+1}$

$f'(1) = -2.\frac{1}{8}(4+\ln(27)) - 1$

4_ Det. $f: R \rightarrow R$ y $c \in R$

a_ $\int_0^x f(t)dt = c - e^{-x^2}$ $\qquad$ evaluar en $x=0$

$\int_0^0 f(t)dt = c - e^{-0^2}$

$0 = c - 1 \Rightarrow \boxed{c=1}$

derivo $\left[\int_0^x f(t)dt\right]' = \left[c - e^{-x^2}\right]'$

$\boxed{f(x) = -2x(-e^{-x^2}) = 2xe^{-x^2}}$

c_ $\int_x^1 f(t)dt = \sqrt{x^4+1} + c$ $\qquad$ evaluo en $x=1$

$\int_1^1 f(t)dt = \sqrt{2} + c = 0 \Rightarrow \boxed{c = -\sqrt{2}}$

$-\int_1^x f(t)dt = \sqrt{x^4+1} + c$ $\qquad$ derivo

$-f(x) = \frac{4x^3}{2\sqrt{x^4+1}} \Rightarrow \boxed{f(x) = \frac{-2x^3}{\sqrt{x^4+1}}}$